Chapter 9

Electroweak Interactions

Joining the electromagnetic force that acts between leptons and quarks there is another important interaction between the same participants — the so-called weak interaction. This is the phenomenon responsible for the fact that a neutron is not stable, but is subject to "beta decay" into a proton, an electron, and a neutrino:

$$n \to p + e^- + \nu_e \,.$$

All reactions that are due to the weak interaction can be attributed to one of the two categories: either to those where the electric charge changes — such as in "beta decay", where a neutral neutron changes into a charged proton (as we just saw above), or to those where there is no change in electric charge, as in neutrino scattering

$$\nu + p \to \nu + p$$

The first category is called a charged-current weak interaction, and the second a neutral-current one. One thing they have in common is the participation of four particles, four fermions. Sometimes one fermion changes into three fermions, as in neutron beta decay, and sometimes two fermions interact in a process where they may form a different fermion pair, as in

$$\nu_\mu + n \to \mu^- + p$$

An identifying feature of all weak interaction processes is that they are characterized by a constant that indicates their strength. This parameter, called the Fermi constant after the Italian physicist Enrico Fermi, has been experimentally determined with great precision. It is a small quantity of about 1.16×10^{-5} GeV^{-2}. Note that it is not a dimensionless parameter such as the "fine structure constant"; rather, it is of dimension inverse energy-squared. Its small numeric value is responsible for the "weakness" of the weak nuclear interaction where we compare its strength with electromagnetism.

Still, theoretical physicists do not like to deal with a dimensional quantity. The implication is that we cannot come up with a consistent theory which is able to describe quantized phenomena. The reason is that it carries a characteristic energy scale of 1.16×10^{-5} GeV^{-2} or (0.294 GeV)$^{-2}$. This means the "weak interaction" is marked by an energy of 294 GeV. And precision studies of this phenomenon tell us that our picture of the weak interaction must break down at energies large in comparison with this energy scale — that something new must come up.

The simplest new feature would be to imagine the weak interaction between four particles to be similar to the electromagnetic interactions between two electrons by the exchange of virtual particles. Let us take the "weak" decay of the neutron and suppose that the proton can be changed into a neutron by emitting a virtual particle which we will call a W^-. It will be very short-lived, being just a virtual entity and will then decay into an electron and an antineutrino.

We can see this process to be quite analogous to the electromagnetic interaction between two electrons, which happens by way of the exchange of a virtual photon. Just replace the virtual photon by a W-boson — but recall that the W is electrically charged, whereas the photon is electrically neutral.

We can describe the processes involving neutral currents quite analogously — we just need a new neutral particle that will be exchanged among the relative fermions; let us call this particle the Z-boson. If, for instance, matter scatters a neutrino, the neutrino remains after having exchanged a Z-boson in a weak interaction process.

Note that the description of weak interaction processes in terms of the exchange of virtual particles simplifies matters considerably: the elementary interaction is no longer the process of an interaction among four fermions, as formulated by Fermi; rather, it is now the interaction between two fermions and the virtual boson. That puts the weak interaction into a formal analogy with electromagnetism. Now we can characterize this interaction by a simple dimensionless number.

But how can we discuss the Fermi constant in the framework of this theory? It is easy to see that the W-boson and the Z-boson have to be massive. The mass of the W-boson is closely related to the Fermi constant: the critical energy of 294 GeV which we mentioned above is simply the mass of the W-boson divided by the constant of interaction between fermions and W-boson.

But before we turn to the masses, we have to consider the strength of the interaction proper. Given that the mediation of the weak interaction is carried by the weak bosons, quite analogous to QCD, we have to answer the question: is the analogy between weak interaction and electromagnetic interaction just a formality, or is there a direct connection between them? Such a connection could exist if, for instance, the weak interaction were not really weak, but showed up as such only at relatively low energies simply because of the large masses of W-bosons and Z-bosons. If, on the other hand, we assume that the elementary interaction of fermions and W-bosons is equal to the electromagnetic interaction of electrons and photons, we simply wind up with a problem. In this case, the mass of the W-boson has to be 37 GeV — a value, which has long been excluded as being way too light.

At this juncture, we have to mention another important difference between electromagnetic and weak interactions as far as space reflection is concerned: Take an arbitrary fermion — say, an electron or a quark. We can construct this fermion from a right-handed and a left-handed fermion. Now, a right-handed fermion is a particle that has its spin pointed in the direction of its momentum — we might compare it with a screw that turns to the right — whereas a left-handed fermion has its spin pointing in the direction opposite to its momentum.

