According to 1st eq of motion

$\frac{dx}{dt} = at + u$

$dx = (at + u)\,dt$

$dx = at\,dt + u\,dt$

Now integrated

$\int dx = \int at\,dt + u\,dt$

$x = \frac{at^2}{2} + ut + B \rightarrow (3)$

According to boundry condition

At $t = 0$ $x = 0$

Now eq (3) becomes

$0 = 0 + 0 + B$

$B = 0$

$x = \frac{at^2}{2} + ut + 0$

$x = ut + a\frac{t^2}{2}$

$\downarrow$ $\downarrow$

$S = v_i$

$\boxed{S = v_i t + \frac{1}{2} at^2}$

Third eq of motion

$a = \frac{dv}{dt}$

÷ and x by dx

$a = \frac{dv}{dt} \times \frac{dx}{dx}$

$\vec{a} = \frac{dv}{dx} \cdot \frac{dx}{dt}$

$\vec{a} = \frac{dv}{dx} \cdot \vec{v}$

$a\,dx = v\,dv$

$\int a\,dx = \int v\,dv$

$ax = \frac{v^2}{2} + c \rightarrow (4)$

at b.c $t = 0$ $x = 0$

$v = u$ $v^2 = u^2$

$0 = \frac{u^2}{2} + c$

$c = -\frac{u^2}{2}$

$ax = \frac{v^2}{2} - \frac{u^2}{2}$

$ax = \frac{v^2}{2} - \frac{u^2}{2}$

$ax = \frac{v^2 - u^2}{2}$

$2ax = v^2 - u^2$

$\downarrow$

$\boxed{2aS = v_f^2 - v_i^2}$

① First Eq of motion :-

⇒ Consider an object is moving with initial velocity "U". After certain velocity became 'V' by defination of acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \frac{d}{dt}\cdot\left(\frac{d\vec{x}}{dt}\right)$$

$$\vec{a}dt = d\left(\frac{d\vec{x}}{dt}\right)$$

⇒ Now integrated both sides

$$\int a\,dt = \int d\left(\frac{d\hat{x}}{dt}\right)$$

$$at + \underset{\substack{\downarrow \\ \text{constant of} \\ \text{integrati}}}{A} = \frac{dx}{dt} \Rightarrow$$

$$\Rightarrow \frac{dx}{dt} = at + A \rightarrow (1)$$

according to boundary condition

$$\Rightarrow \text{at } t = 0 \quad \frac{dx}{dt} = u$$

$$\frac{dx}{dt} = at + A$$

$$U = 0 + A \Rightarrow u = A \rightarrow ②$$

$$\frac{dx}{dt} = at + U \rightarrow ②$$

⇒ At any time 't'

$$\frac{dx}{dt} = V$$

$$V = at + U$$

$$\underset{\substack{\downarrow \\ \text{final}}}{V} = \underset{\substack{\downarrow \\ \text{init}}}{U} + at$$

at .

Projectile Motion:-

- **Projectile:-**

A body or ball which make an angle with the horizantal is know as projectile

- Since angle with the horizental may be above or may be below

- **Projectile Motion:**

The motion of body along two perpendicular axes at the same time is know a projectile motion

- **Trajectory:-**

The path followed by the projectile in its flite is know as trajectory.

- Types of Projectile motion:-

(i) Linear projectile (ii) oblique projectile

- In linear projectile angle is 'Zero'
- In ablique Projectile $\theta > 0$

xample of linear projectile

- ball through at Zero angle that is Linear projectile

Foot ball through its angle $\theta > 0$ is oblique Projectile.

Motion in one dimension:-

- The motion of a body along horizental or vertical, that is along x- axis or along y- axis is known as one dimensional motion.

- Motion of a car along straight rod
- The motion of a ball along gravel either cartesian co-ordinate axis, x or y axes.

Motion in two dimension:-

The motion of a body along two perpendicular axes at the same time that is along two perpendicular axes 'x' and 'y'. is known as motion in two dimension.

A foot ball kicked off by a players.

