

## Introduction to Languages & Theory of Computation

### ⇒ Mathematical induction & Recursive defs

- The principle of M.I
- The Strong principle of M.I
- Recursive definitions of functions with Domain  $\mathbb{N}$
- Recursive definitions of Sets
- Structural Induction
- Read chap-2 of John C. Martin for Review

### ⇒ Regular Languages and Regular Expressions

- The Set  $R$  of regular languages over  $\Sigma$ , and the corresponding regular expressions, are defined as follows.

- FRIDAY 1  
13 محرم
- SUNDAY 3 SATURDAY 2  
14 محرم 15 محرم
- (i)  $\emptyset$  is an element of  $R$  and the corresponding R.E is  $\emptyset$
  - (ii)  $\{\Delta\}$  is an element of  $R$  and the R.E is  $\Delta$
  - (iii) for each  $a \in \Sigma$ ,  $\{a\}$  is an element of  $R$ , and the corresponding R.E is  $a$ .
  - (iv) If  $L_1$  and  $L_2$  are any elements of  $R$ , and  $\delta_1$  and  $\delta_2$  are corresponding R.Es
    - (a)  $L_1 \cup L_2 \in R$  and R.E =  $(\delta_1 + \delta_2)$
    - (b)  $L_1 L_2 \in R$  and its R.E =  $(\delta_1 \delta_2)$
    - (c)  $L_1^* \in R$  and its R.E =  $\delta_1^*$

Hence Languages that can be obtained using (i) - (iv) are regular languages.

→ Simplified rules for languages to R.E  
 → Discard curly brackets  $\{ \}$  or replace them with parenthesis  
 → Replace  $\cup$  by  $+$   
 Some examples are as below.

Language	R.E
$\{ \Lambda \}$	$\Lambda$
$\{ 0 \}$	$0$
$\{ 001 \}$ (i.e. $\{ 0 \} \{ 0 \} \{ 1 \}$ )	$001$
$\{ 0, 1 \}$ (i.e. $\{ 0 \} \cup \{ 1 \}$ )	$0+1$
$\{ 1, \Lambda \} \{ 001 \}$	$(1+\Lambda)001$
$\{ 110 \}^* \{ 0, 1 \}$	$(110)^* (0+1)$
$\{ 0, 10 \}^* (\{ 11 \}^* \cup \{ 001, \Lambda \})$	$(0+10)^* ((11)^* + (001+\Lambda))$

4 MONDAY  
 ١٢ محرم

5 TUESDAY  
 ١٣ محرم

$1^*10$  is an R.E for strings that consist of substring 10 preceded by any number of 1's

Similarly

$$L^2 = LL$$

$$\gamma^2 = \gamma\gamma$$

$$\gamma^+ = ((\gamma^*)\gamma)$$

Evening

→ Precedence (Kleene\*, Concatenation, +)  
So be careful in parenthesizing e.g.

$$a + b^*c = (a + ((b^*)c))$$

→ Are the following R.Es equal or not over  $\{0,1\}$

$$1^*(1 + \Lambda) = 1^*$$

$$1^*1^* = 1^*$$

$$(0^*1^*)^* = (0+1)^*$$

$$\begin{aligned} \text{L.H.S} &= (0^*1^*)^* \Rightarrow ((\Lambda, 0, 00, 000, \dots)(\Lambda, 1, 11, 111, \dots))^* \\ &\Rightarrow ((\Lambda, 1, 11, 111, \dots), (0, 01, 011, 0111, \dots), \\ &\quad \dots (000, 0001, 00011, 000111, \dots), \dots)^* \end{aligned}$$

$$\Rightarrow (\Lambda, 0, 1, 01, 10, 00, 11, \dots) = (0+1)^*$$

$$= \text{R.H.S}$$

→ Examples

✓ (1)  $L \subseteq \{0,1\}^*$  be the language of all strings of even length

$$L = \{00, 01, 10, 11\}^*$$

$$\text{R.E} = (00 + 01 + 10 + 11)^* \text{ or } ((0+1)(0+1))^*$$

(ii) strings with odd number of 1's

$$R.E = 0^*10^*(10^*10^*)^* \checkmark$$

$$\text{OR} = 0^*1(0^*10^*1)^*0^* \checkmark$$

$$\text{OR} = (0^*10^*1)^*0^*10^* \checkmark$$

$$\text{OR} = 0^*(10^*10^*)^*1(0^*10^*1)^*0^* \checkmark$$

Now is it the same R.E

$$(10^*10^*)^*10^*$$

strings with 0 <sup>in beginning</sup> are not producing

another correct version

$$0^*(10^*10^*)^*10^* \checkmark$$

8 FRIDAY

9 SATURDAY

10 SUNDAY

(iii) strings of length 6 or less

$$R.E = (0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$$

$$\text{OR } (0+1)^6$$

$$\text{OR } (0+1+\Delta)^6 \text{ to allow } \Delta$$

✓ (iv) strings ending in 1 and not containing 00

$$R.E = (1+01)^*(1+01)$$

$$\text{OR } (1+01)^+$$

(v) Language for C identifiers

$L = a, b, c, \dots, z, A, B, \dots, Z$

$d = 0, 1, 2, \dots, 9$

R.E =  $(L+d)^*(L+d)^*$

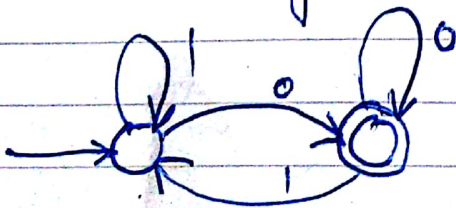
(vi) - Strings ending with 0 e.g.  $(0+1)^*0$   
(remembering only the last digit 0, else is not need to be remembered)