Now let us look at a reflection in a mirror: a left-handed fermion changes into a right-handed fermion. If Nature were invariant under space reflection, left-handed and right-handed fermions would have to show exactly the same interactions — and this in fact is the case for electromagnetic processes, but not for weak ones. As early as in 1956, experimental evidence was found that the weak interaction violates space reflection symmetry; this discovery entered into the annals of physics as "parity violation". It was first observed in the weak decay of cobalt into nickel.

It was soon found that parity violation by leptons and quarks is quite easily described as long as we limit ourselves to interactions of fermions with W-bosons: only left-handed leptons and quarks interact with W-boson; right-handed ones do not. That means the interaction of W-bosons with fermions is quite different from that of the fermions with photons. To this day, we have no notion why Nature chooses to be left-handed when interacting weakly. We have to accept this fact without understanding it.

We mentioned above that W-bosons can change an electron into a neutrino, and vice versa. This fact reminds us of a similar connection in chromodynamics: in QCD, gluons are capable of changing one "color" of quarks into another one — say, making a green quark out of a red one. This works because color transformations act like "charges" of the color group SU(3) — the gauge group of strong interaction theory. Do we have an implication here that W-bosons are gauge bosons in analogy to the gluons? What gauge group would that point to?

This last question can be readily answered: as far as the weak interaction is concerned, leptons and quarks always act as doublets. The left-handed electron neutrinos ν_e and electrons, the up and down quarks, etc.

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \qquad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \text{etc.}$$

or simply all the doublets of left-handed fermions. The implication is that we have to consider all transformations concerning doublets — implying the group SU(2), in contrast to the group SU(3) in strong interactions. The latter concerns three charges. Two of the latter can be identified with the weak charges we have introduced — the charges that change the upper component of a doublet into a lower one, or vice versa — say, $u_L \rightarrow d_L$ and vice versa. These "charges" change the electric charge by one unit. In addition, there is a third "charge", an electrically neutral one. How should this be interpreted? Is it simply a "weak charge" we need for a formal description of a neutral current interaction, or are matters more complicated than that?

Before we address this question, let us deal with another problem: We aim at interpreting the W and Z bosons as the gauge bosons in a gauge theory of the weak interaction, which will be analogous to QCD. But: the gluons of QCD are massless like photons, whereas W's and Z's are quite massive as we have seen, and that difference presents a severe problem. It is impossible simply to enter the massive aspect of gauge bosons into our gauge theory without, as a consequence, coming up with abstruse results.

Fortunately, there is an interesting possibility to hold on to the massive quality of the gauge bosons without introducing it explicitly. It was discussed by a number of theorists as early as in 1965, most explicitly by Peter Higgs. The ruse that does the trick is the introduction of scalar bosons in addition to the vector bosons. They are postulated to interact with the gauge bosons, with the consequence, among other things — that this interaction "gives" the gauge bosons a mass. This process has an added implication: it violates the prevalent gauge symmetry — leading to the concept of "spontaneous symmetry breaking". M. Veltman and G. 't Hooft were able to show, in 1971, that the introduction of mass parameters by means of this mechanism does not lead to senseless results. Their brilliant interpretation was recognized by the Nobel Committee in 1999.

The simplest trick for the construction of a gauge theory of weak interactions implies the use of the gauge group SU(2). That makes the left-handed leptons and quarks doublets, the right-handed ones singlets that do not participate in the interaction. This scenario gives us three gauge bosons, which we take to be W^+, W^- and Z^0. All three of these have the specific property that they interact only with left-handed fermions.

In 1977, new evidence was collected on weak neutral interactions. It showed an interesting feature: This interaction, in contrast to the charged one, also interacts with the right-handed fermions. This new discovery excluded a simple SU(2) theory as we postulated above.

Now, we know that the electromagnetic interaction acts on the left-handed as well as on the right-handed fermions, similar to the neutral-current weak interaction. This observation suggests taking a look at a possibility to unify the electromagnetic and weak interactions in some way. A number of such attempts were made over quite a period of time — by Sheldon Glashow in 1962, by Abdus Salam and John Ward in 1964, by Steve Weinberg in 1967, and again by Salam in 1968. Glashow, Salam and Weinberg were honored for their work by the 1979 Nobel prize. Ever since, it has become clear that the framework of this theory does full justice to the weak interaction.

