

Sensitivity of the Roots and Wilkinson Polynomial

The roots of a polynomial can be very sensitive to small perturbations of the Coefficients of the polynomial.

For example

$$P(x) = x^3 - 2x^2 + 1 \quad x_1 = 1 \quad x_2 = 1.6180$$

$$x_3 = -0.6180$$

The roots are $x_1 = -1, x_2 = -1$

Now, perturb coefficient -2 to -1.9999 , leaving other two unchanged. The roots of perturbed polynomial are $1 \pm 0.01i$

It is true that the multiple roots are prone to perturbations but the roots of a polynomial with well-separated roots can be very sensitive too.

WILKINSON Polynomial

The polynomial $P(x) = (x-1)(x-2) \dots (x-20)$

is the celebrated Wilkinson polynomial.

The roots of Wilkinson polynomial are

$1, 2, 3, \dots, 20$ and thus very well-separated.

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