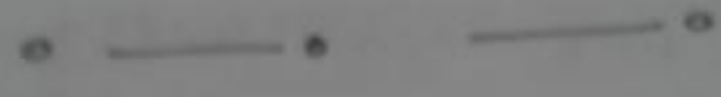


EXAMPLE 3:1

- The sum can take on any value from $30 + 30 = 60$
- Where the vectors point in the same direction $30 - 30 = 0$

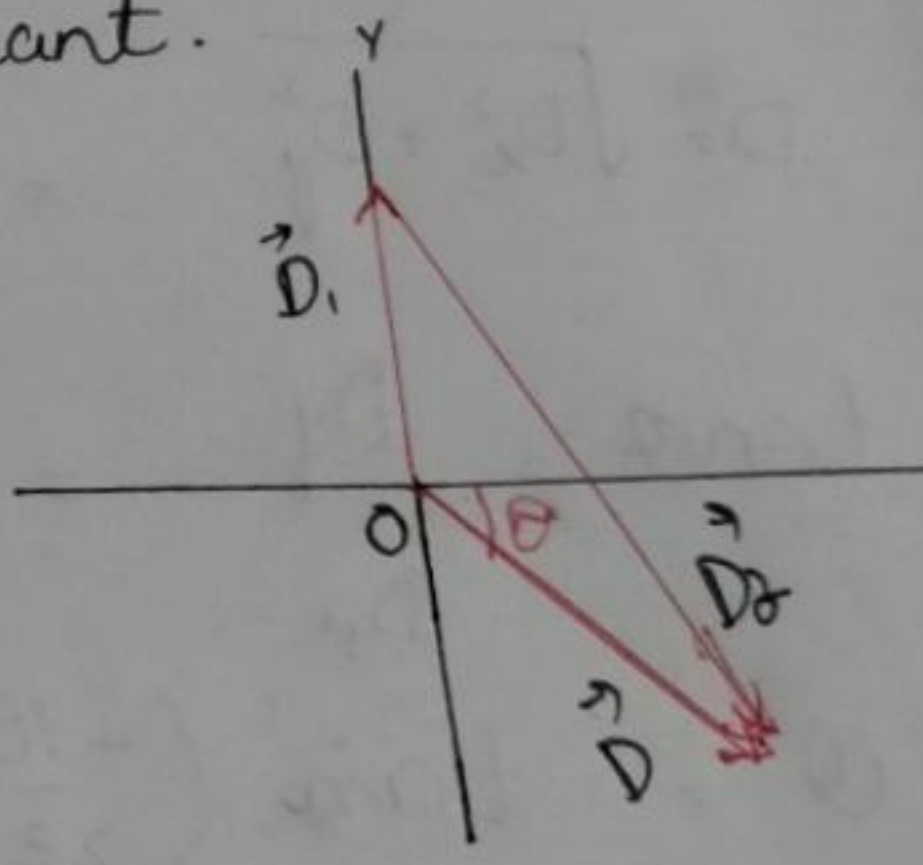
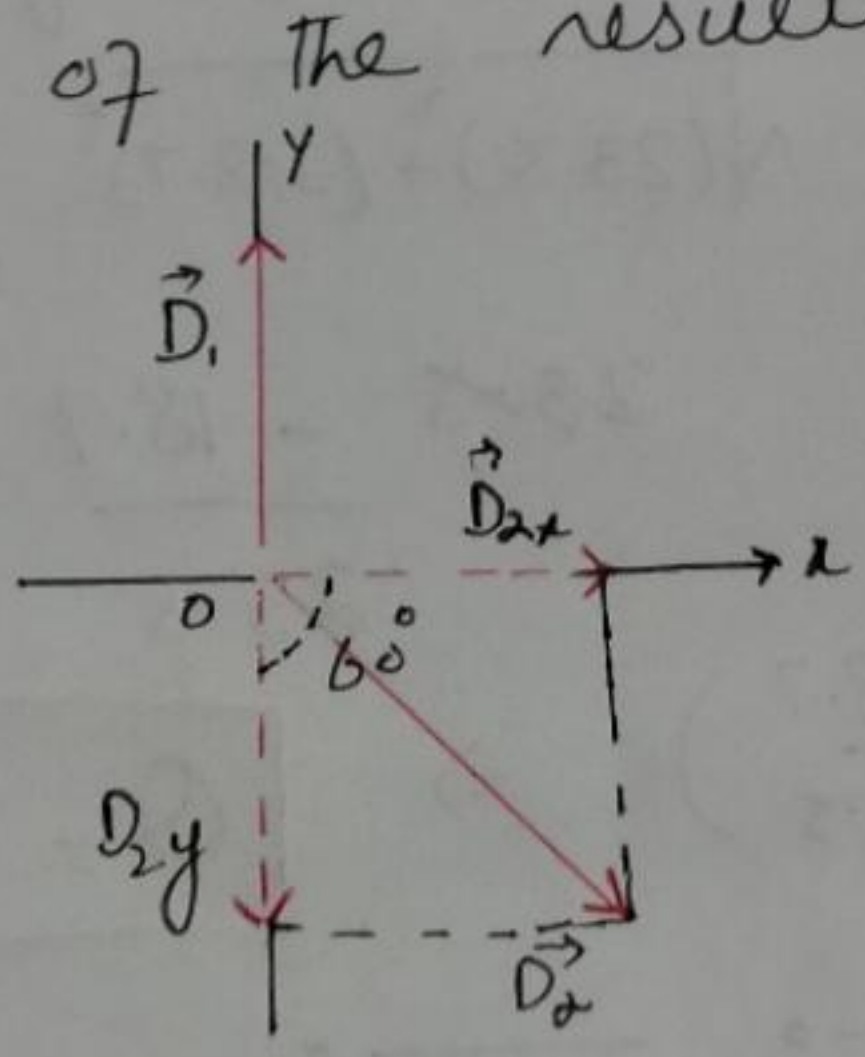
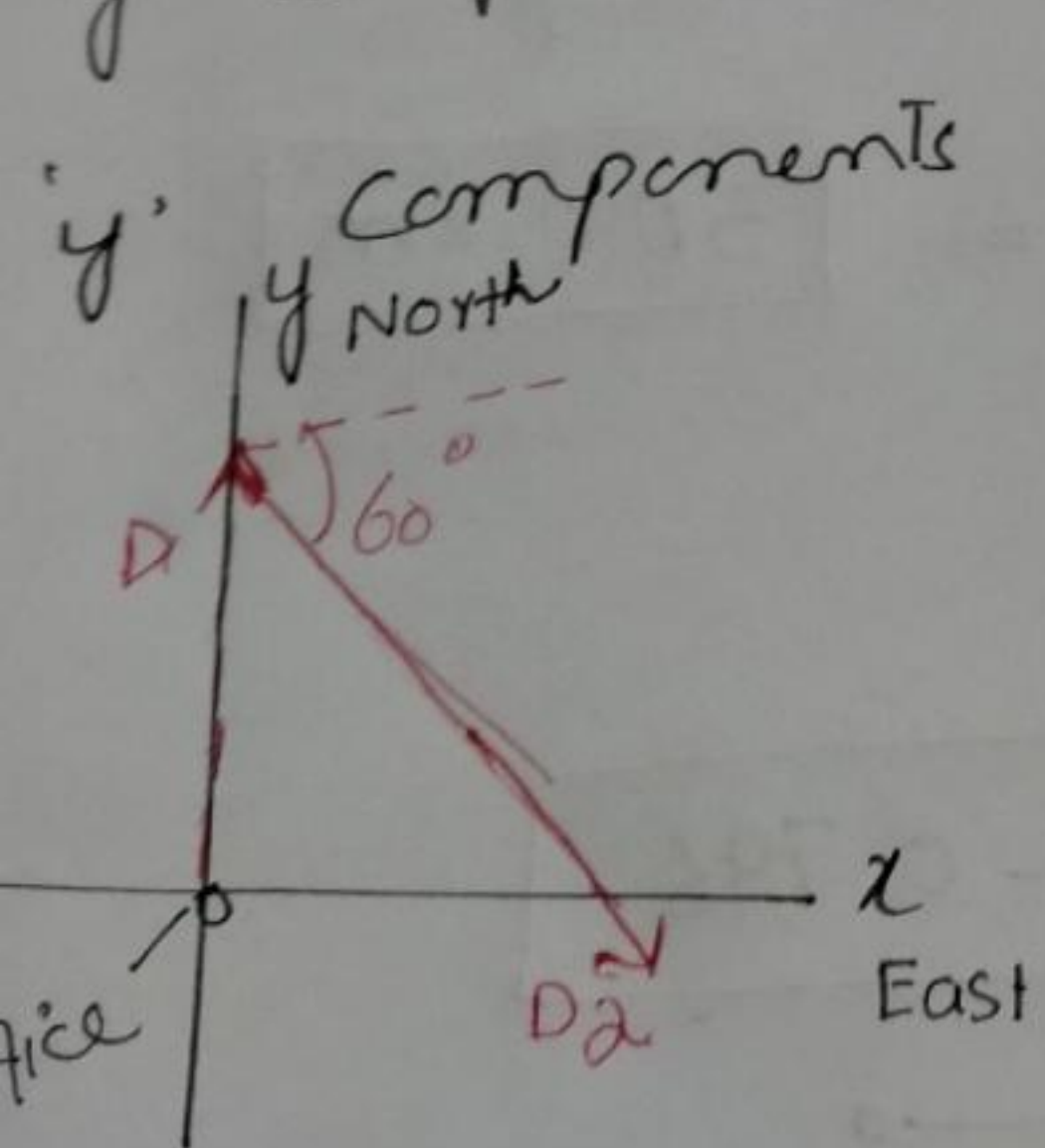
When the vectors are antiparallel