In a unified theory of electromagnetic and weak interactions — or, as we call them today, of electroweak interaction — we need to deal with a total of four gauge bosons: W^--boson, W^+-boson, Z^0-boson and the photon. For that reason, we have to extend the relevant gauge group: the simplest extension is the inclusion of an added U(1) group, leading to the gauge group SU(2) × U(1) — a gauge group consisting of the product of SU(2) and U(1).

Suppose we start from this group and generate the masses of the gauge bosons by means of spontaneous symmetry breaking. In the process, we find some notable implication. The two W-bosons obtain some mass, which initially, can be chosen arbitrarily. The two electrically neutral bosons have an interesting mass spectrum: one of them winds up slightly heavier than the W-boson, whereas the other one remains massless. So it is natural to identify the last one with the photon, whereas the heavy one is the Z^0-boson. That means we wind up with a theory that couples the weak and electromagnetic interactions directly, and establishes a close link between photon and Z-boson.

Given that electromagnetism and weak neutral current are closely connected, it appears natural that the Z-boson interacts with left-handed and right-handed fermions. To be sure, there was no way to predict the weak neutral interaction with any precision. It depends on a parameter that is not implied by the theory, but has to be measured by experiment. We have accustomed ourselves to characterize this parameter as an angle, called θ_w where the subscript w stands either for "weak" or for "Weinberg", after Steven Weinberg, whom we mentioned above. This angle tells us the strong interrelation between electromagnetic and weak interactions. For the special case with $\theta_{\mathrm{w}} = 0$, there would be no connection — but experiment shows us that, in fact, $\theta_w = 28.7$.

Our SU(2) $\times$ U(1) theory of electroweak interactions makes an important prediction: the masses of W and Z bosons are fixed once we know the angle θ_{w}. Both of these bosons were discovered in the year 1984 at the European Center for Nuclear Research, (CERN), in proton–antiproton collisions. By this time, we have precise results and find $m(W) = 80.45$ GeV and $m(Z) = 91.19$ GeV, in close similarity to the theoretical predictions.

To study the Z-boson more precisely, an electron–positron collider, called LEP, was built at the CERN laboratory, starting operations in 1989. Over the next decade, some 20 million Z-decays were observed. In this way, the parameters of the elctroweak interaction were measured with high precision, giving us a close glimpse of the mass of the Z-boson and the Weinberg angle θ_w. Remarkably, the results obtained at LEP were quite close to the predictions of electroweak theory. Many theorists expected experimental results that would differ from their predictions, but that did not happen. Nature justified electroweak symmetry expectation. The LEP research program provided a splendid justification of the SU(2) $\times$ U(1) theory.

Finally, let us mention that new results of neutrino-initiated interactions are hinting at facts that have not been expected in the framework of the simplest SU(2) $\times$ U(1) models. There, we expect neutrinos to be massless. But several experimental indications, like those obtained with the neutrino detector that has been used for years near Kamioka in Japan, "neutrino oscillations" have been observed. This means that, say, a muon neutrino radiated off by the weak decay of a charged pion, can change its identity while flying through space: it may become a tau neutrino (ν_α). Such "neutrino flavor change" is possible only if neutrinos are not massless. In that case, there is no reason why a neutrino, say, a muon neutrino (ν_μ) is a fixed-mass state, with some given mass. It might be a mixture of two or even three mass states. Given that weak decays do not permit us to observe neutrino masses due to their very small size, there is no reason why this scenario should not be accurate.

On the other hand, the different mass states will be propagated at different velocities; this implies a change of neutrino structure as a function of mass states — such that a muon neutrino may well change into a tau neutrino, only to bounce back into the muon neutrino state, and so on. Such processes have been observed, if indirectly, not only near Kamioka in Japan, but also with the neutrino facility close to Sudbury in Canada. We must conclude that neutrino flavor mixtures exist in Nature, and that the relevant mixing angles may well be large. Neutrino masses, of course, must be quite small — on a scale we do not yet know with any assurance. But indications are that neutrino masses are below 1 eV.

