Example $f(x) = x^3 - 2x^2 - 5$

The roots of $f(x)$ are $(0.26906, -0.3453 \pm 1.3187i)$

Try to approximate pair of complex roots

Sol Let $x_0 = -1$, $x_1 = 0$, $x_2 = 1$

Step I $f(x_0) = -8$, $f(x_1) = -5$, $f(x_2) = -6$

Step II compute a, b and c

$$c = f(x_2) = -6$$

a and b are computed by solving 2×2 linear system

$$\begin{bmatrix} (x_0 - x_2)^2 & x_0 - x_2 \\ (x_1 - x_2)^2 & x_1 - x_2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} f(x_0) - c \\ f(x_1) - c \end{bmatrix}$$

$$\begin{bmatrix} (-1-1)^2 & -1-1 \\ (0-1)^2 & 0-1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -8+6 \\ -5+6 \end{bmatrix}$$

$$\begin{pmatrix} 4 & -2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$4a - 2b = -2$$

$$a - b = 1$$

$$\Rightarrow a = -2, \quad b = -3$$