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Quantum Mechanics

Quantum mechanics began in 1900 when the study of light emitted by heating solids was studied, so we begin by discussing ***the nature of light***.

- In 1801Thomas Young gave convincing experimental evidence for the wave nature of light by showing that light exhibited diffraction and interference when passed through two adjacent pinholes.
- In 1860 Maxwell developed Maxwell's equations predicted that an accelerated electric charge would radiated energy in the form of electromagnetic waves.
- light is a type of energy and has wave properties

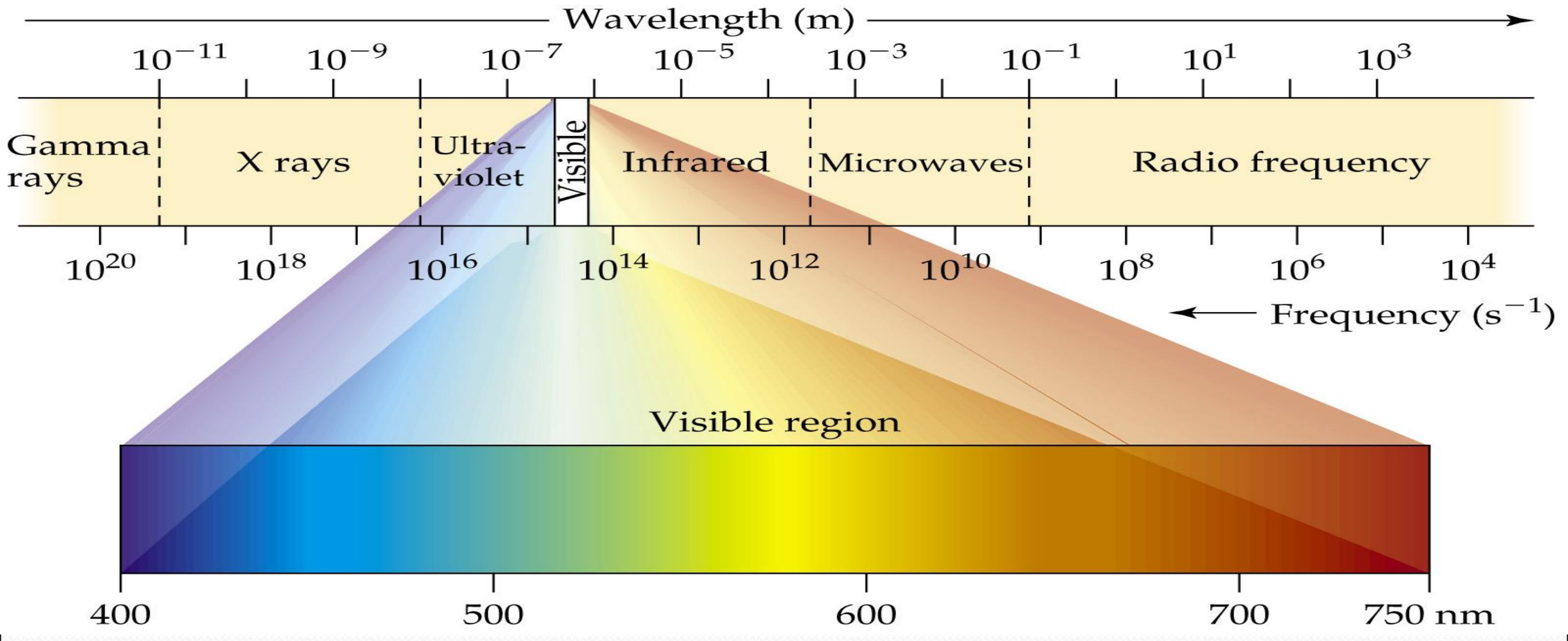
- Electromagnetic (EM) radiation travels through space as electric energy and magnetic energy.
- Wavelength λ and frequency ν are related by:

$$\lambda \text{ x } \nu = c = 3.00 \text{ x } 10^{8} \text{ m/s}$$

- Frequency (or wavelength) determines the type of radiation All electromagnetic waves travel at speed c= 3.00×10^{10} cm/sec in vacuum.
- As a wave, we can describe the energy by its wavelength, which is the distance from the crest of one wave to the crest of the next wave.
- The wavelength of electromagnetic radiation can range from miles (radio waves) to inches (microwaves in a microwave oven) to millionths of an inch (the light we see) to billionths of an inch (x-rays).

- The wavelength of light is more commonly stated in nanometers (nm). One nanometer is one billionth of a meter.
- Visible light has wavelengths of roughly 400 nm to roughly 700 nm. This range of wavelengths is called the visible spectrum.

