For dynamical system, the Hamilton
defined as

$$H = \sum_{p=1}^{n} \dot{q}_p p_p - L$$

$$\Rightarrow L = \dot{q}_p p_p - H$$

the Lagrangian is invariant under a pt.
formation

$$\dot{q}_p p_p - H = \dot{Q}_p P_p - K$$

$$p_p \, dq_p - H dt = P_p \, d\bar{Q}_p - K dt \quad (1)$$

$$q_p = \frac{\partial q_p}{\partial Q_j} dQ_j + \frac{\partial q_p}{\partial t} dt$$

in eq (1), we get

$$\frac{\partial q_p}{\partial Q_j} dQ_j + p_p \frac{\partial q_p}{\partial t} dt - H dt = P_p d\bar{Q}_p - K dt$$

$$= P_j d\bar{Q}_j - K dt$$

comparing!

$$P_j = p_p \frac{\partial q_p}{\partial Q_j}$$

$$-K = -H + p_p \frac{\partial q_p}{\partial t}$$

$$K = H - p_p \frac{\partial q_p}{\partial t} \quad \leftarrow \text{(Hamilton in term of pt. transf.)}$$

the pt. transf. from q to Q
independent of time 't' then $K = H$

ample:-

A dynamical system (D.S) is

described by the generalized co-ordinates by the Hamiltonian fun. ~~$H(q, p; t)$~~

$H(q, p; t) = ap^2 + bp(q+t)^2$

where 'a & b' are consts. A pt. transf. is made a new generalized co-ordinate $Q = q + t$

Find Hamiltonian fun. $K = K(Q, P; t)$

Solution:

As Hamilton's in terms of point transformation is given by

$$K = H - p_p \frac{\partial q_p}{\partial t} \quad (1)$$

As

$$H(q, p, t) = ap^2 + bp(q+t)^2 \quad (2)$$

As $$P_j = p_p \frac{\partial q_p}{\partial Q_j} = p_p(+1)$$
$$= p_p$$

So eq. (2) becomes

$$H = a(P_j)^2 + b(P_j)(Q)^2 \cancel{t} \qquad \because q + t = Q$$

$$H = aP_j^2 + bP_j Q^2$$

so eq. (1) becomes,

$$K = aP_j^2 + bP_j Q^2 - P_j \frac{\partial q_p}{\partial t}$$

$$K = aP_j^2 + bQ^2 P_j - P_j(-1)$$

So

$$K = aP_j^2 + bQ^2P_j + P_j$$

As Req.