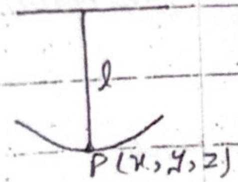
14-02-2011

nalytic Dynamics :-
It is the relationship b/w
tion of bodies and its causes, namely -us
ces acting on the bodies.
e analytic dynamics is body movement
and force action.

onstraints :-
Constraint mean restriction &
ts motion is called constrainted motion. nt

e.g.
Consider a billiard ball on the table. Its motion is restricted by the boundries of the table and it moves on the surface of the table. Its motion is called constrainted or restricted motion. Simple pendulum is an nt example of restricted motion.

2:- A simple pendulum moves in a vertical plane. If $P(x, y, z)$ are co-ordinate of bob then $z = c$ & $x^2 + y^2 = l^2$
which are restriction over constraints of a bob.



Generalized Co-ordinates :- (x, y, z) these are cartesian co-ordinates having dimensions of the length "L". A set of co-ordinates denoted by q_i $(i = 1, 2, \ldots\ldots n)$ which specify configuration (position) of system are called generalized co-ordinates.

$$q_i = (q_1, q_2, \ldots\ldots, q_n)$$

$$(x, y, z) = (q_1, q_2, q_3)$$

Generalized Velocities :-

$\dot{q}_i = \frac{dq_i}{dt}$ represents generalized velocities of the corresponding generalized co-ordinates.

Workless Constraint :- The constraint (eq.s.) due to which work done is zero is called workless constraint.

Holonomic Constraint :- If the conditions due to which constraint can be expressed as a fun.

$$f(q_1, q_2, \ldots\ldots, q_n, t) = 0$$

then the constraint is said to be holonomic otherwise non-holonomic.

$$f(q_1, q_2, \ldots\ldots, q_n, t) < 0 \quad \text{(within Boundary)}$$

Types of constraint:-

There are two main types of constraint.

1.- Workless constraint.
2.- Holonomic constraint

which are discuss above.

Other Types of Constraint:-

1:- Scleronomous:- The constraint which do not depend upon time are called scleronomous or time - independent constraint.

2:- Rhonomous:- The constraint which depends upon time are called rhonomous or time - dependent constraint.

Virtual Displacement:-

A hypothetical displacement of a system in which constraint forces remains unchange during (so small) time is called virtual displacement i.e

$$\delta r_i \simeq 0$$

Principle of Virtual Work (PVW):-

Necessary & sufficient conditions for a system of particles to be in equilibrium is that "~~Workdone~~ Virtual workdone by the applied forces is zero".

$$\sum \vec{F}_i^{(ext)} \cdot \delta \vec{r}_i = 0$$

or

$$\sum \vec{F}_i^{(a)} \cdot \delta \vec{r}_i = 0$$

where $\vec{F}_i^{(a)}$ = Applied forces on a particle

$\delta \vec{r}_i$ = Virtual disp. of the particle.

$\vec{F}_i^{(a)}$, $\vec{F}_i^{(ext)}$, $\vec{F}_i$ (ext) represents the same meaning i.e applied force. So

D' Alembert's Principle :- As we know that

$$\vec{F} = m\vec{a} = m\dot{\vec{v}} = \dot{\vec{p}}$$

so $\dot{\vec{p}} = \vec{F}_i = \vec{F}_{i,ext} + \vec{f}_i$

$$\vec{F}_{i,ext} + \vec{f}_i - \dot{\vec{p}} = 0$$

By the principle of virtual work.

$$\Sigma \vec{F}_i \;\delta\vec{r}_i = 0$$

$$\Sigma (\vec{F}_{i,ext} + \vec{f}_i - \dot{\vec{p}}) \cdot \delta\vec{r}_i = 0$$

$$\because \vec{f}_i \cdot \delta\vec{r}_i = 0$$

So

$$\Sigma (\vec{F}_{i,ext} - \dot{\vec{p}}) \cdot \delta\vec{r}_i = 0$$

which is called D' Alemberts principle.

Imp. Note :-

⇐ اگر کسی Body پر force لگائی جائے اور اس کا disp بہت ہی کم ہو تو اسے ہم virtual displacement کہتے ہیں i.e $d \approx 0$

⇒ Workdone of constraint $= 0$

⇒ Internal forces are constraint forces

$\therefore$ so $\vec{f}_i \cdot \delta\vec{r}_i = 0$. where $\vec{f}_i$ is the internal force.

⇒ We mostly use Newton's 2nd law for motion of any particle.

⇒ The machine in which pully system exist is called "Atwoods machine".