The Wave Nature of Light

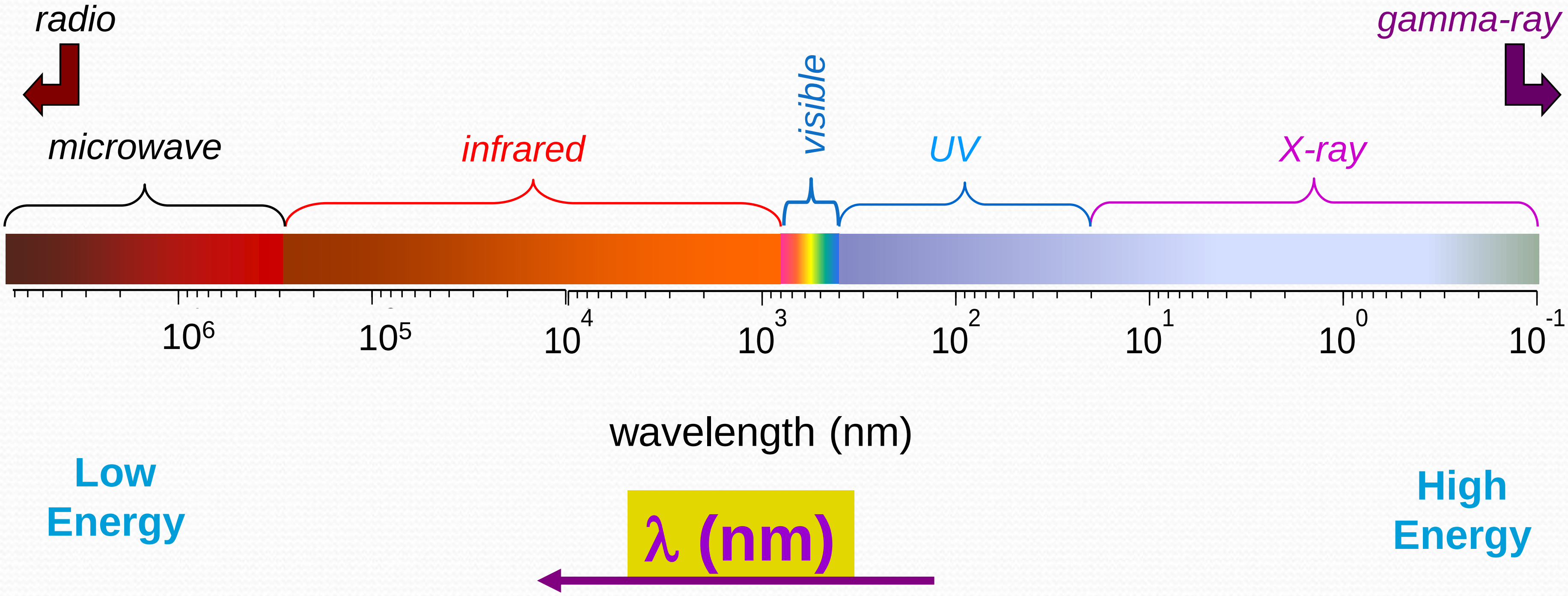


$$c = \lambda \cdot \nu$$

$$E = h \cdot \nu$$

The speed of light is constant

The wavelength λ of Electromagnetic Spectrum

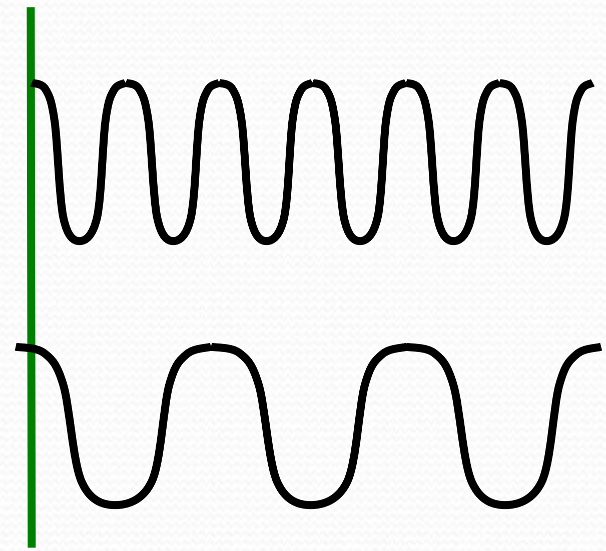


The speed of EM waves

- Observe what happens when a radio wave and a visible wave move through space (at same speed of c)

Visual 6 Hz

Radio 3 Hz



- The longer the wavelength, the smaller the frequency has to be to keep c constant

The speed of EM waves

Q1 - Which of the following has the higher frequency

1. visible light or UV (choose one)
2. X-rays or radio waves (choose one)

Q2- Which of the following pairs has the longer wavelength:

1. Infrared or Ultraviolet (choose one)
2. Gamma rays or Radio waves (choose one)

Energy and Matter

Size of Matter	Particle Property	Wave Property
Large – macroscopic	Mainly	Unobservable
Intermediate – electron	Some	Some
Small – photon	Few	Mainly

For matter $E = m c^2$

For waves $E = h \nu$

Exercises:

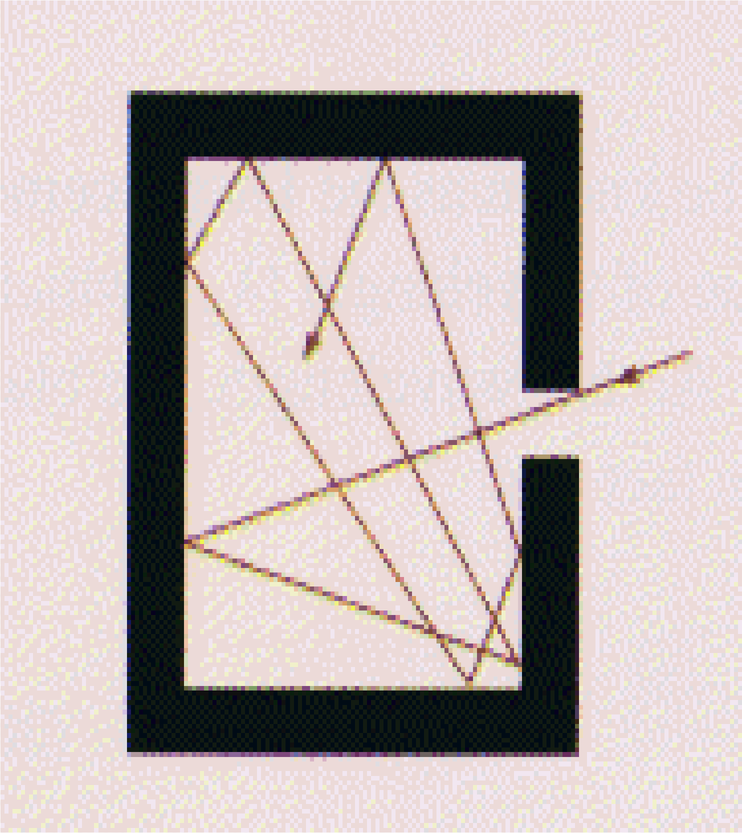
1. Photons have a wavelength of 500 nm. (The symbol nm is defined as a nanometer = 10^{10} m) What is the energy of this photon, and what is the type of the electromagnetic wave in this case?

