PROPERTIES OF EXPONENTS

1. $a^n a^m = a^{n+m}$ Example : $a^{-9}a^4 = a^{-9+4} = a^{-5}$

2. $\left(a^n\right)^m = a^{nm}$ Example : $\left(a^7\right)^3 = a^{(7)(3)} = a^{21}$

3. $\dfrac{a^n}{a^m} = \begin{cases} a^{n-m} \\ \dfrac{1}{a^{m-n}} \end{cases}, \quad a \neq 0$ Example : $\dfrac{a^4}{a^{11}} = a^{4-11} = a^{-7}$, $\dfrac{a^4}{a^{11}} = \dfrac{1}{a^{11-4}} = \dfrac{1}{a^7} = a^{-7}$

4. $(ab)^n = a^n b^n$ Example : $(ab)^{-4} = a^{-4}b^{-4}$

5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, \ b \neq 0$ Example : $\left(\dfrac{a}{b}\right)^8 = \dfrac{a^8}{b^8}$

6. $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n = \dfrac{b^n}{a^n}$ Example : $\left(\dfrac{a}{b}\right)^{-10} = \left(\dfrac{b}{a}\right)^{10} = \dfrac{b^{10}}{a^{10}}$

7. $(ab)^{-n} = \dfrac{1}{(ab)^n}$ Example : $(ab)^{-20} = \dfrac{1}{(ab)^{20}}$

8. $\dfrac{1}{a^{-n}} = a^n$ Example : $\dfrac{1}{a^{-2}} = a^2$

9. $\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}$ Example : $\dfrac{a^{-6}}{b^{-17}} = \dfrac{b^{17}}{a^6}$

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10. $\left(a^n b^m\right)^k = a^{nk} b^{mk}$ Example : $\left(a^4 b^{-9}\right)^3 = a^{(4)(3)} b^{(-9)(3)} = a^{12} b^{-27}$

11. $\left(\dfrac{a^n}{b^m}\right)^k = \dfrac{a^{nk}}{b^{mk}}$ Example : $\left(\dfrac{a^6}{b^5}\right)^2 = \dfrac{a^{(6)(2)}}{b^{(5)(2)}} = \dfrac{a^{12}}{b^{10}}$

12 $b^{\frac{m}{n}} = \left(b^{\frac{1}{n}}\right)^m$ OR $b^{\frac{m}{n}} = \left(b^m\right)^{\frac{1}{n}}$

Problem 1:

$$\frac{5x^{-1}y^{-4}}{\left(3y^{5}\right)^{-2}x^{9}}=\frac{5\left(3y^{5}\right)^{2}}{xy^{4}x^{9}}=\frac{5(9)y^{10}}{xy^{4}x^{9}}=\frac{45y^{6}}{x^{10}}$$

Problem 2:

$$\left(\frac{24a^{3}b^{-8}}{6a^{-5}b}\right)^{-2}=\left(\frac{4a^{3}a^{5}}{b^{8}b}\right)^{-2}=\left(\frac{4a^{8}}{b^{9}}\right)^{-2}$$

Problem 3:

$$\left(\frac{x^{2}y^{-\frac{2}{3}}}{x^{-\frac{1}{2}}y^{-3}}\right)^{-\frac{1}{7}}=\left(\frac{x^{2}x^{\frac{1}{2}}y^{3}}{y^{\frac{2}{3}}}\right)^{-\frac{1}{7}}=\left(\frac{x^{2+\frac{1}{2}}y^{3-\frac{2}{3}}}{1}\right)^{-\frac{1}{7}}=\left(x^{\frac{5}{2}}y^{\frac{7}{3}}\right)^{-\frac{1}{7}}$$

PROPERTIES OF RADICALS

If n is a positive integer greater than 1 and both a and b are positive real numbers then,

1. $\sqrt[n]{a^n} = a$
2. $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$
3. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

RATIONALIZING THE DENOMINATOR

Problem 1:

$$\frac{4}{\sqrt{x}} = \frac{4}{\sqrt{x}}\frac{\sqrt{x}}{\sqrt{x}} = \frac{4\sqrt{x}}{\sqrt{x^2}} = \frac{4\sqrt{x}}{x}$$