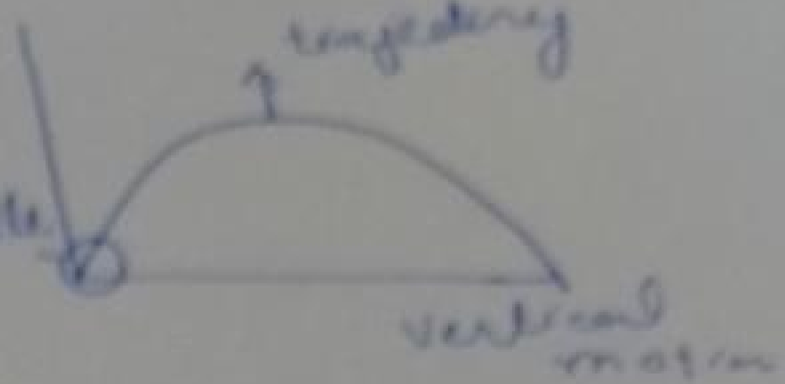
A stone thrown in air

A Mesile lanched by lanching pad.

Ball thrown by crikete.

factors effect projectile motion.

- (i) air friction
- (ii) acceleration due to force of gravity

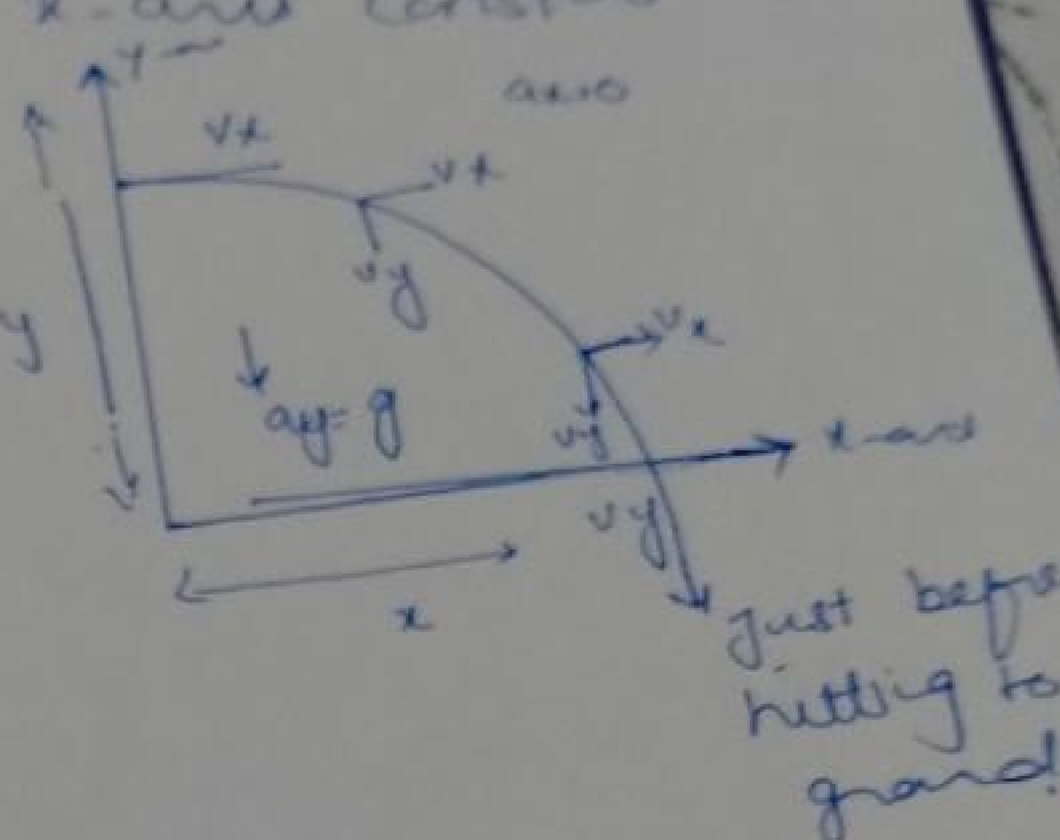


→ Horizontal and vertical distances.-

- Horizontal distance:-

velocity of projectile along x-axis constant

$S = vt \rightarrow (i)$



$$V_{av} = \frac{V_i + V_f}{2}$$

As velocity remains constant along horizontal $V_i = V_f = V_x$

$$V_{av} = \frac{V_x + V_x}{2} = \frac{2V_x}{2}$$

$$V_{av} = V_x$$

using (i)