Black body radiation

- A black body is a theoretical object that absorbs 100% of the radiation that hits it. Therefore it reflects no radiation and appears perfectly black.
- Roughly we can say that the stars radiate like blackbody radiators. This is important because it means that we can use the theory for blackbody radiators to infer things about stars.
- At a particular temperature the black body would emit the maximum amount of energy possible for that temperature.
- Blackbody radiation does not depend on the type of object emitting it. Entire spectrum of blackbody radiation depends on only one parameter, the temperature, T.

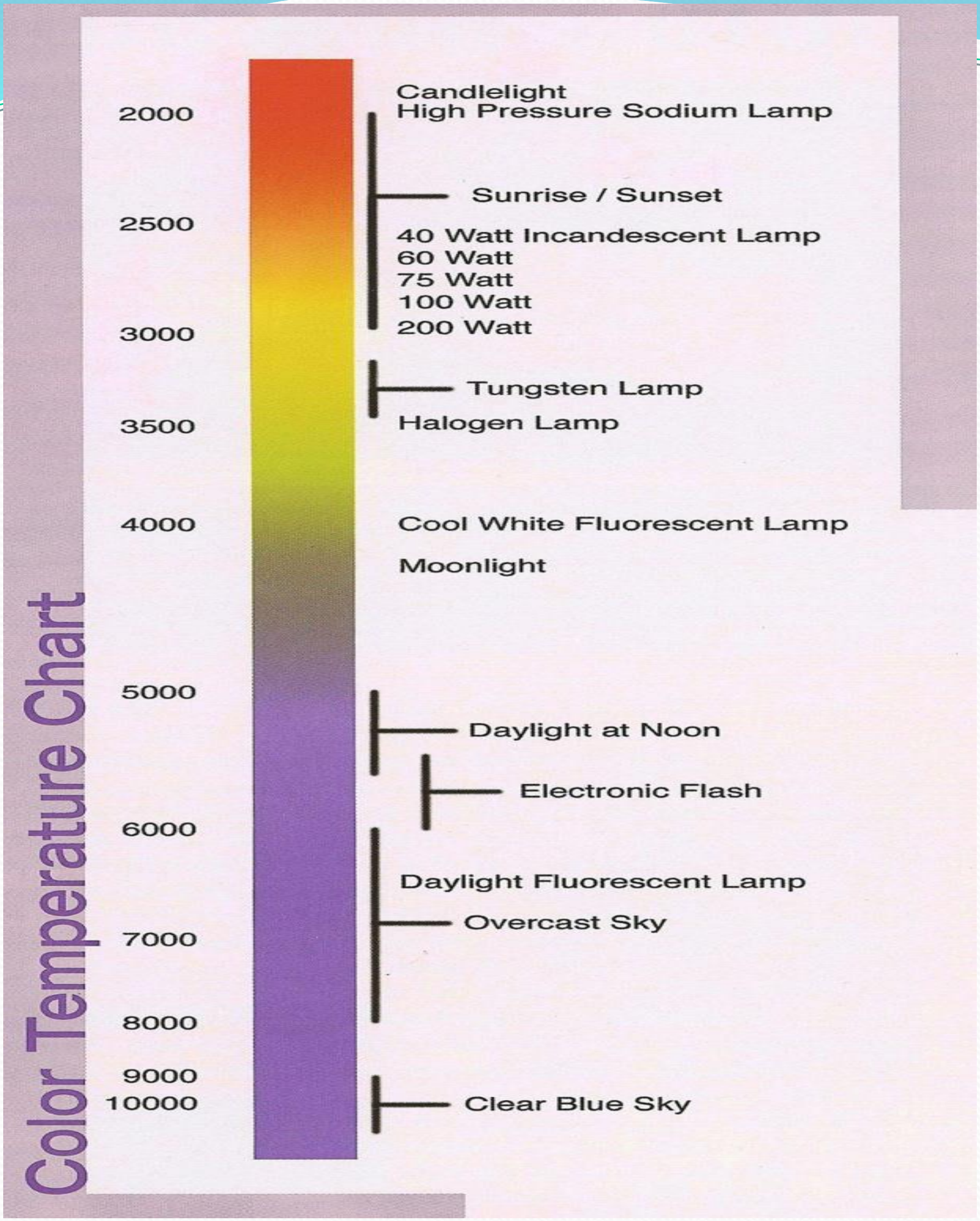
Definition of a black body

A black body is an ideal body which allows the whole of the incident radiation to pass into itself (without reflecting the energy) and absorbs within itself this whole incident radiation (without passing on the energy). This propety is valid for radiation corresponding to all wavelengths and to all angels of incidence. Therefore, the black body is an ideal absorber of incident radaition.



The relationships between temperature, wavelength, and energy emitted by an ideal thermal radiator (blackbody).

1. Based on everyday observations, the bodies at different temperatures emit radiation (heat energy) of different wavelengths or colors. For example, the wires in a heater begin to glow red when heated then the color of the wire will changed by increasing the temperature.
2. Blackbody radiation is the theoretical maximum radiation expected for temperature-related thermal self-radiation.
3. This radiation can have a peak energy distribution in the infrared, visible, or ultraviolet region of the electromagnetic spectrum.
4. The hotter the emitter, the more energy emitted and the shorter the wavelength. An object at room temperature has its peak radiation in the infrared while the sun has its peak in the visible region.
5. The equations for calculating radiation based on temperature use the Kelvin temperature scale. (Be sure to use the Kelvin scale for all calculations).