Chapter 10

Grand Unification

It has become quite obvious that we have witnessed a major breakthrough for modern physics as the theories of electroweak interactions (or, SU(2) $\times$ U(1)) and of the strong interaction (or, SU(3)) were developed. Their framework provides ways to describe almost all elementary particle phenomena, covering weak, electromagnetic, and strong interactions. Still, there are more observed features that are not covered. One of these is the fact that the electric charges of leptons and quarks are fixed — or, as we say today, quantized. The electric charges of the electrons and muons are -1, those of the quarks $2/3$ or $-1/3$. It appears as though a law we do not yet know forced the charges to assume these values, including the feature that the fractional charges apply to the quarks which are color triplets, whereas the electrons and muons — both color singlets — have integer charges.

It is easy to see that the SU(2) $\times$ U(1) theory will admit arbitrary charges due to its inclusion of a free parameter, the "Weinberg angle" θ_w. It would be easy to change the charges of u-quarks to $2/\pi$ instead of $2/3$, a quantization of charges can be imposed only by fixing the Weinberg angle.

There is another problem: the strength of the "strong" interaction when we compare it with the electroweak phenomena. We might ask ourselves: Is there a theory which provides a correct description of the strong and electroweak interactions including their relative strength? Notice that the chromodynamic analogue to the fine structure constant α_s at the energy $E = m(Z)$ is about 0.12 (or almost exactly 1/8), whereas the electromagnetic fine structure constant α at the same $E = m(Z)$ value amounts to 1/128, or a 16th of the previous value. A unified theory of particle interactions would have to explain that fact.

So, let us now try and build up a theory that describes the strong and electroweak interactions jointly. The theory must contain the product of three gauge groups, SU(3) × SU(2) × U(1). A unified theory of all interactions can be built up if we embed these three gauge groups into a larger group. Mathematically, this is not so hard — but we have to take care that the fermions are properly represented: the leptons have to appear as color singlets, the quarks as color triplets. This necessity puts a severe squeeze on possible groups that will satisfy this condition.

The smallest group that contains the color group SU(3) and the group SU(2) × U(1) which also doing justice to the fermions, is the group SU(5). Let us recall that the fermions of the first lepton/quark family are

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} (e^+) \begin{pmatrix} uuu \\ ddd \end{pmatrix} (\bar{u}\bar{u}\bar{u})(\bar{d}\bar{d}\bar{d}) \,,$$

where we have explicitly mentioned the colors of the quarks. The fermions of the other two families are similarly listed, so that we wind up with a total of 15 fermions per family. Let us now group the fermions into two systems

$$\begin{pmatrix} \nu_e & \vdots & \\ & & \bar{d}\bar{d}\bar{d} \\ e^- & \vdots & \end{pmatrix} , \quad \begin{pmatrix} uuu & \vdots & \\ & & \bar{u}\bar{u}\bar{u}, e^+ \\ ddd & \vdots & \end{pmatrix}$$

The first of these systems contains five fermions, the second ten. Now, it turns out that these two fermion systems are two different representations of the group SU(5); this implies that every SU(5) transformation has to follow mathematical prescriptions for the transformations of its components.

These prescriptions also determine the electric charges: the group SU(5) has a total of $5^2 - 1 = 24$ charges (note that the larger a group, the more different charges it contains: the group U(1) has just one charge, SU(2) has three, SU(3) of the color space has eight charges).

Interestingly, the charges within a group add up to zero. E.g., the electric charges of the five fermions we are considering have to cancel each other once we add them up: Since the electric charge of the neutrino is zero, this gives us a relation of the electric charges of the electron and that of the d-quark.

$$Q(e^-) = \frac{1}{3} Q(d)$$

So, the group structure imposes precisely the electric charges we observe in Nature. And, of course, the factor 3 is the number of colors the quarks may have. Analogously, we observe the electric charge of the u-quark to be $2/3$ — because is has to be just one unit larger than that of the d-quark.

The SU(5) theory also permits to calculate the "weak angle" and the chromodynamic fine structure constant for which we find:

$$\theta_w = 37.8^\circ \,, \qquad \alpha_s = 8/3\alpha \sim 1/51$$

These two values present a real problem: they do not at all agree with the values that were determined experimentally: θ_w is about 28.7° and α_s is not as small as indicated above.

There is another problem concerning the SU(5) theory: it has 24 gauge bosons, corresponding to the 24 charges. There are the eight gluons of QCD, the W^+, W^-, Z^0, and the photon, adding up to 12 gauge bosons. That leaves 12 more gauge bosons that must exist and cause new interactions hitherto unseen. Not only unseen, but they look odd; they could, e.g., transform a lepton into a quark. To be sure, that is possible in principle — given the SU(5) representations which contain quarks as well as leptons: elements of one and the same representation can always be transformed into each other by a group transformation.