$$S = V_x t$$

but $S = X$

$$\boxed{X = V_x t} \dashrightarrow (a)$$

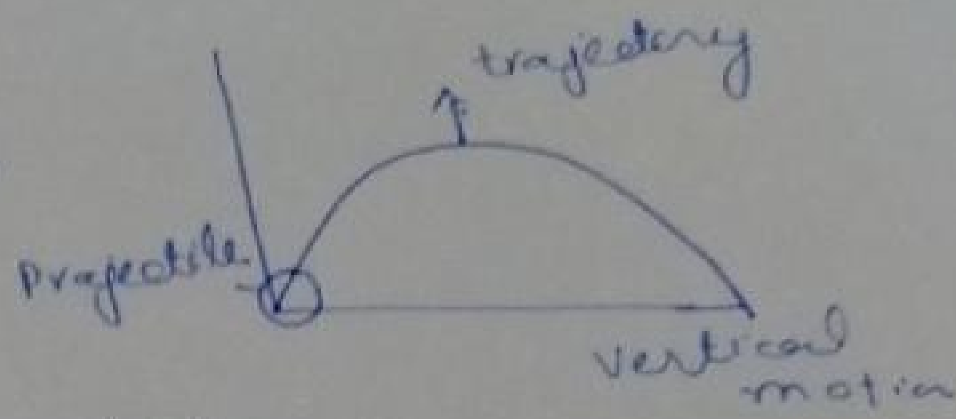
vertical distance:

As we know

$$S = v_i t + \frac{1}{2} a t^2$$

→ **Factors effect projectile motion:-**

- (i) air friction
- (ii) acceleration due to force of gravity.



