Step II Solve quadratic equation representing parabola P and take x_3 as the root closest to x_2.

Note. Even x_0, x_1 and x_2 are real, x_3 can be complex.

Detail for computing x_3:

Step I Start by taking x_0, x_1 and x_3 as initial guess

let the equation of parabola P be

$$P(x) = a(x-x_2)^2 + b(x-x_2) + c \quad \text{—} \quad (1)$$

Since $P(x)$ passes through $(x_0, f(x_0))$, $(x_1, f(x_1))$ and $(x_2, f(x_2))$, Therefore, we have

$$P(x_0) = a(x_0-x_2)^2 + b(x_0-x_2) + c = f(x_0) \quad \text{—} \quad (2)$$

$$P(x_1) = a(x_1-x_2)^2 + b(x_1-x_2) + c = f(x_1) \quad \text{—} \quad (3)$$

$$P(x_2) = c = f(x_2)$$

Step II Find a and b by solving (2) and (3)

$$\left.\begin{aligned} a(x_0-x_2)^2 + b(x_0-x_2) &= f(x_0) - c \\ a(x_1-x_2)^2 + b(x_1-x_2) &= f(x_1) - c \end{aligned}\right]$$

or
$$\begin{pmatrix} (x_0-x_2)^2 & (x_0-x_2) \\ (x_1-x_2)^2 & (x_1-x_2) \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} f(x_0)-c \\ f(x_1)-c \end{pmatrix}$$

Step III Knowing a, and b we can now solve