...range's Eq. of Motion In Terms of Quasi Co-ordinates :-

...oof :-

Consider a dynamical system of ... particles whose configuration at any ...me 't' by using Quasi co-ordinates.

By D'Alemberts principle,

$$(F_i - m_i \ddot{r}_i)\, \delta r_i = 0$$

$$(m_i \ddot{r}_i - F_i)\, \delta r_i = 0 \qquad (1)$$

...here $r_i = r_i(\pi_1, \pi_2, \ldots, \pi_s, t)$

$$\delta r_i = \frac{\partial r_i}{\partial \pi_s}\, \delta \pi_s$$

So eq. (1) becomes

$$\left(m_i \ddot{r}_i - F_i\right) \frac{\partial r_i}{\partial \pi_s} \delta \pi_s = 0$$

$$\left(m_i \ddot{r}_i \frac{\partial r_i}{\partial \pi_s} - F_i \frac{\partial r_i}{\partial \pi_s}\right) \delta \pi_s = 0 \quad \text{--- (2)}$$

Now, $$\frac{dr_i}{dt} = \frac{\partial r_i}{\partial \pi_s} \cdot \frac{d\pi_s}{dt} + \frac{\partial r_i}{\partial t} \quad \text{--- (3)}$$

$$\dot{r}_i = \frac{\partial r_i}{\partial \pi_s} \dot{\pi}_s + \frac{\partial r_i}{\partial t} \quad \text{--- (4)}$$

$$\therefore \frac{\partial \dot{r}_i}{\partial \dot{\pi}_s} = \frac{\partial r_i}{\partial \pi_s} \quad \text{--- (5)}$$

so eq (4) becomes (Taking $\frac{\partial}{\partial \pi_s}$ on both sides)

$$\frac{d}{dt}\left(\frac{\partial r_i}{\partial \pi_s}\right) = \frac{\partial}{\partial \pi_s}\left(\frac{\partial r_i}{\partial \pi_s}\right)\frac{d\pi_s}{dt} + \frac{\partial}{\partial t}\left(\frac{\partial r_i}{\partial \pi_s}\right)$$

$$= \frac{\partial^2 r_i}{\partial \pi_s^2} \dot{\pi}_s + \frac{\partial^2 r_i}{\partial \pi_s \partial t}$$

$$= \frac{\partial}{\partial \pi_s}\left[\frac{\partial r_i}{\partial \pi_s} \dot{\pi}_s + \frac{\partial r_i}{\partial t}\right]$$

[Using $$\frac{d}{dt}\left(\frac{\partial r_i}{\partial \pi_s}\right) = \frac{\partial}{\partial \pi_s}(\dot{r}_i)$$

$$= \frac{\partial}{\partial \pi_s}\left(\frac{dr_i}{dt}\right) \quad \text{--- (6)}]$$

Now, using

$$\frac{d}{dt}\left(\frac{\partial r_i}{\partial \pi_s} \dot{r}_i\right) = \ddot{r}_i \frac{\partial r_i}{\partial \pi_s} + \dot{r}_i \frac{d}{dt}\left(\frac{\partial r_i}{\partial \pi_s}\right)$$

$$\ddot{r}_i \frac{\partial r_i}{\partial \pi_s} = \frac{d}{dt}\left(\dot{r}_i \frac{\partial \dot{r}_i}{\partial \dot{\pi}_s}\right) - \dot{r}_i \frac{\partial \dot{r}_i}{\partial \pi_s}$$

(By (5) & (6))

$$\ddot{r}_i \frac{\partial r_i}{\partial \pi_s} = m_i \frac{d}{dt}\left(\dot{r}_i \frac{\partial \dot{r}_i}{\partial \dot{\pi}_s}\right) - m_i \dot{r}_i \frac{\partial \dot{r}_i}{\partial \pi_s} \quad \text{—(}$$

$$T = \frac{1}{2} m_i \dot{r}_i^2$$

$$= \dot{r}_i m_i \frac{\partial \dot{r}_i}{\partial \pi_s}$$

$$\frac{\partial T}{\partial \dot{\pi}_s} = m_i \dot{r}_i \frac{\partial \dot{r}_i}{\partial \dot{\pi}_s}$$

Using in eq (7),

$$\ddot{r}_i \frac{\partial r_i}{\partial \pi_s} = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\pi}_s}\right) - \frac{\partial T}{\partial \pi_s} \quad \text{—(8}$$

&

$$F_i \frac{\partial r_i}{\partial \pi_s} = Q_s$$

$$\dot{\pi}_s = \frac{d\pi_s}{dt} = \omega_s \quad \text{—(}$$

Using eq (8) & (9) in (2),

$$\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \omega_s}\right) - \frac{\partial T}{\partial \pi_s} - Q_s\right] \delta \pi_s = 0$$

For conservative system,

$$Q_s = -\frac{\partial V}{\partial \pi_s}$$

$$\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \omega_s}\right) - \frac{\partial T}{\partial \pi_s} + \frac{\partial V}{\partial \pi_s}\right] \delta \pi_s = 0$$

Now,

$$L = T - V$$

$$\frac{\partial L}{\partial \omega_s} = \frac{\partial T}{\partial \omega_s}$$

eq (10) becomes

$$\left[\frac{d}{dt}\left(\frac{\partial L}{\partial \omega_s}\right) - \frac{\partial}{\partial \pi_s}(T - V)\right]\delta \pi_s = 0$$

$$\left(\frac{d}{dt}\left(\frac{\partial L}{\partial \omega_s}\right) - \frac{\partial L}{\partial \pi_s}\right)\delta \pi_s = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \omega_s}\right) - \frac{\partial L}{\partial \pi_s} = 0$$

As Req.