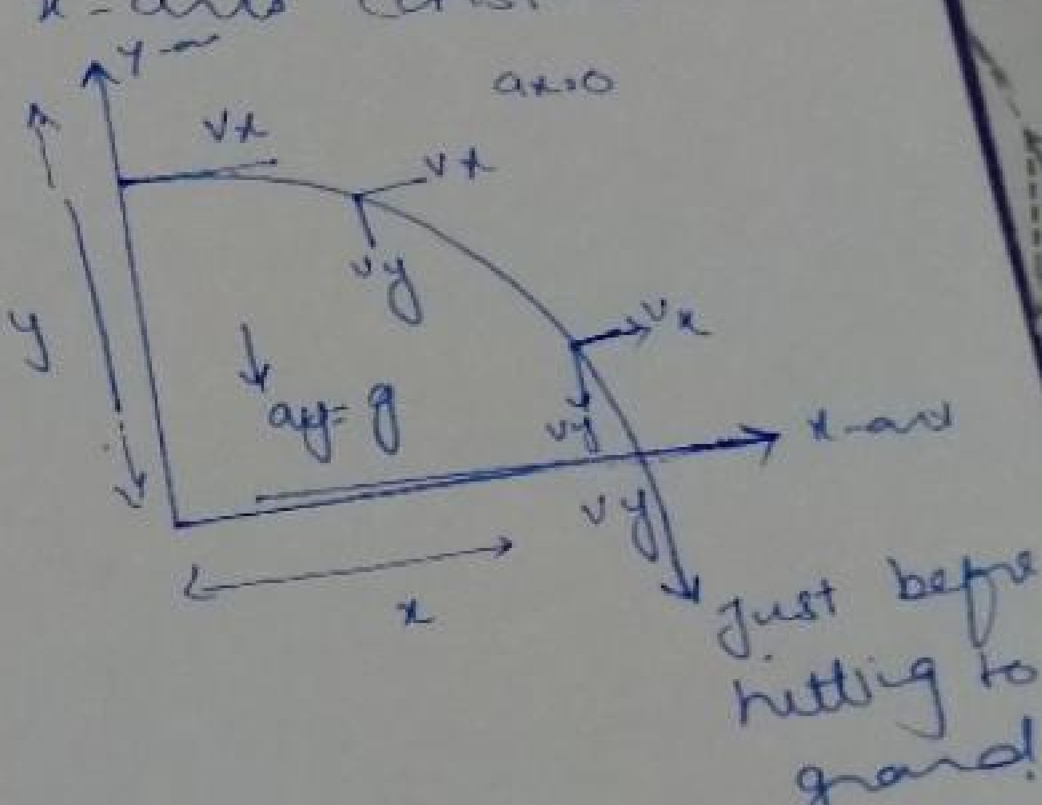
→ Horizental and vertical distance:-

• Horizental distance:-

velocity of projectile along x-axis constant

$S = vt \rightarrow (i)$

$V_{av} = \frac{V_i + V_f}{2}$



As velocity remains constant along horizental $V_i = V_f = V_x$

$V_{av} = \frac{V_x + V_x}{2} = \frac{\cancel{2}V_x}{\cancel{2}}$

$V_{av} = V_x$

using (i)

$S = V_x t$

but $S = X$

$\boxed{X = V_x t}$ ---> (a)

vertical distance:

As we know

$S = v_i t + \frac{1}{2} a t^2$

- for vertical motion

$S_y = V_{iy}t + \frac{1}{2} a_y t^2$ $\quad \because a = g$

initially $V_{iy} = 0$

$y = 0(t) + \frac{1}{2} a_y t^2$

$\boxed{y = \frac{1}{2} g t^2} \rightarrow$ (b)

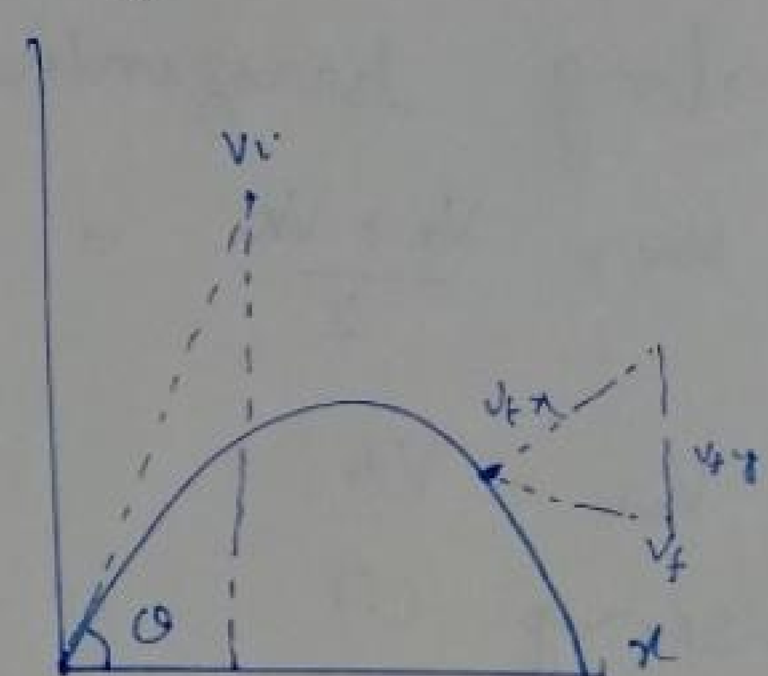
velocity of Projectile at any instant:-

known Vi and θ. find velocity at time 't'

mangnitude and direction

Horizental component of velocity:-

$V_{ix} = V_i \cos\theta$



By first Eq of motion

horizental motion

$V_{fx} = V_{ix} + a_x t$ $\quad \because$ newton first law no acceleration in horizental $a_x = 0$

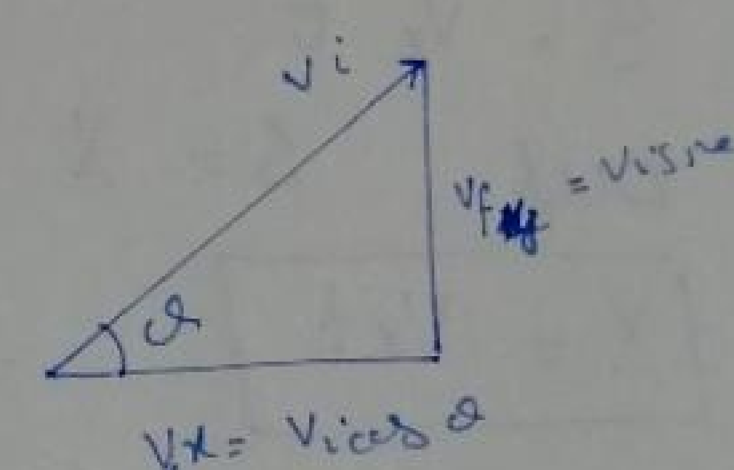
$a_x = 0$

$V_{fx} = V_{ix} + 0$

$V_{fx} = V_{ix}$

$\boxed{V_{fx} = V_{ix} = V_i \cos\theta}$

$\boxed{V_{fx} = V_i \cos\theta}$



Now vertical components of velocity:-

Whereas the vertical components of velocity decreases as body moves up and it become zero at highes point 'A'

