EXAMPLE 3.1.

- The sum can take on any value from

$30 + 30 = 60$

- Where the vectors point in the same direction

$30 - 30 = 0$

- When the vectors are antiparallel

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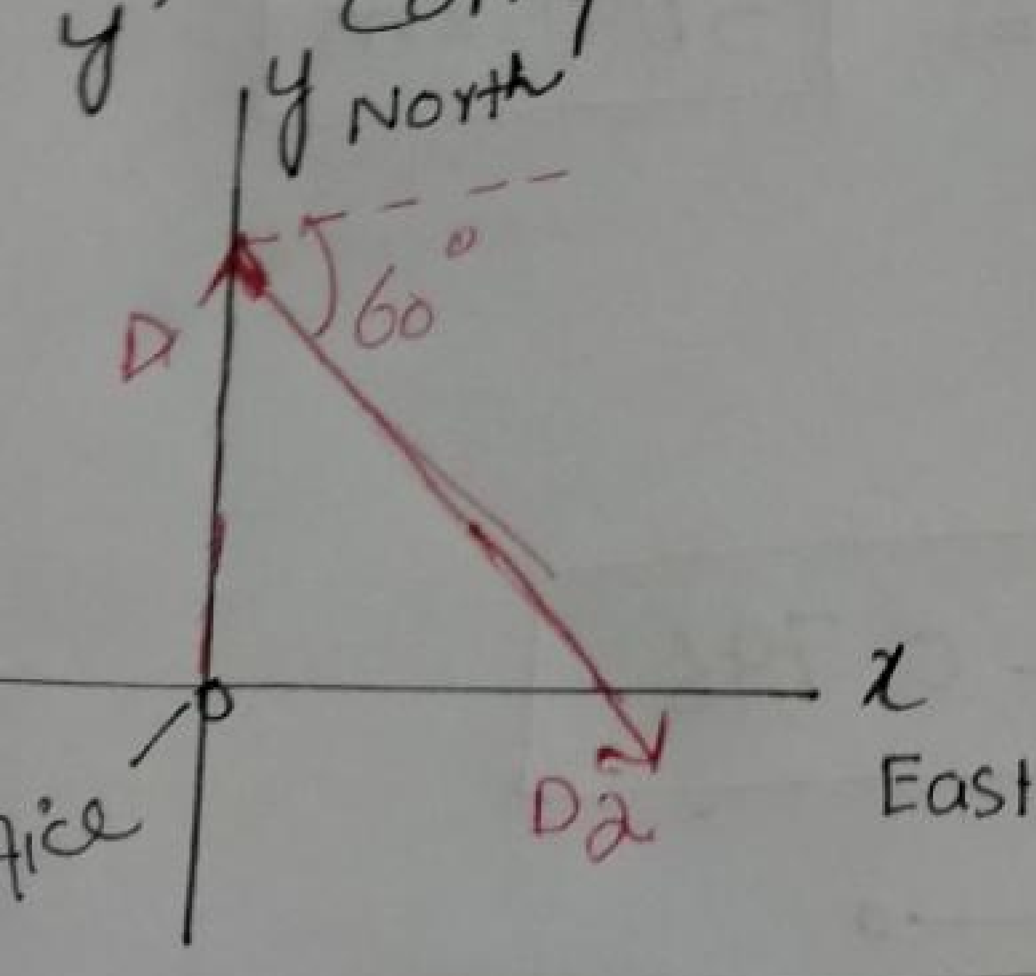
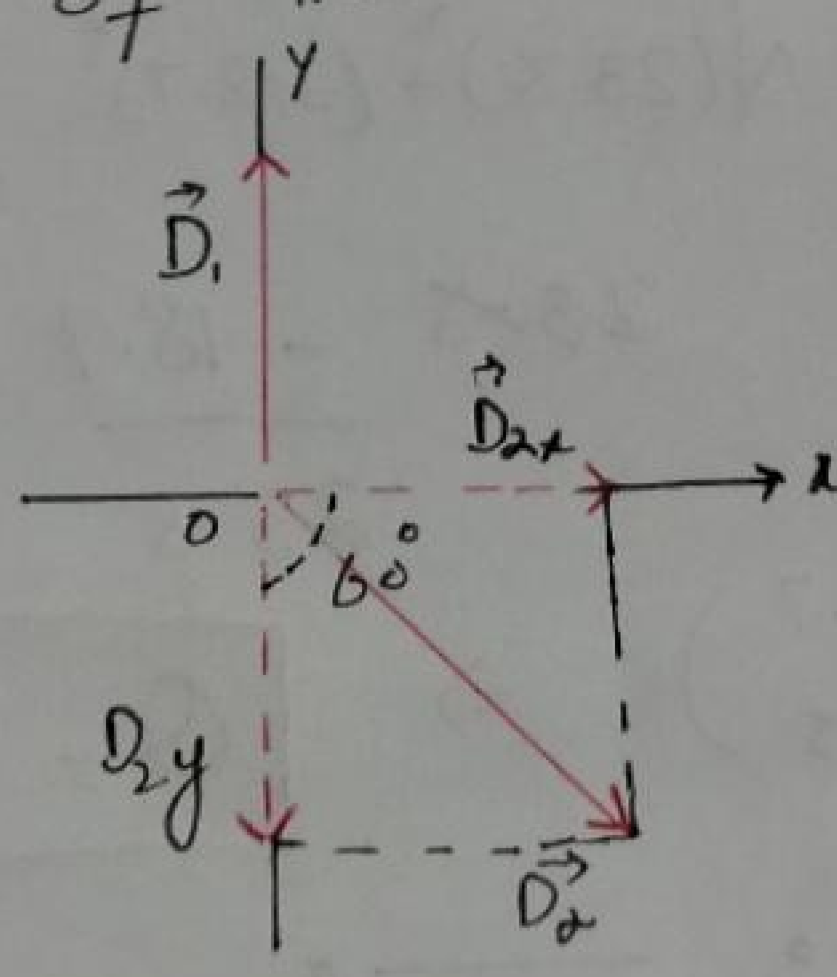
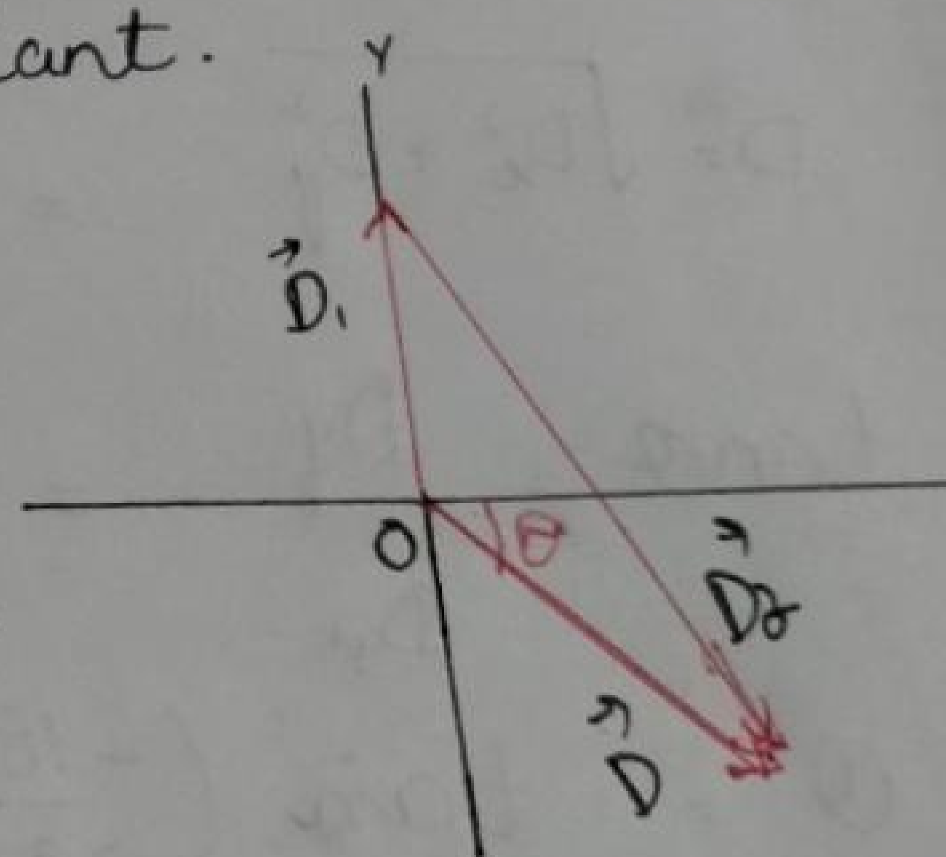
EXAMPLE 3:2

