Definition of finite Automaton

A finite automaton, or finite state machine (FA) is a five tupple $(Q, \Sigma, q_0, A, \delta)$ where

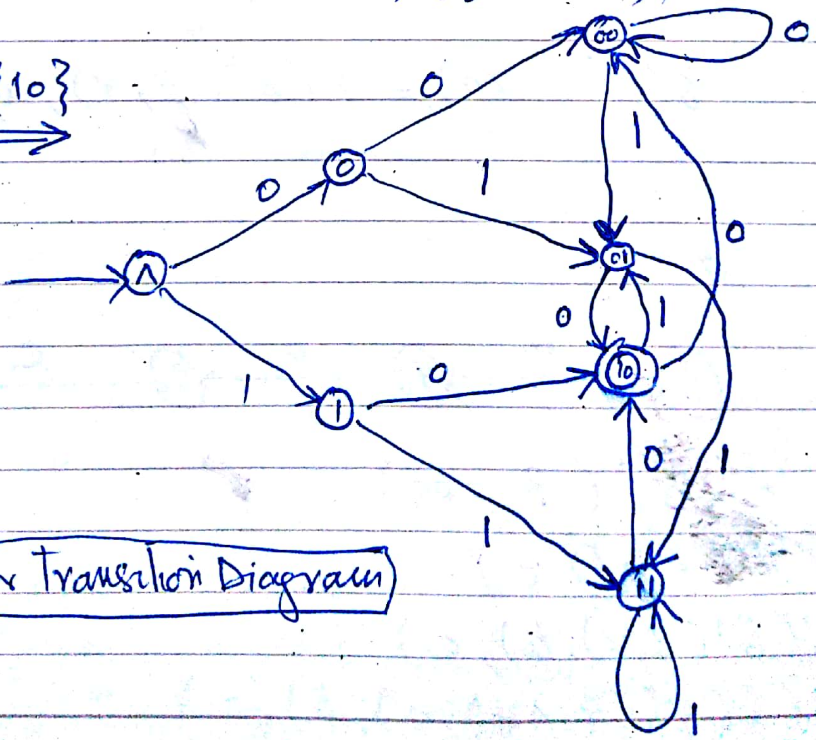
- Q is finite set of states
- Σ is finite alphabet of input symbols
- $q_0 \in Q$ (the initial state)
- $A \subseteq Q$ (the set of accepting states)
- δ is a function from $Q \times \Sigma$ to Q (the transition function)

for any element q of Q and any symbol $a \in \Sigma$, we interpret $\delta(q, a)$ as the state to which the FA moves, if it is in state q and receives the input a.

Then Mathematically we can say That

$$M = (Q, \Sigma, q_0, A, \delta) \text{ be an FA}$$

for $\{0,1\}^*\{10\}$

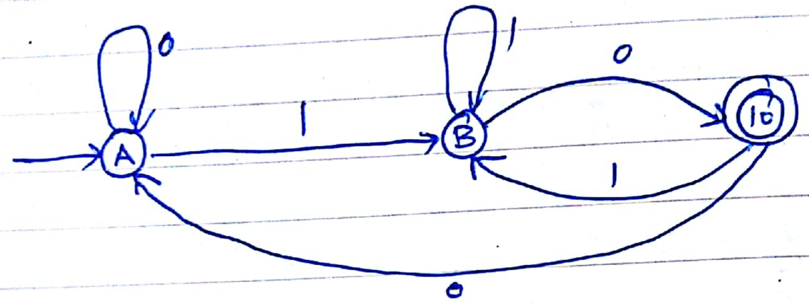


FA or Transition Diagram

states	input 0	input 1
Λ	0	1
0	00	01
1	10	11
00	00	01
01	10	11
10	00	01
11	10	11

Transition Table

– Simplified or reduced form



Recursive defn

0 → <u>Defn</u>: <u>The extended Transition function δ^*</u>

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an FA, we define the function

$$\delta^* : Q \times \Sigma^* \to Q$$

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as follows

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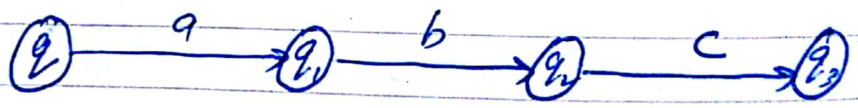
1. for any $q \in Q$, $\delta^*(q, \Lambda) = q$
2. for any $q \in Q$, $y \in \Sigma^*$, and $a \in \Sigma$