Color temperature

A light spectrum of Blackbodies is often characterized in terms of its temperature even if it's not exactly a blackbody.

Thermal radiation spectral radiance

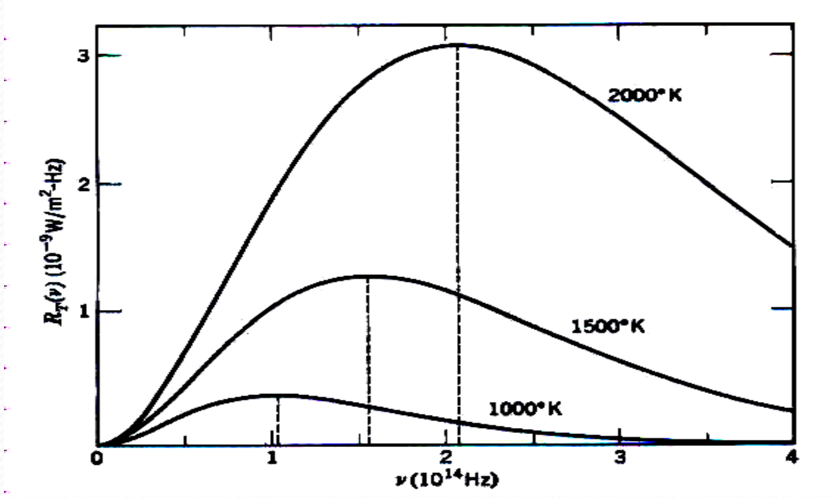
thermal radiation

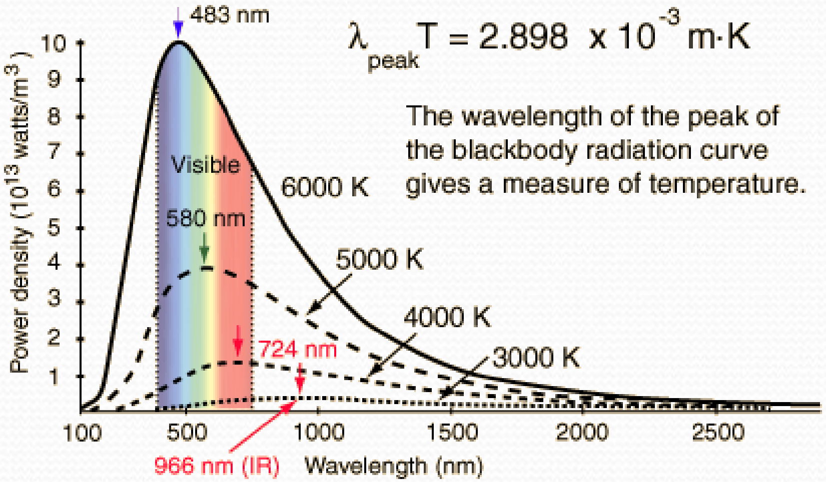
● **Thermal radiation: The radiation emitted by a body as a result of temperature.**

● **Blackbody : A body that its surface absorbs and emit all the thermal radiation incident on them.**

● **Spectral radiance: The spectral distribution of blackbody radiation.**

$R_T(\nu)d\nu$: $R_T(\nu)$ **represents the emitted energy from a unit area per unit time between** ν **and** $\nu+d\nu$ **at absolute temperature T.**

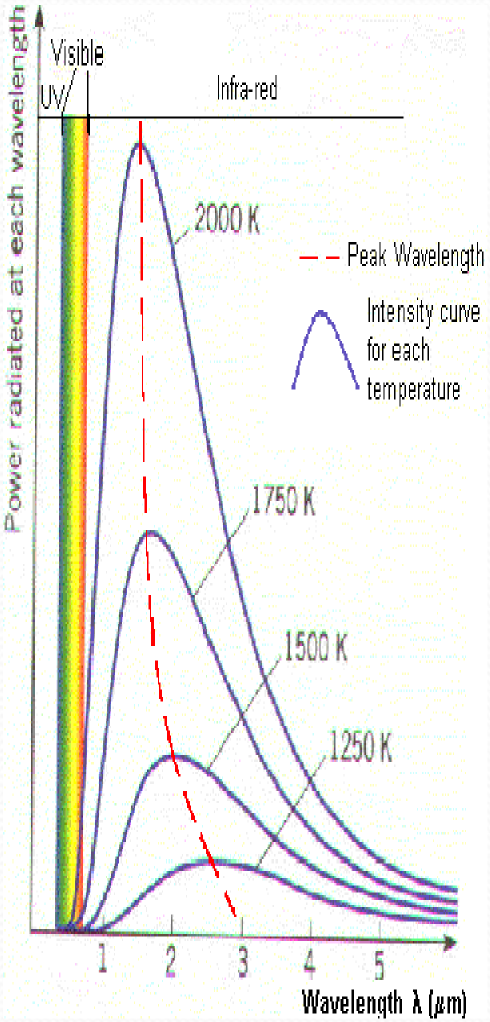




The spectral radiance of blackbody radiation shows that:

1. The higher the temperature, the more the emission and the shorter the average wavelength.
2. The power radiation is most intense at certain wavelength λ_{max} or υ_{max} for particular temperature.
3. λ_{max} increases linearly with increasing temperature.

→ →



Now it is very important to find mathematical equations shows the following relationships:

1. Inverse proportionality between λ_{max} and T.

Classical quantum mechanics of the black body radiation:

The power radiated per unit surface area of the radiator is given by the Stefan-Boltzmann law.

The Stefan-Boltzmann Law

* The amount of energy radiated is proportional to the temperature of the object raised to the fourth power.