The vertical component of velocity

$v_{iy} = v_i \sin\theta$

$v_{fy} = v_{iy} + a_y t$

$v_{fy} = v_i \sin\theta + a_y t$

$a_y = -g$ for upward motion

$v_{fy} = v_{iy} - gt \Rightarrow \boxed{v_{fy} = v_i \sin\theta - gt}$

for downward motion

$\boxed{v_{fy} = v_{iy} + gt}$

The magnitude of velocity will be

$v = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(v_i\cos\theta)^2 + (v_i\sin\theta - gt)^2}$

$v = \sqrt{v_i^2\cos^2\theta + v_i^2\sin^2\theta + g^2t^2 - 2v_i\sin\theta}$

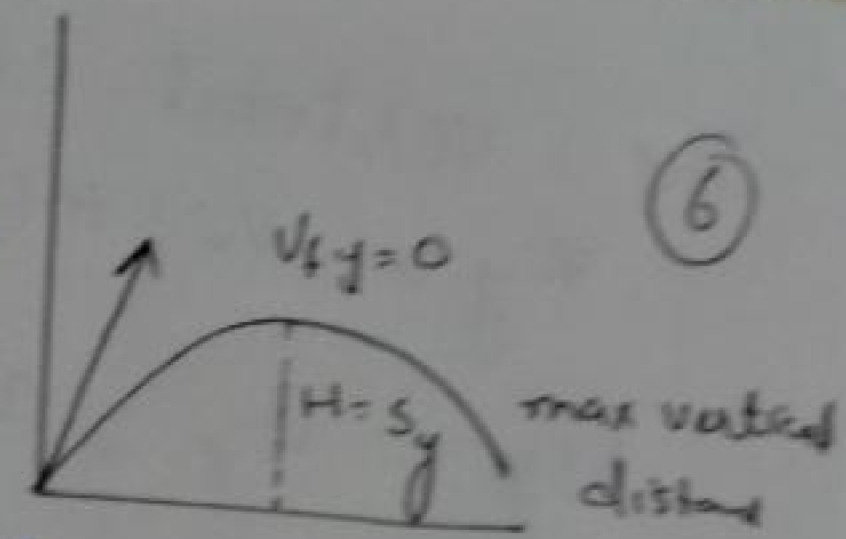
$v = \sqrt{v_i^2(\underbrace{\cos^2\theta + \sin^2\theta}_{=1}) + g^2t^2 - 2v_i\sin\theta}$

$v = \sqrt{v_i^2 + g^2t^2 - 2v_i \sin\theta\, g}$

Now

$\tan\phi = \dfrac{v_{fy}}{v_{fx}} = \left(\dfrac{v_i\sin\theta - gt}{v_i\cos\theta}\right)$

ii) Height of Projectile:-



⑥

vertical ~~distance~~ displacement travells by the Projectile in its flight as height of Projectile