$$\delta^*(q, ya) = \delta(\delta^*(q, y), a)$$

<u>Example</u>

Let

Fig 3.4



$$\delta^*(q, abc)$$

$$= \delta(\delta^*(q, ab), c)$$

$$= \delta(\delta(\delta^*(q, a), b), c)$$

$$= \delta(\delta(\delta(\delta^*(q, \Lambda), a), b), c)$$

$$= \delta(\delta(\delta(q, a), b), c)$$

$$= \delta(\delta(q_1, b), c)$$

$$= \delta(q_2, c)$$

$$= q_3 \checkmark$$

Now extended Concept for x and y string which Contains alphabets

$$\delta^*(q, xy) = \delta^*(\delta^*(q, x), y)$$

→ <u>Definition 3.4 : Acceptance by an FA</u>

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an FA. A string $x \in \Sigma^*$ is accepted by M if $\underbrace{\boxed{\delta^*(q_0, x)}}_{\text{state comes}} \in A$. If a string is not accepted, we say it is rejected by M. The language accepted by M, or the language recognized by M, is the set

$$L(M) = \{x \in \Sigma^* \mid x \text{ is accepted by } M\}$$

If L is any language over Σ, L is accepted or Recognized by M if and only if $L = L(M)$

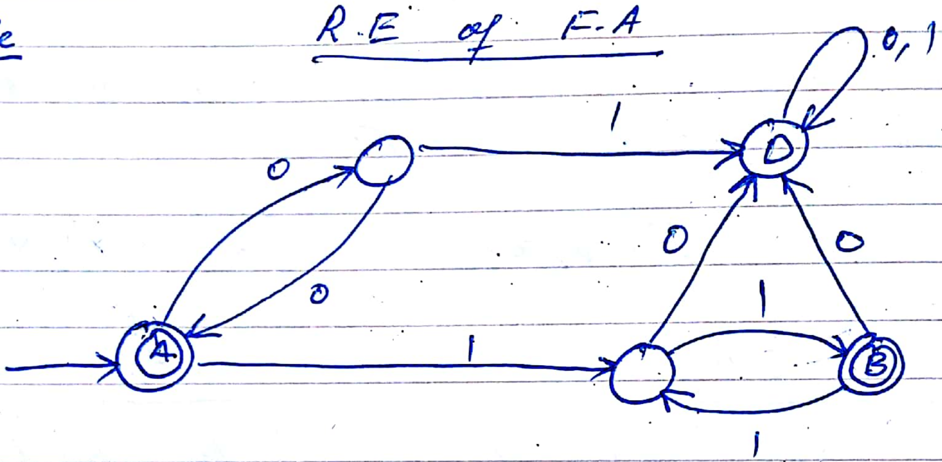
★ "The Power of machine does not lie in the number of strings it accepts" but <u>in accepting some and rejecting others.</u>

Theorem

A language L over the alphabet Σ is regular iff there is an FA with input alphabet Σ that accepts L.