EXAMPLE 3:2

- We choose the positive x axis to be east and the positive y axis to be north, since those are the compass directions used on most maps.
- The origin of the xy coordinate system is at the post office.

We resolve each vectors into its x, y components we add the x components together, and then y components together, giving us the 'x' and 'y' components of the resultant.



- \vec{D}_1 has magnitude 22.0 km and points north, it has only a y-component. (2)

$$D_{1x} = 0 \quad D_{1y} = 22.0 \text{ km}$$

- \vec{D}_2 has both 'x' and 'y' components.

$$D_{2x} = +(47.0 \text{ km})(\cos 60^\circ) = (47.0 \text{ km})(0.500) = \boxed{+23.5 \text{ km}}$$

$$D_{2y} = -(47.0 \text{ km})(\sin 60^\circ) = -(47.0)(0.866) = \boxed{-40.7 \text{ km}}$$

- D_{2y} is negative because this vector component points along the negative y-axis

- The resultant vector, \vec{D} has components.

$$D_x = D_{1x} + D_{2x}$$

$$D_x = 0 + 23.5 = \boxed{23.5 \text{ km}}$$

$$D_y = D_{1y} + D_{2y}$$

$$D_y = 22.0 + (-40.7) = \boxed{-18.7 \text{ km}}$$

This specifies the resultant vector completely

$$D_x = 23.5 \text{ km}$$

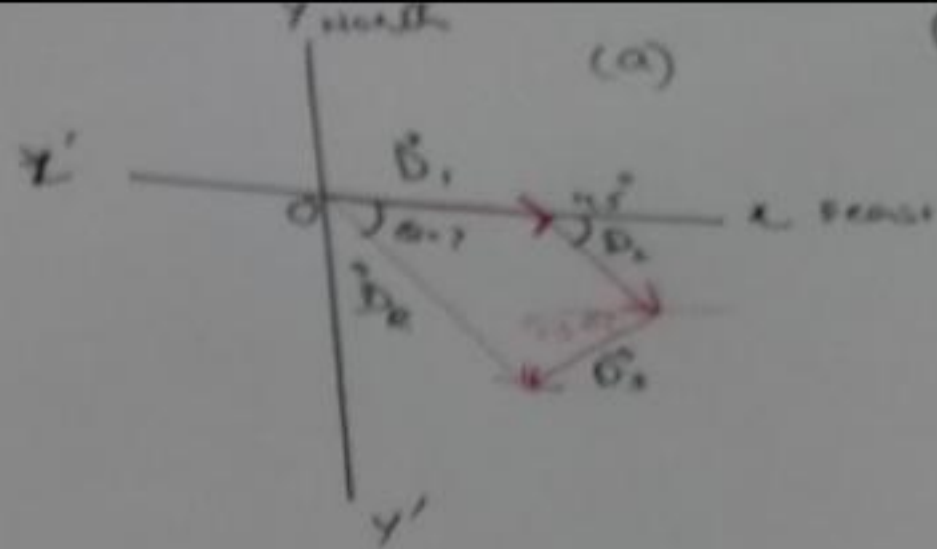
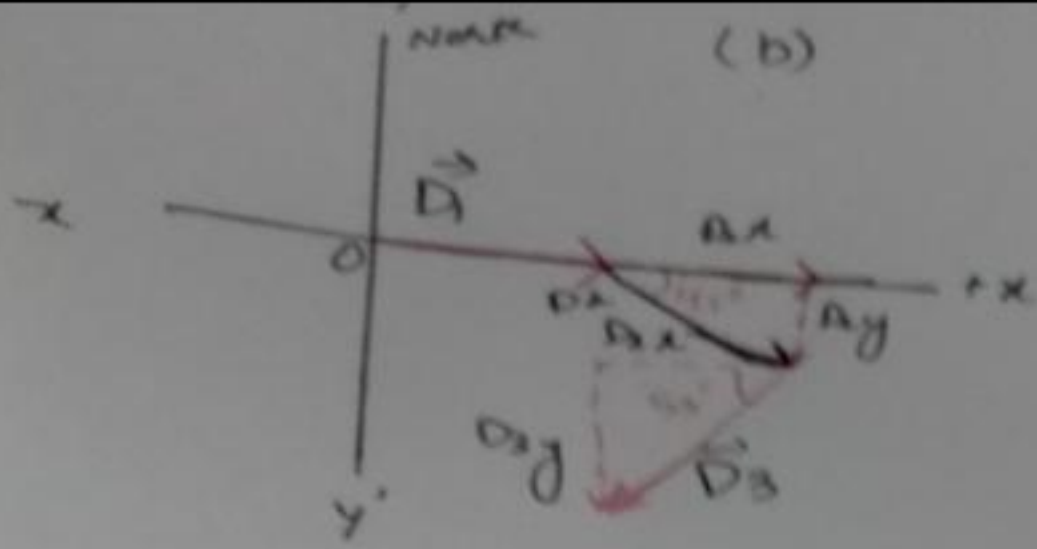
$$D_y = -18.7 \text{ km}$$

Resultant vector by giving its magnitude