- We choose the positive x axis to be east and the positive y-axis to be north, since those are the compass direction used on most maps.
- The origin of the xy coordinate system is at the post office.

We resolve each vectors into its x, y components, and then we add the x components together, and then the x components together, giving us the 'x' and 'y' components of the resultant.







- $\vec{D}_1$ has magnitude 22.0 Km and points north, it has only a y-component. (2)

$D_{1x} = 0$ $\quad D_{1y} = 22.0$ Km

- $\vec{D}_2$ has both 'x' and 'y' components.

$D_{2x} = +(47.0\text{km})(\cos 60^\circ) = (47.0\text{km})(0.500) = \boxed{+23.5\text{km}}$

$D_{2y} = -(47.0\text{ Km})(\sin 60^\circ) = -(47.0)(0.866) = \boxed{-40.7\text{Km}}$

- D_{2y} is negative because this vector components points along the negtive y-axis

- The resultant vector $\vec{D}$ has components.

$D_x = D_{1x} + D_{2x}$

$D_x = 0 + 23.5 = \boxed{23.5\text{ km}}$

$D_y = D_{1y} + D_{2y}$

$D_y = 22.0 + (-40.7) = \boxed{-18.7\text{ Km.}}$

This specifies the resultant vector completely

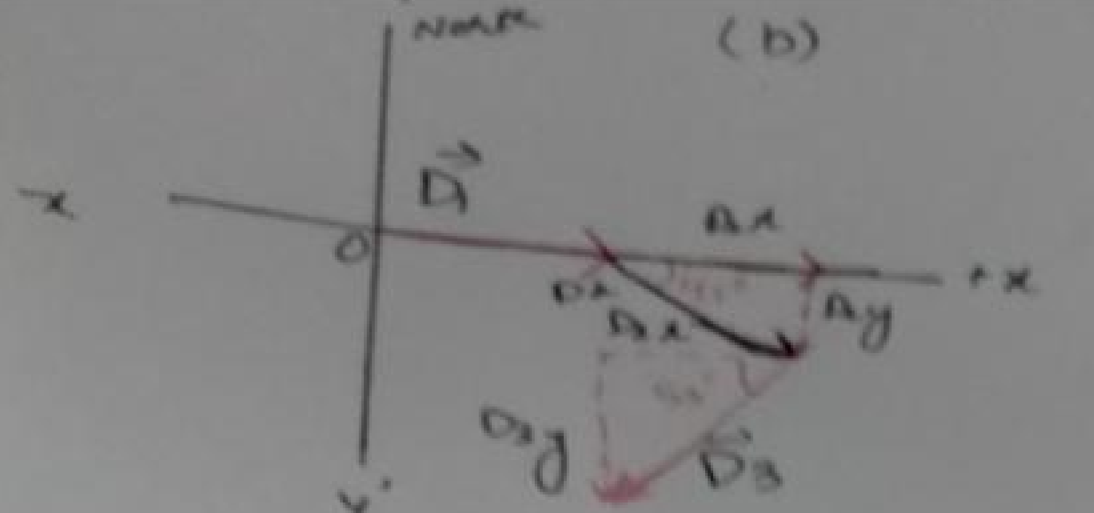
$D_x = 23.5$ km $\quad D_y = -18.7$ Km

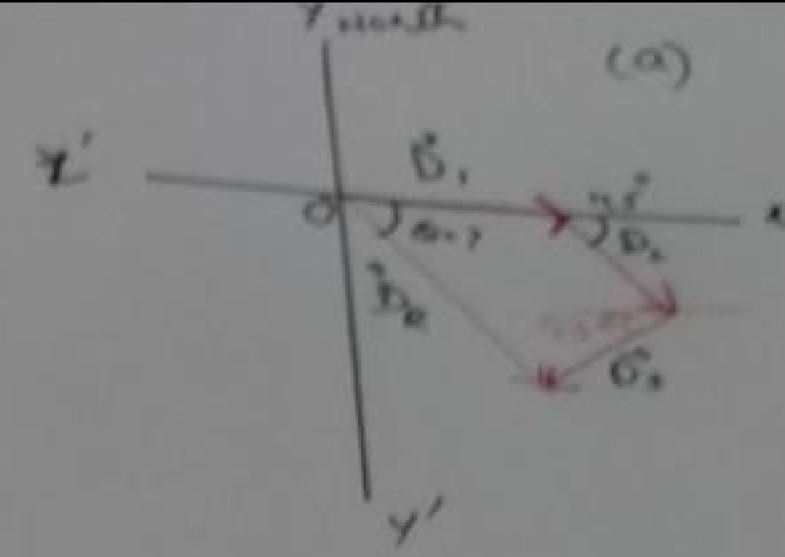
Resultant vector by giving its magnitude

$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(23.5)^2 + (-18.7)^2} = \boxed{30.0\text{ km}}$