(vii) - Algorithm for strings with next to last symbol 0

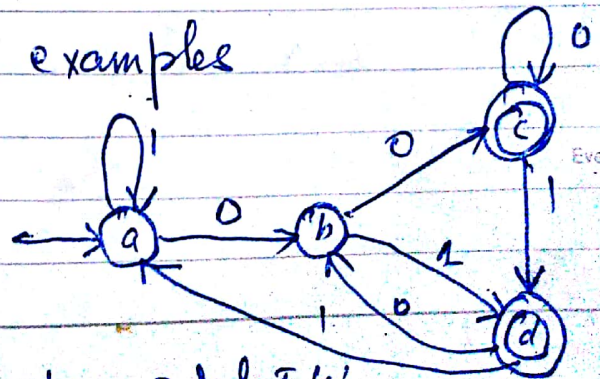
- a. The strings is 1 or 1 or end with 11
  - b. The strings is 0 or ends with 10
  - c. The strings with 00
  - d. The strings end with 01
- } not allowed
- } allowed

(viii) - Algorithm for (iv) which is ending in '1'  
- Containing 00 ?

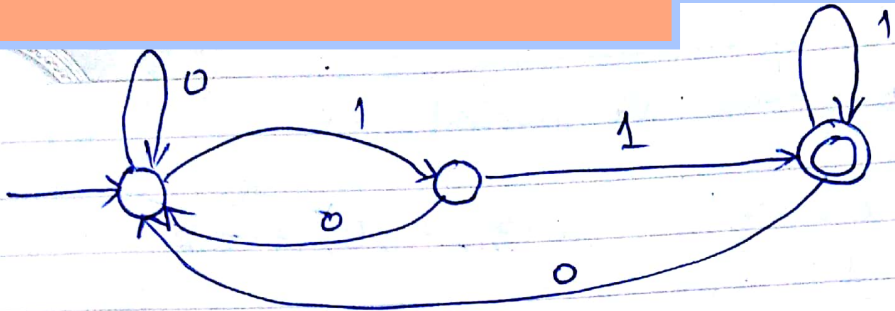
→ Flow Diagrams for examples



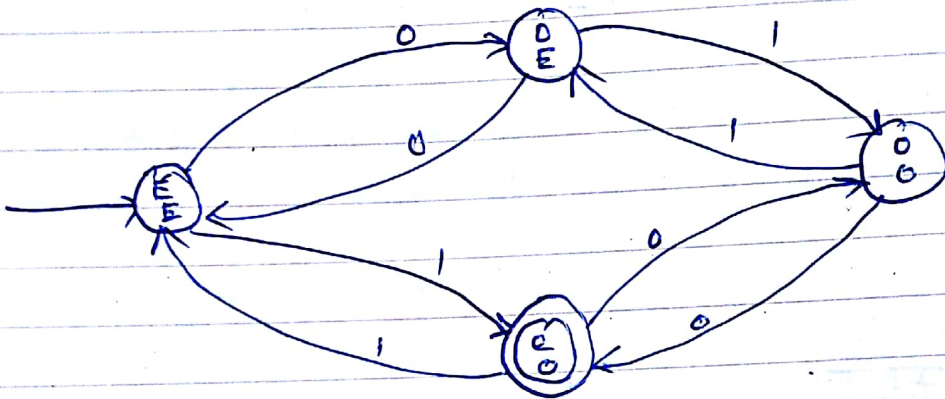
(a) ending in '0'



(b) next to last '0'



(c) ending with '11'

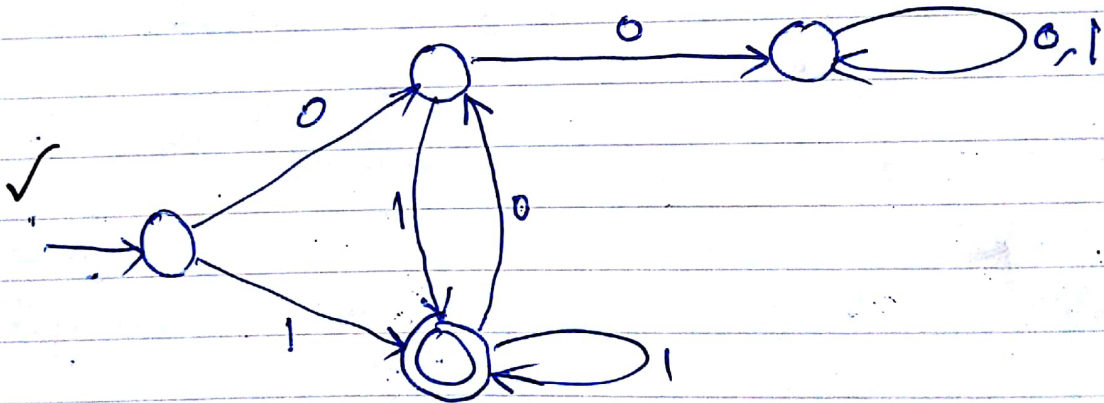


Evening

13 WEDNESDAY

(d) no even ed n, odd

14 THURSDAY



(e) edg in '1' not containg 00

Evening

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