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(23.5)^2 + (-18.7)^2} = \boxed{30.0 \text{ km}}$$

$$\tan \theta = \frac{D_y}{D_x} = \frac{23.5}{-18.7}$$

$$\theta = \tan^{-1}\left(\frac{-18.7}{23.5}\right) \Rightarrow \boxed{\theta = -0.796}$$



3

• \vec{D}_1 Components

$$D_{1x} = D_1 \cos 30^\circ = 620 \text{ km}$$

$$D_{1y} = D_1 \sin 30^\circ = 0 \text{ km}$$

• \vec{D}_2 Components

$$D_{2x} = +D_2 \cos 45^\circ = (440)(0.707) = 311 \text{ km}$$

$$D_{2y} = -D_2 \sin 45^\circ = -(440)(0.707) = -311 \text{ km}$$

• \vec{D}_3 Components

$$D_{3x} = -D_3 \cos 53^\circ = -(550)(0.602) = -331 \text{ km}$$

$$D_{3y} = -D_3 \sin 53^\circ = -(550)(0.799) = -439 \text{ km}$$

Now add 'x' components and 'y' components

$$D_x = D_{1x} + D_{2x} + D_{3x}$$

$$D_x = 620 + 311 - 311 = 600 \text{ km}$$

$$D_y = D_{1y} + D_{2y} + D_{3y}$$

$$D_y = 0 \text{ km} - 311 - 439 = -750 \text{ km}$$

bw magnitude and direction

$$D_R = \sqrt{D_x^2 + D_y^2} = \sqrt{(600)^2 + (-750)^2} = 960 \text{ km}$$

and $\frac{D_y}{D_x} = \frac{-750}{600} \Rightarrow \theta = -51^\circ$

EXAMPLE 3.4 :-

- We use the components we found in expt 3.3

$$D_{1x} = 0 \quad D_{1y} = 22.0 \text{ km} \quad \text{and} \quad D_{2x} = 23.5 \text{ km} \quad D_{2y} = -40.7 \text{ km}$$