➡ The Stefan Boltzmann equation

$$\mathbf{F = \sigma T^4}$$

F = flux of energy (W/m^2)

T = temperature (K)

$\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$ (Stefan-Boltzmann constant)

This law gives the total energy being emitted at all wavelengths by the blackbody (which is the area under the Planck Law curve).

* Explains the growth in the height of the curve as the temperature increases. Notice that this growth is very abrupt.

Classical quantum mechanics of the black body radiation:

Basic Laws of Radiation

1) All objects emit radiant energy.

2) Hotter objects emit more energy than colder objects (per unit area). The amount of energy radiated is proportional to the temperature of the object.

3) The hotter the object, the shorter the wavelength (λ) of emitted energy.

➡This is Wien's Law

$$\lambda_{max} \cong \frac{3000\ \mu m}{T(K)}$$

Classical quantum mechanics of the black body radiation:

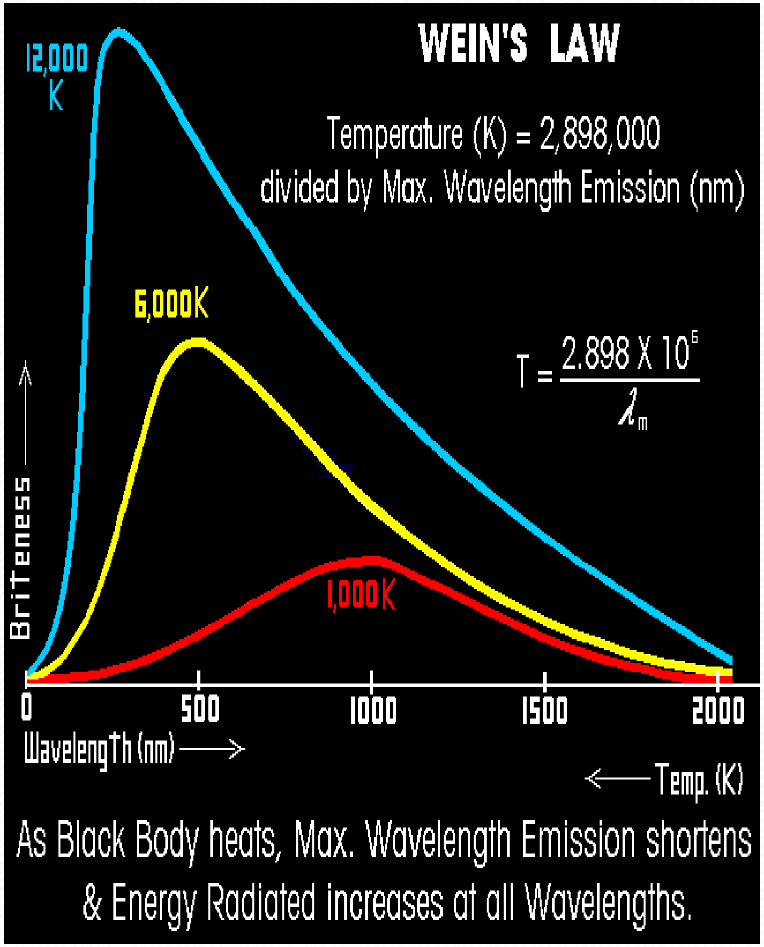
$$\lambda_{max} = \frac{b}{T}$$

Wein Displacement Law

- It tells us as we heat an object up, its color changes from red to orange to white hot.
- You can use this to calculate the temperature of stars.

The surface temperature of the Sun is 5778 K, this temperature corresponds to a peak emission = 502 nm = about 5000 Å.

- b is a constant of proportionality, called Wien's displacement constant and equals 2.897 768 × 10^{-3} m K = 2.897768 × 10^{6} nm K.



Classical quantum mechanics of the black body radiation:

➡ Stefan Boltzmann Law.

$$F = \sigma T^4$$

F = flux of energy (W/m^2)
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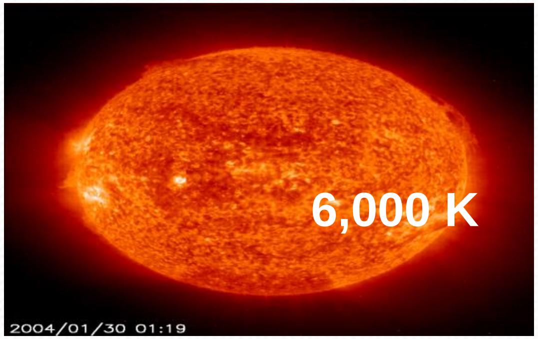
➡ Wien's Law

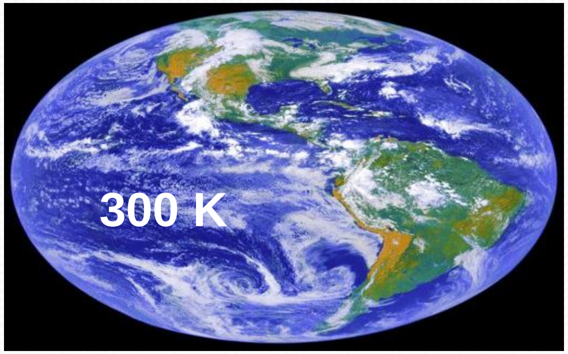
$$\lambda_{max} \cong \frac{3000\ \mu m}{T(K)}$$

λ_{peak} vs Temperature

T	$\lambda_{peak} = \dfrac{2.9 \times 10^{-3}\ m}{T(Kelvin)}$	
310^0K (body temp)	$\dfrac{2.9 \times 10^{-3}\ m}{310^0}$	$=9 \times 10^{-6}$m infrared light
5800^0K (Sun's surface)	$\dfrac{2.9 \times 10^{-3}\ m}{5800^0}$	visible light $=0.5 \times 10^{-6}$m

We can use these equations to calculate properties of energy radiating from the Sun and the Earth.