Example

R.E of F.A



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$R.E = (00)^*(11)^*$

Example

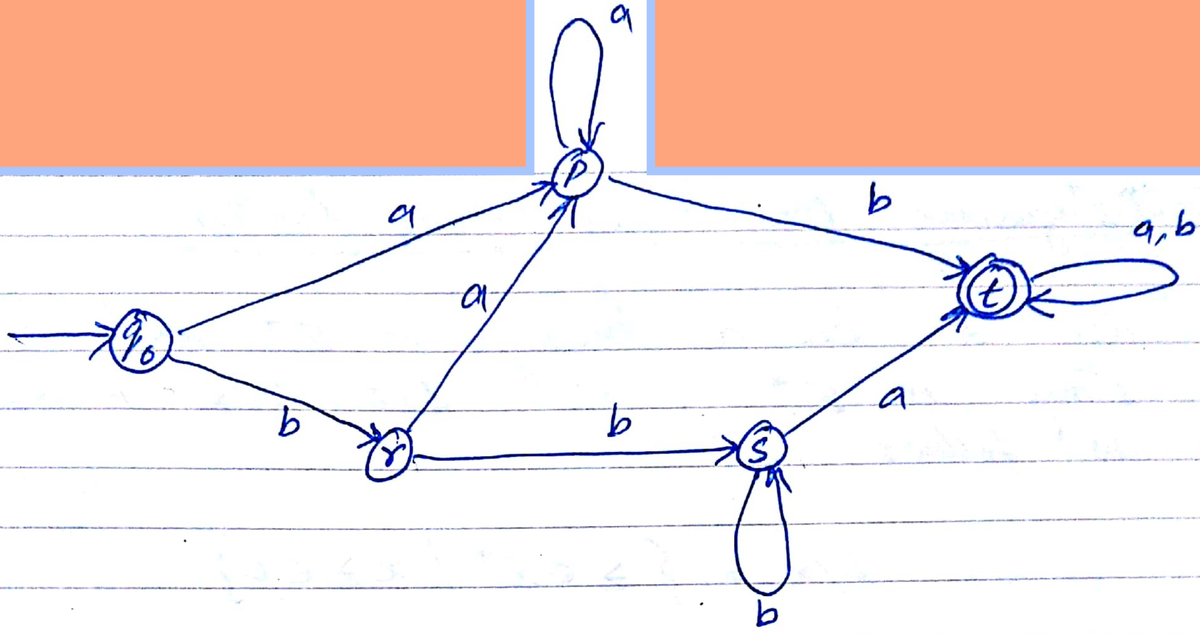
A finite Automaton M accepting $\{a,b\}^*\{baaa\}$ can be seen in book.

Example

R.E to F.A

Language L of All strings in $(a,b)^*$ That contain at least one of two substring ab and bba

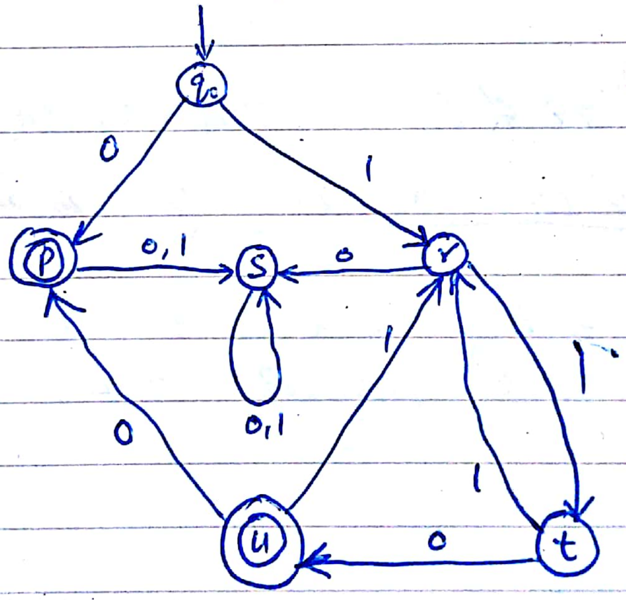
oR $(a+b)^*(ab+bba)(a+b)^*$.



<u>**Example**</u> $r = (11+110)^*0$

$\Sigma = \{0,1\}$

(i) – $0 \in L$ & $1 \notin L$, $10 \notin L$, $110 \in L$, $1110 \notin L$



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Distinguishing One String from Another

Let L be a language in Σ^* and x any string in Σ^*. The set L/x is defined as follows

$$L/x = \{ z \in \Sigma^* \mid xz \in L \}$$

Two strings x and y are said to be distinguishable with respect to L if $L/x \neq L/y$. Any string z That is in one of two sets but not the other (i.e. for which $xz \in L$

and $yz \notin L$, or vice versa) is said to

distinguish x and y with respect to L.