we use Eq of motion

$$2as = v_f^2 - v_i^2$$

$$S = h$$

$$a = -g \qquad v_{iy} = v_i \sin\theta \qquad v_{fy} = 0$$

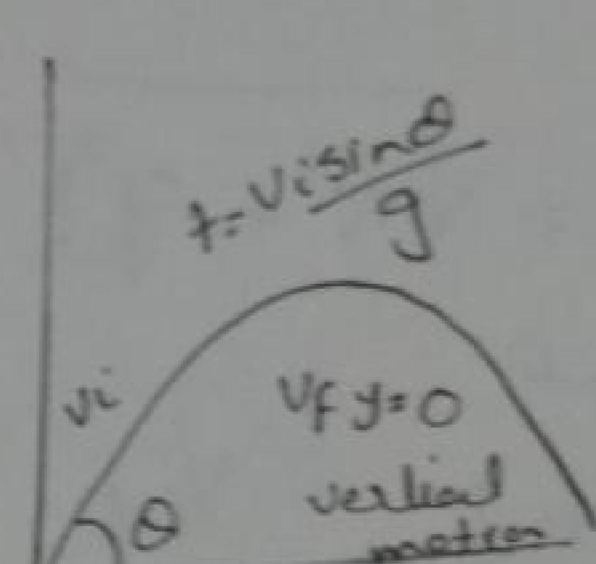
$$2a_y S_y = v_{fy}^2 - v_{iy}^2$$

$$(2(-g)(h)) = 0 - v_i^2 \sin^2\theta$$

$$\cancel{-}2gh = \cancel{-}v_i^2 \sin^2\theta$$

$$h = \frac{v_i^2 \sin^2\theta}{2g}$$

Time of Flight:-



~~is The time takes by projectile reach at~~

The Time taken by projectile to move from its point of Projection to point

- It is the total time for which Projectile remain in air (7)

$V_{iy} = V_i \sin\theta \quad , \quad a_y = -g$

$S = 0 \qquad t = T$

Using second eq of motion

$S = V_i t + \frac{1}{2} a t^2$

$h = V_{iy} t + \frac{1}{2}(-g) t^2$

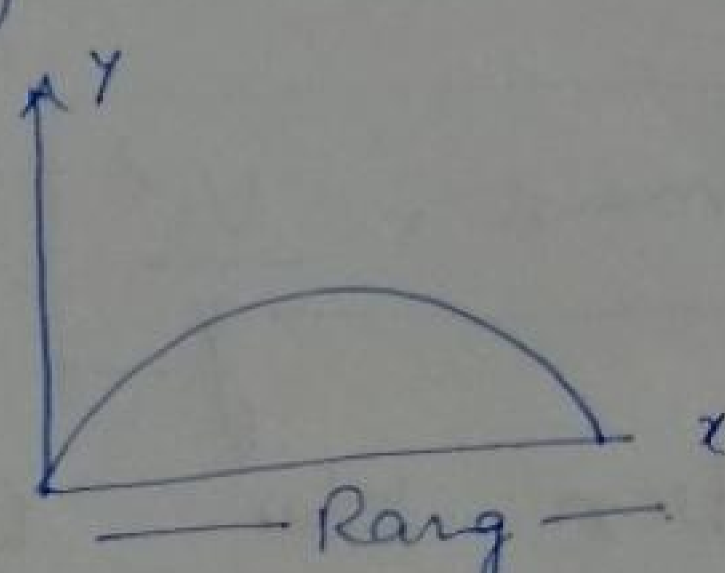
$0 = V_i \sin\theta \, T + \frac{1}{2}(-g) T^2$

$\frac{1}{2} g T^{\cancel{2}} = V_i \sin\theta \cancel{T}$

$$\boxed{T = \frac{2 V_i \sin\theta}{g}}$$

Range of Projectile:

It is the maximum horizontal distance covers by Projectile its time of flight



$V_{ix} = V_i \cos\theta$

$t = T = \frac{2 V_i \sin\theta}{g}$

(

$S = V_x t$

$R = V_{fx} t$

$= v_i \cos\theta \cdot \frac{2v_i \sin\theta}{g}$

$R = \frac{2v_i^2 \sin\theta \cos\theta}{g}$

$2\sin\theta\cos\theta = \sin 2\theta$

$$\boxed{R = \frac{v_i^2 \sin 2\theta}{g}}$$

If $v_i^2 = \text{constant}$

$R \propto \sin 2\theta$

$\sin 2\theta$

$2\theta = \sin^{-1}(1)$

$2\theta = 90^\circ \Rightarrow \theta = 90/2 \Rightarrow \theta = 45^\circ$

$\because$ maximum range the angle of projection must be equal to 45°

$R = \frac{v_i^2 \sin 2\theta}{g}$

$R_{max} = v_i^2 \frac{\sin 2(45)}{g}$

$$\boxed{R_{max} = \frac{v_i^2}{g}}$$

Relation b/w Range and height is

$$\boxed{R = 4h}$$