We have

$$\vec{D}_1 = 0 \hat{i} + 22.0 \text{ km } \hat{j}$$

$$\vec{D}_2 = 23.5 \hat{i} - 40.7 \text{ km } \hat{j}$$

$$\vec{D} = \vec{D}_1 + \vec{D}_2$$

$$= (0 + 23.5) \text{ km } \hat{i} + (22.0 - 40.7) \text{ km } \hat{j}$$

$$\vec{D} = 23.5 \hat{i} - 18.7 \hat{j}$$

$$D_x = 23.5$$

$$D_y = 18.7$$

$$D = \sqrt{(23.5)^2 + (18.7)^2}$$

$$= 30.0 \text{ km}$$

EXAMPLE 3.4

- We use the components we found in expt 2.2
 $D_{1x} = 0$ $D_{1y} = 22.0 \text{ km}$ and $D_{2x} = 23.5 \text{ km}$ $D_{2y} = -40.7 \text{ km}$

We have

$$\vec{D}_1 = 0\hat{i} + 22.0 \text{ km} \hat{j}$$

$$\vec{D}_2 = 23.5\hat{i} - 40.7 \text{ km} \hat{j}$$

$$\vec{D} = \vec{D}_1 + \vec{D}_2$$

$$= (0 + 23.5) \text{ km} \hat{i} + (22.0 - 40.7) \text{ km} \hat{j}$$

$$\vec{D} = 23.5 \hat{i} - 18.7 \hat{j}$$

• $D_x = 23.5$

$D_y = 18.7$

$$D = \sqrt{(23.5)^2 + (18.7)^2} = 30.0 \text{ km}$$

Vectors Kinematics :-

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 \rightarrow (a)$$

$$\Delta t = t_2 - t_1 \rightarrow (b)$$

\hat{i} unit vector notation

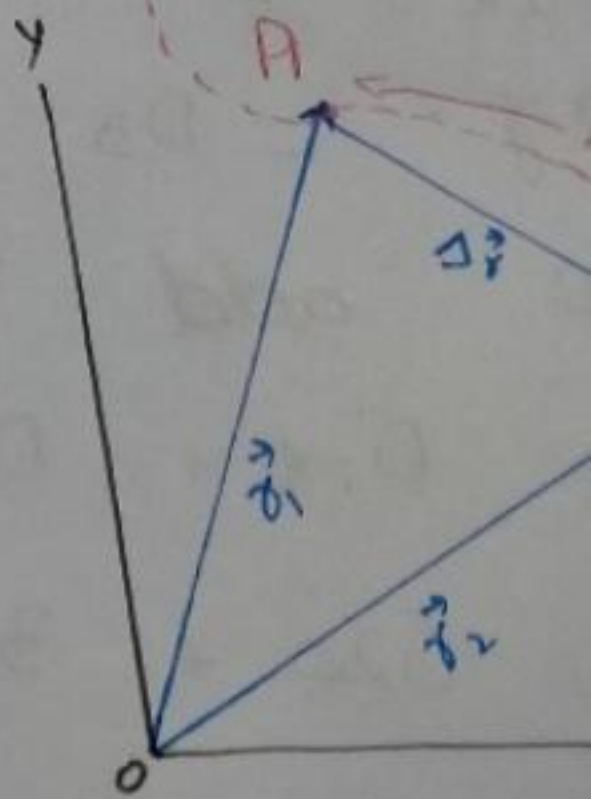
$$\vec{r}_1 = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k} \rightarrow (c)$$

x_1, y_1, z_1 coordinate of point P_1

$$\vec{r}_2 = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k} \rightarrow (d)$$

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta \vec{r} = (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j} + (z_2 - z_1) \hat{k}$$



Similar

• If the motion along the z-axis, then (5)
 $y_2 - y_1 = 0, z_2 - z_1 = 0$ and magnitude of displacement

$$\Delta \vec{r} = x_2 - x_1$$

• The average velocity vector over the time interval
 $\Delta t = t_2 - t_1$

average velocity = $\frac{\Delta \vec{r}}{\Delta t} \rightarrow (f)$

• Let us consider shorter and shorter intervals
 let Δt approach zero, so distance points
 P_2 and P_1 also approaches zero.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \rightarrow (g)$$

where $v_x = dx/dt, v_y = dy/dt, v_z = dz/dt$

Average acceleration = $\frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

instantaneous acceleration vector.