If $L/x = L/y$, x and y are indistinguishable with respect to L

Example

the language L of all strings in $(0,1)^*$ That end in 10.

strings $\overset{x}{00}$ and $\overset{y}{01}$ are distinguishable with respect to L because if $z = 0$, Then $\underbrace{00}_{x}\underbrace{0}_{z} \notin L$ and $\underbrace{01}_{y}\underbrace{0}_{z} \in L$

which gives

$$xz \notin L \neq yz \in L$$

$$L/x \neq L/y$$

Similarly

if two strings are $\underset{x}{0}$ and $\underset{y}{00}$

then L/x and L/y are equal and indistinguishable (Now in this case the property of ending in '10' depends on 'z' string)

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<u>Lemma</u>

Suppose $L \subseteq \Sigma^*$ and $M = (Q, \Sigma, q_0, A, \delta)$ is an FA recognizing L. If x and y are two strings in Σ^* that are distinguishable with respect to L, then

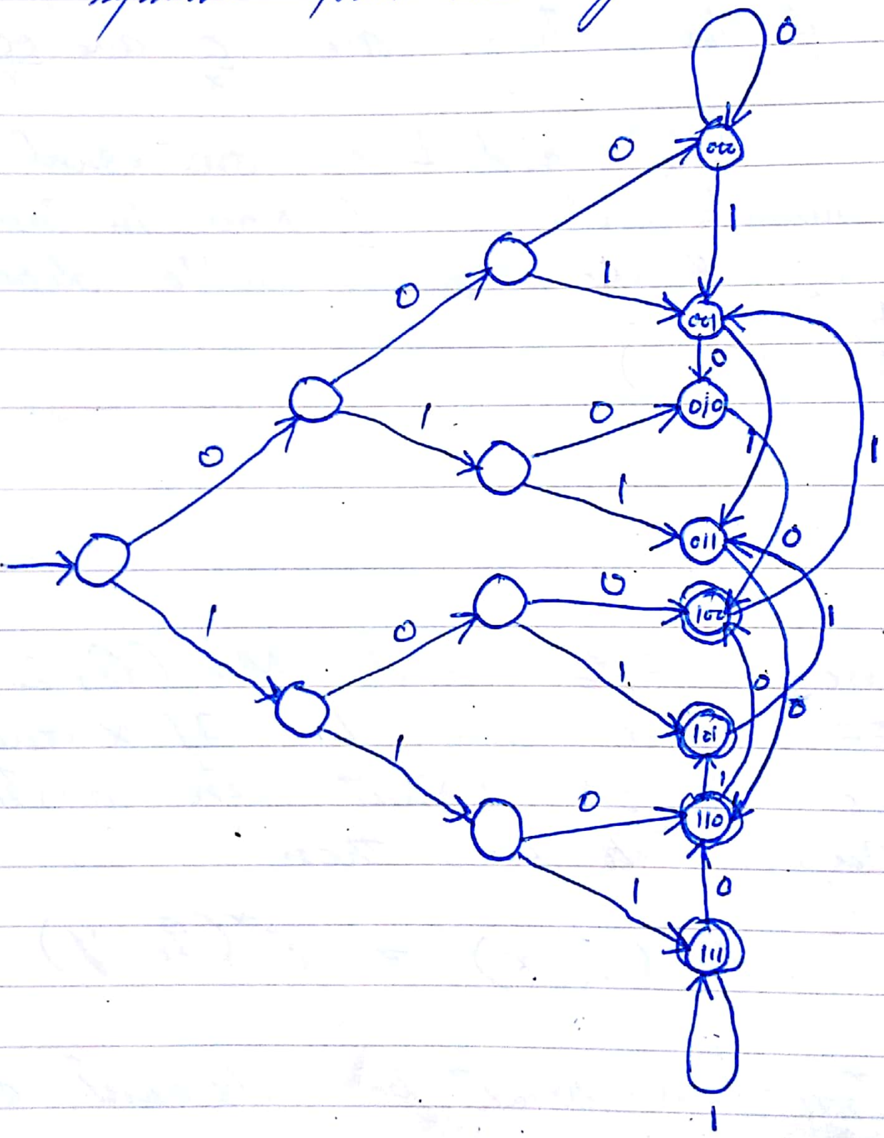
$$\delta^*(q_0, x) \neq \delta^*(q_0, y)$$

(their transitions must be different on FA)

FA Example of language L_n

Suppose $n \geq 1$ and let

$$L_n = \{x \in (0,1)^* \mid |x| \geq n \text{ and the } n\text{th symbol from the right in } x \text{ is } 1\}$$



<u>Theorem</u> the language pal of pallindromes over The alphabet $\{0,1\}$ Can not be accepted by any FA and is Therefore not regular