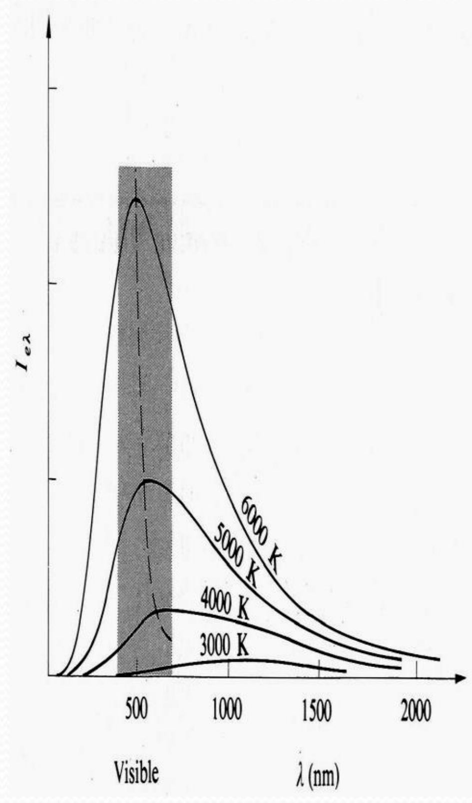


	T (K)	λ_{max} (μm)	region in spectrum	F (W/m^2)
Sun	6000	0.5	Visible (green)	7×10^7
Earth	300	10	infrared	460

Classical quantum mechanics of the black body radiation:

Wien's Law

- To find a relation describe the relation between the intensity of black body radiation versus wavelengths for several temperatures as given in the following figure. Many theoretical physicists tried to derive expression but they were all unsuccessful. One expression that is derived according to the laws of nineteenth century is ***Wien Law***



$$E_\lambda \, d\lambda = \frac{C_1}{\lambda^5} \, e^{-C2/\lambda T} \, d\lambda$$

$C_2 = h\,C/\,k$, $C_1 = 8\pi\, h\, C$

And it was found that for small wavelengths (high frequencies), the previous equation fitted the experimental data but it deviate at large wavelengths infrared waves.

Classical quantum mechanics of the black body radiation:

The Rayleigh-Jeans Law

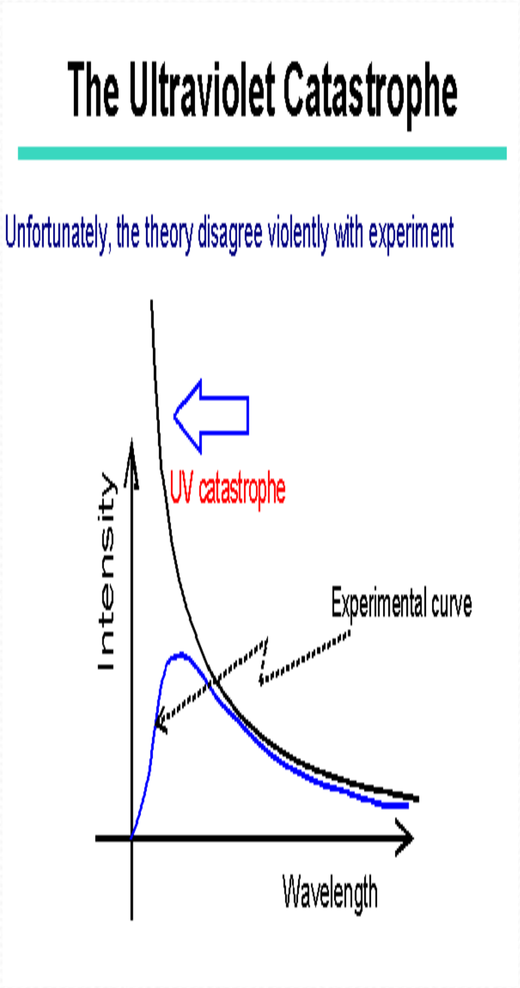
- The second atempt was by Rayleigh and Jeans how use some assumptions to obtain the following equation:

$$E_\lambda \, d\lambda = \frac{C_1}{C_2} \frac{T}{\lambda^4} \, d\lambda$$

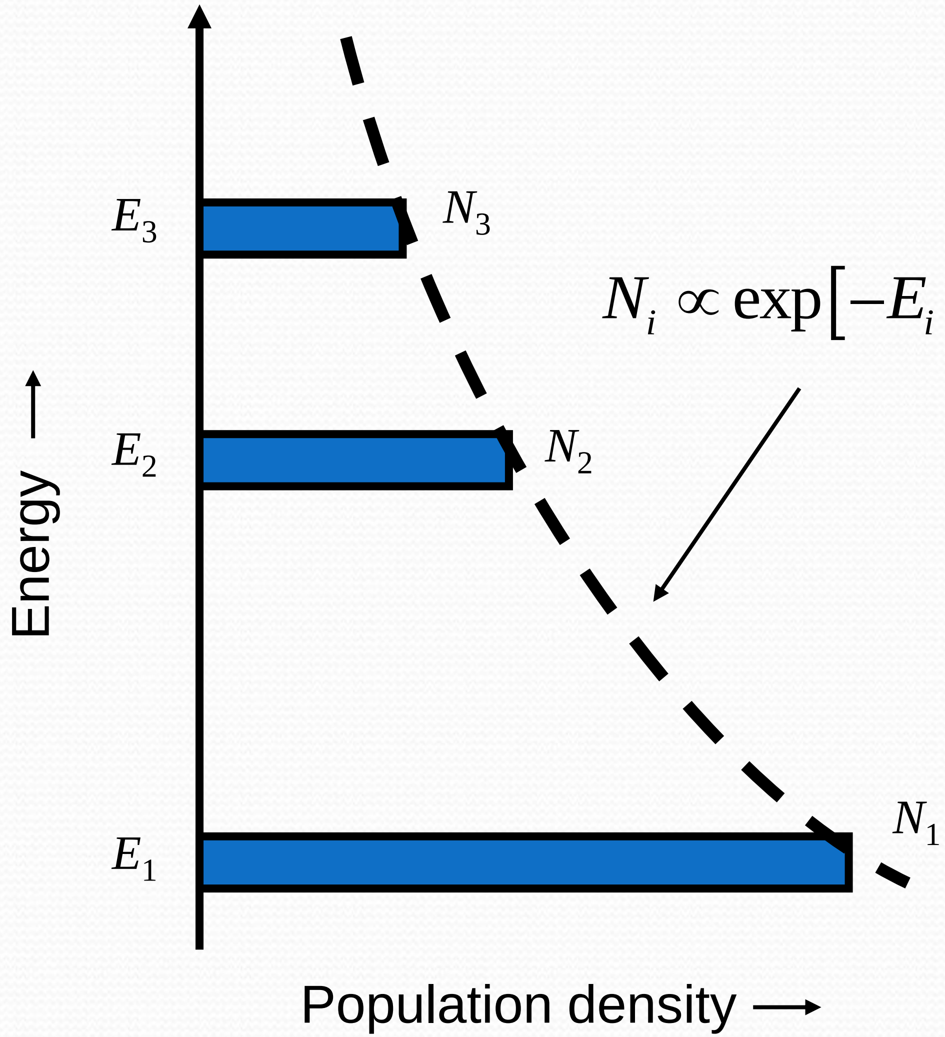
- This equation agrees with experimental measurements for long wavelengths (low frequencies) and failure at short wavelengths.

* It predicts an energy output that diverges towards infinity as wavelengths grow smaller.

* The failure at short wavelengths has become known as the ***ultraviolet catastrophe***.



Boltzmann Population Factors



$$N_i \propto \exp\left[-E_i / k_B T\right]$$

N_i is the number density of molecules in state i (i.e., the number of molecules per cm^3).

T is the temperature, and k_B is Boltzmann's constant.

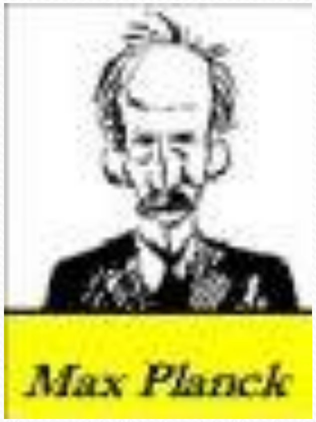
Quantum mechanics of the black body radiation

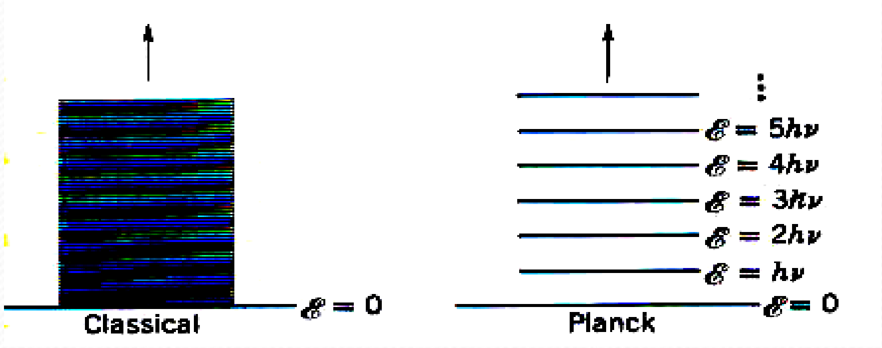
Planck's Postulate and its implication

Planck's postulate: **Planck assumed that the radiation emitted by the body was due to the oscillations of the electrons in the constituent particles of the material body.**

(i.e., simple harmonic oscillation can posses only total energy

$$\varepsilon = nh\nu \qquad n = 1, 2, 3 \ldots$$





<u>*Quantum mechanics of the black body radiation*</u>

Planck's solution

EM energy cannot be radiated or absorbed in any arbitrary amounts, but only in discrete "quantum" amounts.

The energy of a "quantum" depends on frequency as

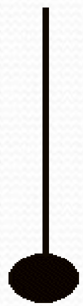
$$E_{quantum} = h\,\upsilon$$

$h = 6.6 \times 10^{-34}$ Js

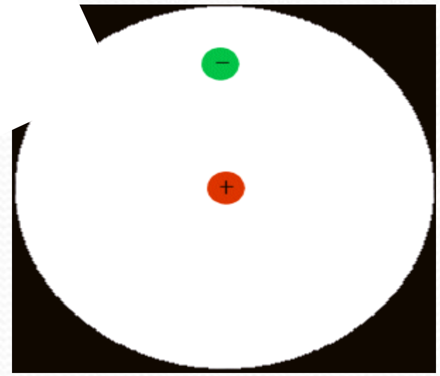
"Planck's constant"

Planck's quantum is small for "ordinary-sized" objects but large for atoms etc

"ordinary" pendulum
$\upsilon = 1$ Hz



Hydrogen atom
$\upsilon \approx 2 \times 10^{14}$ Hz



about the same as the electron's KE

$E_{quant} = h\upsilon$

$= (6.6 \times 10^{-34} Js) \times (2 \times 10^{14} Hz)$

$= (6.6 \times 2) \times 10^{-34+14} J$

$= 1.3 \times 10^{-19} J$

$E_{quant} = h\upsilon \quad = 6.6 \times 10^{-34} Js \times 1 Hz$

$= 6.6 \times 10^{-34} J$ ← very tiny

Classical vs ***Quantum*** world

In everyday life, quantum effects can be safely ignored

This is because Planck's constant is so small

At atomic & subatomic scales, quantum effects are dominant & must be considered

Laws of nature developed without consideration of quantum effects do not work for atoms