One of the consequences of these new interactions implies that the proton is unstable and can decay into lighter particles — say, into a positron that takes over the positive electric charge, and a π^0 meson. Now, this kind of decay violates baryon number conservation. This decay occurs because the new interactions permit two quarks in the proton to transform into a positron and an antiquark.

In SU(5) theory, the lifetime of the proton depends on the masses of the new gauge bosons. The observed great stability of the proton implies that the mass of these particles must be enormous — at least 10^{15} GeV. Only if we accept that mass we assure a proton stability corresponding to experimental evidence indicating some 10^{31} years. Notice that this number is many orders of magnitude larger than the age of the universe which, today, is being estimated at 14 billion years (or 10^{10} years). The fact that we feel we can infer a proton lifetime much larger than the proven age of the universe is due to the observational method employed: proton decay has been searched for by looking at large amounts of protons, e.g., at many tons of water.

Given the large mass of such new gauge bosons, we are constrained to assume a possible unification of the strong, electromagnetic, and weak interactions at energies no lower than 10^{16} GeV, that is where SU(5) theory may turn effective. There is an interesting implication to the onset of a new energy scale (of 10^{16} GeV): should SU(5) theory be correct, the coupling strengths of the strong, electromagnetic, and weak interactions will all be the same at energies of more than 10^{16} GeV, since the different interactions are nothing but different manifestations of one and the same unified theory. We then have to expect that the weak mixing angle θ_w is almost precisely 38°, and the relation $\alpha_s = 8/3\alpha$ characterizes the strengths of strong and electromagnetic interactions.

Still, we have to keep in mind that our knowledge of coupling strengths is based on experiments performed at relatively low energies, say, below 10 GeV. There is no justification to equating these to coupling strengths we expect at energies of some 10^{16} GeV. We do expect, in the framework of quantum field theory, that the coupling strengths change slowly as a function of the prevailing energy scale. The QCD coupling strength diminishes with increasing energy because of the features of asymptotic freedom, whereas that of U(1) theory increases slowly.

Experiments at the LEP rings at CERN permitted a precise determination of the parameters of SU(2) × U(1) theory. The fairly precise knowledge of the relative coupling strengths thereby permits us an extrapolation to higher energies. We find that the three coupling strengths approach each other at about 10^{15} GeV; but their energy dependence does not intersect in one point, as theory would suggest.

The dependence of coupling constants as a function of energy does indicate that the idea of a unification of fundamental forces makes sense. Still, it appears that somewhere on the long path from energies accessible today to unification energy, something must be happening beyond the slow change in coupling parameters, as a result of prevailing interactions. For instance, it might well be that, at energies of about 1000 GeV, new symmetries, and thereby, new interactions, appear.

Today, a number of theorists are discussing the onset of what is called supersymmetry. Symmetries as we have known them — such as, say, isospin symmetry — group particles of one and the same spin: a symmetry transformation can, for instance, easily change a proton into a neutron, but not a spin $-1/2$ proton into a spin -0 meson. But in the framework of supersymmetry, there may be a possibility to change a fermion into a boson. There may, for instance, be a spin $1/2$ quark that will be changed into a particle with spin 0 which, of course, does not exist in the observed spectrum. So, we are implying that there be a new particle with a mass high enough so it has eluded observation. This hypothetical particle is assigned the name squark, short form for supersymmetric partner of the quark. In fact, the supersymmetric extension of our Standard Model adds a new boson to every "old" fermion, and a new fermion to every "old" boson. The supersymmetric partner of the spin 1 photon is the hypothetical spin $-1/2$ photino.

To this day, we do not know whether Nature realized supersymmetry or whether it is just the theorists' extravaganza. If it exists, there must be a critical energy scale where this new type of symmetry sets in. That could also be the energy defining the mass scale for supersymmetric particles. The prevailing assumption is that supersymmetry sets in, if at all, at about 1000 GeV.