$\tan\theta = \dfrac{D_y}{D_x} = \dfrac{-18.7}{23.5}$

$\theta = \tan^{-1}\left(\dfrac{-18.7}{23.5}\right) \Rightarrow \boxed{\theta = -0.796}$





- $\vec{D}_1$ Components $D_1 = 620km$

$D_{1x} = D_1 \cos 0° = \boxed{620km}$

$D_{1y} = D_1 \sin 0° = \boxed{0km}$

- $\vec{D}_2$ Components

$D_{2x} = +D_2 \cos 45° = (440)(0.707) = \boxed{311 km}$

$D_{2y} = -D_2 \sin 45° = -(440)(0.707) = \boxed{-311 km}$

- $\vec{D}_3$ Components

$D_{3x} = -D_3 \cos 53° = -(550)(0.602) = \boxed{-331 km}$

$D_{3y} = -D_3 \sin 53° = -(550)(0.799) = \boxed{-439 km}$

Now add 'x' components and 'y' components

$D_x = D_{1x} + D_{2x} + D_{3x}$

$D_x = 620 + 311 - 311 = \boxed{600 km}$

$D_y = D_{1y} + D_{2y} + D_{3y}$

$D_y = 0 km - 311 - 439 = \boxed{-750 km}$

Now magnitude and direction

$D_R = \sqrt{D_x^2 + D_y^2} = \sqrt{(600)^2 + (-750)^2} = \boxed{960 km}$

and $D_y/D_x = -750/600 \Rightarrow \boxed{\theta = -51°}$

EXAMPLE 3.4 :-

- We use the components we found in exp#3.3
  $D_{1x} = 0 \quad D_{1y} = 22.0\,\text{Km}$ and $D_{2x} = 23.5\,\text{Km} \quad D_{2y} = -40.7\,\text{Km}$

We have

$$\vec{D_1} = 0\hat{i} + 22.0\,\text{km}\,\hat{j}$$

$$\vec{D_2} = 23.5\hat{i} - 40.7\,\text{km}\,\hat{j}$$

$$\vec{D} = \vec{D_1} + \vec{D_2}$$

$$= (0 + 23.5)\,\text{km}\,\hat{i} + (22.0 - 40.7)\,\text{km}\,\hat{j}$$

$$\vec{D} = 23.5\,\hat{i} - 18.7\,\hat{j}$$

- $\boxed{D_x = 23.5}$ $\boxed{D_y = 18.7}$

$$\vec{D} = \sqrt{(23.5)^2 (18.7)^2} = \boxed{30.0\ \text{km}}$$

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EXAMPLE 3.4:-

(4)

- We use the components we found in expr. 2
$D_{1x} = 0$ $D_{1y} = 22.0\,km$ and $D_{2x} = 23.5\,km$ $D_{2y} = -40.7\,km$

We have

$$\vec{D_1} = 0\hat{i} + 22.0\,km\,\hat{j}$$

$$\vec{D_2} = 23.5\hat{i} - 40.7\,km\,\hat{j}$$

$$\vec{D} = \vec{D_1} + \vec{D_2}$$

$$= (0 + 23.5)\,km\,\hat{i} + (22.0 - 40.7)\,km\,\hat{j}$$

$$\vec{D} = 23.5\hat{i} - 18.7\hat{j}$$

- $\boxed{D_x = 23.5}$ $\boxed{D_y = 18.7}$

$$\vec{D} = \sqrt{(23.5)^2 (18.7)^2} = \boxed{30.0\ km}$$

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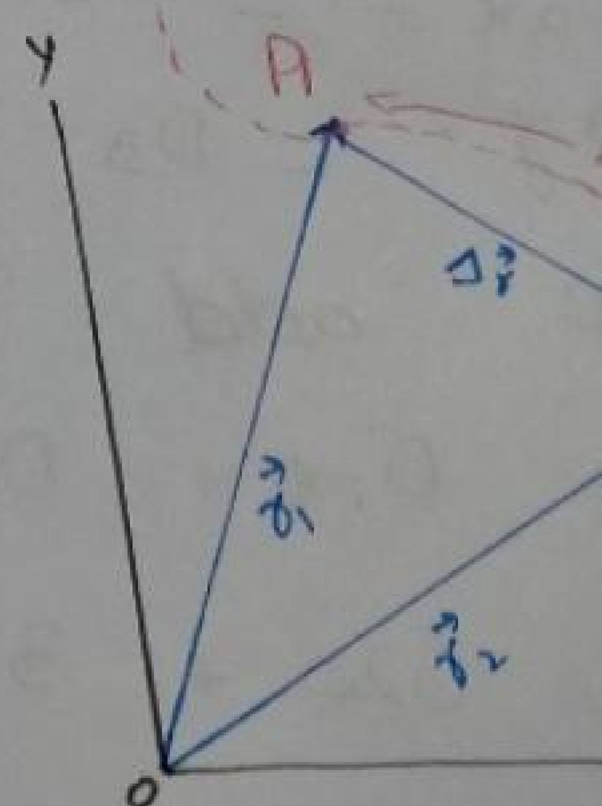
Vectors Kinematics :-