The Planck Function

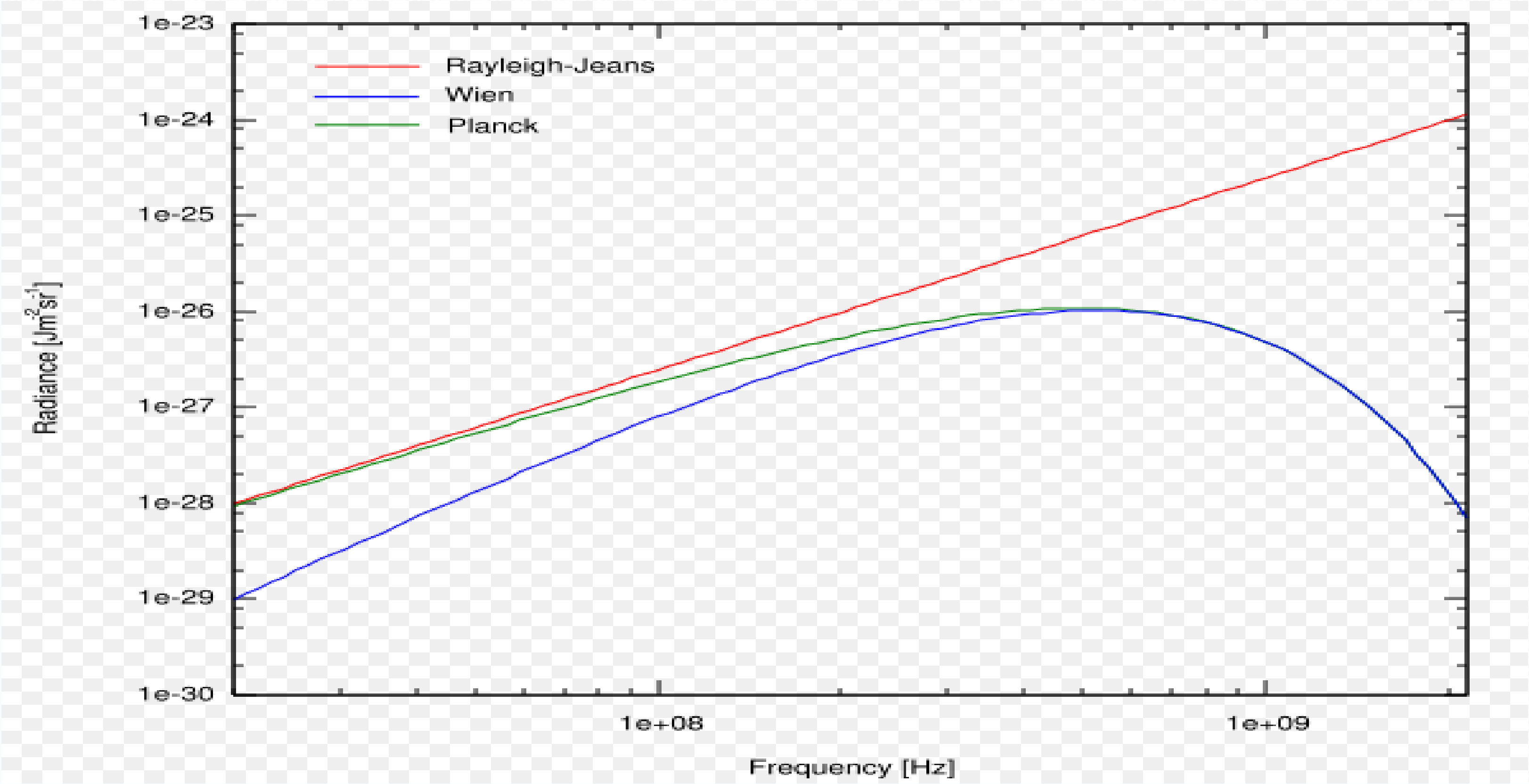
$$E_\lambda \, d\lambda = \frac{c_1}{\lambda^5} \frac{d\lambda}{e^{c_2/\lambda T} - 1}$$

$C_2 = h\,C/\,k$, $C_1 = 8\pi\, h\, C$

$h = 6.6262 \times 10^{-34}$ joule sec
$k = 1.3806 \times 10^{-23}$ joule deg^{-1}
$c = 2.99793 \times 10^{+8}$ m/s
T = object temperature in Kelvins

- **Blackbody radiation follows the Planck function at all the wavelength range**

Black-Body Radiation Laws



Comparison of Rayleigh-Jeans law with Wien's law and Planck's law, for a body of 8 mK temperature.

- Planck equation was applicable with practical results at short wavelengths and long wavelengths. It is important to note that:
- *when λ is small*, Planck equation can be reduced to Wien equation by eliminating (-1) compared with the high value of the exponential in the denominator in Planck equation,
- Thus $$E_\lambda d\lambda = \frac{C_1}{\lambda^5} \frac{d\lambda}{e^{C_2/\lambda T} - 1} \longrightarrow E_\lambda d\lambda = \frac{C_1}{\lambda^5} e^{-C2/\lambda T} d\lambda$$
- *While at large λ,* Planck equation can be devolve into Rayleigh-Jeans equation when $e^{C2/\lambda T}$ compensated by the amount $(1 + C_2/\lambda T)$
- Thus $$E_\lambda d\lambda = \frac{C_1}{\lambda^5} \frac{d\lambda}{e^{C_2/\lambda T} - 1} \longrightarrow E_\lambda d\lambda = \frac{C_1}{C_2} \frac{T}{\lambda^4} d\lambda$$