Problem 2:

$$\frac{1}{3-\sqrt{x}} = \frac{1}{\left(3-\sqrt{x}\right)}\frac{3+\sqrt{x}}{\left(3+\sqrt{x}\right)} = \frac{3+\sqrt{x}}{\left(3-\sqrt{x}\right)\left(3+\sqrt{x}\right)} = \frac{3+\sqrt{x}}{9-x}$$

Problem 3:

$$\frac{5}{4\sqrt{x}+\sqrt{3}} = \frac{5}{\left(4\sqrt{x}+\sqrt{3}\right)}\frac{\left(4\sqrt{x}-\sqrt{3}\right)}{\left(4\sqrt{x}-\sqrt{3}\right)} = \frac{5\left(4\sqrt{x}-\sqrt{3}\right)}{\left(4\sqrt{x}+\sqrt{3}\right)\left(4\sqrt{x}-\sqrt{3}\right)} = \frac{5\left(4\sqrt{x}-\sqrt{3}\right)}{16x-3}$$

POLYNOMIALS

*Polynomial comes from **poly-** (meaning **"many"**) and **-nomial** (in this case meaning **"term"**) ... so it says "many terms"*

*A **polynomial** is an expression consisting of variables (also called indeterminates) and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponentiation of variables*

General expression is: $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \ldots.. + a_n$

Monomial: $3x^2y$ *Binomial:* $5x - 1$ *Trinomial:* $3x + 5y^2 - 3$

These are polynomials

- **3x**
- **x − 2**
- $-6y^2 - (79)x$
- $3xyz + 3xy^2z - 0.1xz - 200y + 0.5$
- $512v^5 + 99w^5$
- **5**

These are not polynomials

- $3xy^{-2}$ is not, because the exponent is "-2" (exponents can only be 0,1,2,...)
- **2/(x+2)** is not, because dividing by a variable is not allowed
- **1/x** is not either
- **√x** is not, because the exponent is "½"

Degree of Polynomial

1. *The degree of a polynomial in one variable is the largest exponent in the polynomial.*

$5x^{12} - 2x^6 + x^5 - 198x + 1$	degree : 12
$x^4 - x^3 + x^2 - x + 1$	degree : 4
$56x^{23}$	degree : 23
$5x - 7$	degree : 1
-8	degree : 0

2. *The degree of each term in a polynomial in two variables is the sum of the exponents in each term and the **degree** of the polynomial is the largest such sum.*

$x^2y - 6x^3y^{12} + 10x^2 - 7y + 1$	degree : 15
$6x^4 + 8y^4 - xy^2$	degree : 4
$x^4y^2 - x^3y^3 - xy + x^4$	degree : 6
$6x^{14} - 10y^3 + 3x - 11y$	degree : 14

FACTORING POLYNOMIALS

(A) Greatest common factor

1. $x^3y^2+3x^4y+5x^5y^3=x^3y\left(y+3x+5x^2y^2\right)$

2. $3x^6-9x^2+3x=3x\left(x^5-3x+1\right)$

3. $9x^2(2x+7)-12x(2x+7)=3x(2x+7)(3x-4)$

(B) By Grouping

1. $x^5-3x^3-2x^2+6=x^3\left(x^2-3\right)-2\left(x^2-3\right)=\left(x^2-3\right)\left(x^3-2\right)$

2. $x^5+x-2x^4-2=\left(x^4+1\right)(x-2)$

(C) Factoing Quadratic Polynomials

(a) $x^2+2x-15$

(b) $x^2-10x+24$

(c) x^2+6x+9

(d) x^2+5x+1

(e) $3x^2+2x-8$

(f) $5x^2-17x+6$

(D) Special Forms

Important Formulas

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$x^2 - 20x + 100 = (x-10)^2$$

$$25x^2 - 9 = (5x+3)(5x-3)$$

$$8x^3 + 1 = (2x+1)(4x^2 - 2x + 1)$$