$$\Delta\vec{r} = \vec{r_2} - \vec{r_1} \rightarrow (a)$$

$$\Delta t = t_2 - t_1 \rightarrow (b)$$

In unit vector notation

$$\vec{r_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \rightarrow (c)$$

x_1, y_1, z_1 coordinate of point P_1 Similarl

$$\vec{r_2} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \rightarrow (d)$$

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

- If the motion along the x-axis, then $y_2 - y_1 = 0$, $z_2 - z_1 = 0$ and magnitude of displacement

$\Delta \vec{r} = x_2 - x_1$

- The average velocity vector over the time interval $\Delta t = t_2 - t_1$

average velocity $= \dfrac{\Delta \vec{r}}{\Delta t} \rightarrow$ (f)

- Let us consider shorter and shorter intervals let Δt approach zero, so distance points P_2 and P_1 also approches zero.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \qquad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \qquad \rightarrow (g)$$

where $v_x = dx/dt$, $v_y = dy/dt$, $v_z = dz/dt$

Average acceleration $= \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$.

Instantaneous acceleration vector.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \qquad \rightarrow (h)$$

where $a_x = \frac{dv_x}{dt}$

$v_x = \frac{dx}{dt}$ then $\frac{dv_x}{dt} = \frac{d^2x}{dt^2}$

$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k} \rightarrow (i)$$

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Example 3.5:-

The position of particle as a function of time

$$\vec{r} = [(5.0\,m/s)t + (6.0\,m/s)t^2]\hat{i} + [(7.0\,m) - (3.0\,m/s^3)t^3]\hat{j}$$

$t_1 = 2.0s$ $t_2 = 3.0s$

(a) At $t_1 = 2.0s$

$$\vec{r}_1 = [(5.0)(2.0) + (6.0)(2.0)^2]\hat{i} + [(7.0) - (3.0)(2.0)^3]\hat{j}$$

$$\boxed{\vec{r}_1 = (34m)\hat{i} - (17m)\hat{j}}$$

Similarly at $t_2 = 3.0s$

$$r_2 = (15m + 54m)\hat{i} + (17.0m - 81m)\hat{j} = (69m)\hat{i} - (74m)$$

Thus $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$$= (69m - 34m)\hat{i} + (-74m + 17m)\hat{j}$$

$$\Delta\vec{r} = (35m)\hat{i} - (57m)\hat{j}$$

$$\boxed{\Delta x = 35\,m} \qquad \boxed{\Delta y = -57m}$$

Now to find velocity we take derivative of the given $\vec{r}$ with respect to time.

$\frac{d(t^2)}{dt} = 2t$ $\qquad$ $\frac{d(t^3)}{dt} = 3t^2$ $\qquad$ 7

$$v = \frac{d\vec{r}}{dt} = [5.0\,m/s + (12\,m/s^2)t]\hat{i} + [0 - (9.0\,m/s^3)t^2]\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = (12\,m/s^2)\hat{i} - (18\,m/s^3)t\,\hat{j}$$

Thus $a_x = 12\,m/s^2$ is constant
but $a_y = -(18\,m/s^3)t$ depends linearly on time
increasing in magnitude with time in the
negative y- direaction

(c) we substitute $t = 3.0\,s$ into the equation
we just derived for $\vec{v}$ and $\vec{a}$

$$\vec{v} = (5.0\,m/s + 36\,m/s)\hat{i} - (81\,m/s)\hat{j}$$

$$\vec{v} = (41\,m/s)\hat{i} - (81\,m/s)\hat{j}$$

$$\vec{a} = (12\,m/s^2)\hat{i} - (54\,m/s^2)\hat{j}$$

Their magnitude at $t = 3.0s$

$$v = \sqrt{(41\,m/s)^2 + (81\,m/s)^2} = 91\,ms^{-1}$$

$$\boxed{v = 91\,m/s}$$

Now

$$\vec{a} = \sqrt{(12\,m/s^2)^2 + (54\,m/s^2)^2}$$

$$\boxed{\vec{a} = 55\,m/s^2}$$