Should supersymmetry exist, the coupling strengths will change at energies where it appears — due to the fact that the supersymmetric partners will participate in interaction, contributing to the changes in c-coupling parameters. It can be shown that the existence of supersymmetric partners at about 1000 GeV sees to it that all coupling strengths approach each other at about 1.5×10^{16} GeV (see Fig. 10.1). That means, we can formulate a supersymmetric variant of SU(5) theory that is consistent with the experimental data observed to date. We also find in their connection that the supersymmetric version of SU(5) theory leaves the proton unstable, but a bit less so than the non-supersymmetric version — with a lifetime of about 10^{33} years. This relatively long lifetime is compatible with data obtainable now.

We should not see more in SU(5) theory than an example for a theory that actually unifies the three observed interactions. But

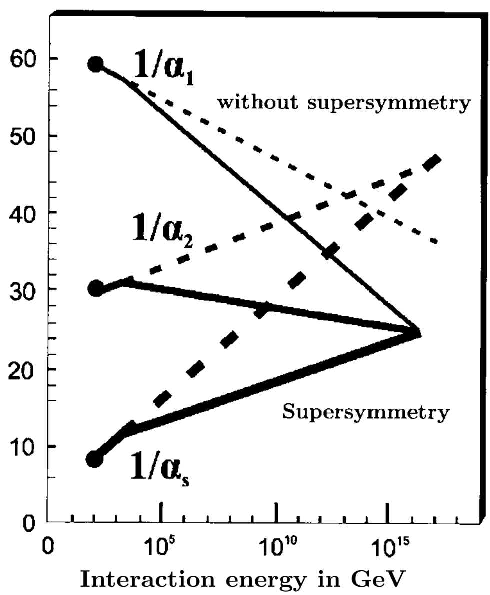

Fig. 10.1. The behavior of coupling constants with and without supersymmetry.

it is not the only theory to do so. In particular, there is a theory based on the symmetry group SO(10) which describes the symmetry in ten-dimensional space. Compare this with SO(3) symmetry that, in three-dimensional space, describes all rotations. Now, SO(10) symmetry has the remarkable property that it has a representation containing 16 elements. This fact opens up the possibility to describe all fermions of a "family", including their antiparticles, in one and the same symmetry representation. Take, for instance, the fermions of the first family

$$\begin{pmatrix} \nu_e & \vdots & uuu & \vdots & \bar{u}\bar{u}\bar{u} & \vdots & \nu_e \\ e^- & \vdots & ddd & \vdots & \bar{d}\bar{d}\bar{d} & \vdots & e^+ \end{pmatrix}$$

Clearly, the SO(10) theory is more comprehensive than the SU(5) version. Rather, it contains the latter as a partial symmetry, and it has 45 gauge bosons. There is also the interesting fact that we can attain a unification of coupling constants without having recourse to supersymmetry. This is due to the fact that SO(10) has more gauge bosons than SU(5) — including partners of the W-bosons that act on right-handed fermions. We have to add that these particles are considerably heavier than the customary W-bosons, so that they are not observable with presently existing means. At high energies, however, these particles are present to modify the coupling parameters. We arrive at a convergence of the coupling parameters if the masses of those new gauge bosons are of order 1000 GeV. This fact implies that SO(10) theory is as consistent with today's observables as the supersymmetric version of SU(5). The future will tell whether Nature actually has recourse to these theoretical possibilities. These theories share the feature that the proton is unstable, with a lifetime not much larger than the limit imposed by experimental evidence obtainable today.

Should the proton actually be able to decay into leptons and photons, the implication would be that baryon number is not exactly conserved. That would help to explain one of the oldest phenomena in our universe. Matter in our world consists mostly of nucleons, and thereby of quarks. Antimatter is made up of antiquarks, but is not observed in our stable galactic system. We also have hints that distant galaxies are made up of matter — not of antimatter. That tells us that the baryon number of the visible universe is enormously large. And we know that the universe such as we observe it originated some 14 billion years ago, in the Big Bang — when matter was created in an extremely hot phase. If baryon numbers were strictly conserved, it would have been the same at the end of the Big Bang as it is today. That means, our cosmos must have been born with a large baryon number. But that does not really make sense: it would be more comprehensible if the baryon number had started from zero at the beginning, if there had been equal numbers of quarks and antiquarks at that time. Just this scenario is possible if baryon number is not strictly conserved as in SU(5) or SO(10) theory. The newfangled forces that act in these theories saw to it that the baryon number zero, as it existed at the start, grew to a huge level by our time; that means today's baryon number is a product of history. As we look toward the Universe in its distant future, the baryon number will again assume very different values.