

Latin square design

The Latin square design is for a situation in which there are two extraneous sources of variation. If the rows and columns of a square are thought of as levels of the the two extraneous variables, then in a Latin square each treatment *appears exactly once in each row and column.*

Example *Methods in Behavioral Research*,
p. 317

Researchers studied the effect of background music on production of workers performing a complex operation in rug manufacture. Four music programs (A, B, C, D) and one no-music program (E) were presented to the first and second shifts in the Latin square design shown on the next page.

The extraneous variables in this study were “days of week” and “week.”

Average Hourly Production in a Day

Weeks	Days					
M	T	W	T	F		
1	A	B	C	D	E	
	133	139	140	140	145	
2	B	C	D	E	A	
	136	141	143	146	139	
3	C	A	E	B	D	
	140	138	142	139	139	
4	D	E	A	C	B	
	129	132	137	136	140	
5	E	D	B	A	C	
	132	144	143	142	142	

Squares of sizes 5 to 9 are generally used. For $t \leq 4$, there are too few error degrees of freedom. The opportunity to use large squares does not often arise. (Number of treatments is usually not very large.)

There are $t!(t - 1)!$ distinct Latin squares for t treatments. One chooses one of these randomly in a particular experiment. This may be accomplished by writing down any legitimate Latin square and then randomly permuting rows and columns.

Cochran and Cox, *Experimental Designs*, pp. 145-146, give Latin squares up to 12×12 .

Analysis of a Latin square design

Parametric model:

$$X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijk},$$

where each of i , j and k ranges from 1 to t .

α_i : effect due to treatment i

β_j : effect due to row j

γ_k : effect due to column k

Usual assumptions about the error terms ϵ_{ijk} .

Sums of squares

SS_T , SS_R and SS_C are the treatment, row and column sums of squares, respectively.

$$SS_T = t \sum_i (\bar{X}_{i..} - \bar{X})^2$$

$$SS_R = t \sum_j (\bar{X}_{.j.} - \bar{X})^2$$

$$SS_C = t \sum_k (\bar{X}_{..k} - \bar{X})^2$$

$$TSS = SS_T + SS_R + SS_C + SS_E$$

ANOVA Table for Latin Square

Source of variation	SS	df	MS	F
Treatments	SS_T	$t - 1$	MS_T	F_T
Rows	SS_R	$t - 1$	MS_R	F_R
Columns	SS_C	$t - 1$	MS_C	F_C
Error	SS_E	$t^2 - 3t + 2$	MS_E	
Total	TSS	$t^2 - 1$		

ANOVA for Latin Square in SPSS

The procedure is basically the same as for the RBD. Use *Analyze* → *Descriptive Statistics* → *Explore* to obtain descriptive statistics.

To do the ANOVA for a Latin square:

- Choose *Analyze* → *General Linear Model* → *Univariate*.
- Move your response variable into the “Dependent Variable” box, and move the treatment, row and column factors into the “Fixed Factor(s)” box.
- Click *Model* and then the *Custom* button. Move the treatment factor and the row and column factors into the model box.
- Click *Continue* and then *OK*.

Latin square example

Four machines are to be tested to see whether they differ significantly in their ability to produce a manufactured part. Different operators and different time periods in the work day are known to have an effect on production.

A Latin square design is used in which 4 operators are “columns” and 4 time periods are “rows.” Machines are assigned at random to the 16 cells of the square with the restriction that each machine is used only once by each operator and in each time period. The following Latin square was obtained.

		Operator			
		1	2	3	4
Time Period	1	C	D	A	B
	2	D	A	B	C
	3	B	C	D	A
	4	A	B	C	D

Advantages of Latin square

1. Greater power than the RBD when there are two external sources of variation.
2. Easy to analyze.

Disadvantages

1. The number of treatments, rows and columns must be the same.
2. Squares smaller than 5×5 are not practical because of the small number of degrees of freedom for error.
3. The effect of each treatment must be approximately the same across rows and columns.