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \rightarrow (h)$$

where $a_x = \frac{dv_x}{dt}$
 $v_x = \frac{dx}{dt}$ then $\frac{dv_x}{dt} = \frac{d^2x}{dt^2}$

$$\vec{a} = \frac{d^2x}{dt^2} \hat{i} + \frac{d^2y}{dt^2} \hat{j} + \frac{d^2z}{dt^2} \hat{k} \rightarrow (i)$$

Example 3:5:-

The position of particle as a function of time

$$\vec{r} = [(5.0 \text{ m/s})t + (6.0 \text{ m/s})t^2] \hat{i} + [(7.0 \text{ m}) - (3.0 \text{ m/s}^3)t^3] \hat{j}$$

$t_1 = 2.0 \text{ s}$ $t_2 = 3.0 \text{ s}$

(a) At $t_1 = 2.0 \text{ s}$

$$\vec{r}_1 = [(5.0)(2.0) + (6.0)(2.0)^2] \hat{i} + [(7.0) - (3.0)(2.0)^3] \hat{j}$$

$$\vec{r}_1 = (34 \text{ m}) \hat{i} - (17 \text{ m}) \hat{j}$$

Similarly at $t_2 = 3.0 \text{ s}$

$$\vec{r}_2 = (15 \text{ m} + 54 \text{ m}) \hat{i} + (7.0 \text{ m} - 81 \text{ m}) \hat{j} = (69 \text{ m}) \hat{i} - (74 \text{ m}) \hat{j}$$

Thus $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$
 $= (69 \text{ m} - 34 \text{ m}) \hat{i} + (-74 \text{ m} + 17 \text{ m}) \hat{j}$

$$\Delta \vec{r} = (35 \text{ m}) \hat{i} - (57 \text{ m}) \hat{j}$$

$$\Delta x = 35 \text{ m}$$

$$\Delta y = -57 \text{ m}$$

Now to find velocity we take derivative of the given \vec{r} with respect to time.

$$\frac{d(x^i)}{dt} = at$$

$$\frac{d(y^i)}{dt} = at^2$$

③

$$v = \frac{dv^i}{dt} = \left[5.0 \text{ m/s} + (12 \text{ m/s}^2)t \right] \hat{i} + \left[0 - (18 \text{ m/s}^2)t \right] \hat{j}$$

$$a^i = \frac{dv^i}{dt} = (12 \text{ m/s}^2) \hat{i} - (18 \text{ m/s}^2)t \hat{j}$$

Then $a_x = 12 \text{ m/s}^2$ is constant
but $a_y = -(18 \text{ m/s}^2)t$ depends linearly on time
increasing in magnitude with time in the
negative y -direction

④ We substitute $t = 3.0 \text{ s}$ into the equations
we just derived for \vec{v} and \vec{a}

$$\vec{v} = \left[5.0 \text{ m/s} + 36 \text{ m/s} \right] \hat{i} - (81 \text{ m/s}) \hat{j}$$

$$\vec{v} = (41 \text{ m/s}) \hat{i} - (81 \text{ m/s}) \hat{j}$$

$$\vec{a} = (12 \text{ m/s}^2) \hat{i} - (54 \text{ m/s}^2) \hat{j}$$

Their magnitude at $t = 3.0 \text{ s}$

$$v = \sqrt{(41 \text{ m/s})^2 + (81 \text{ m/s})^2} = 91 \text{ m/s}$$

$$v = 91 \text{ m/s}$$

Now

$$a^i = \sqrt{(12 \text{ m/s}^2)^2 + (54 \text{ m/s}^2)^2}$$

$$a^i = 55 \text{ m/s}^2$$