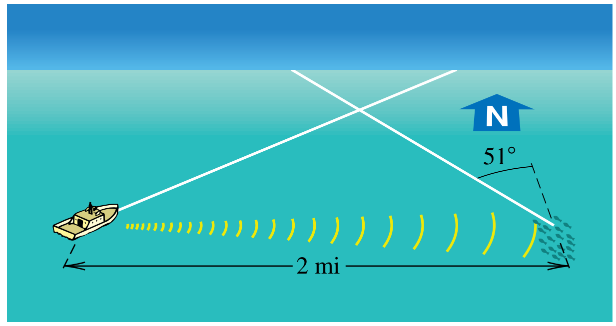
Figure 11

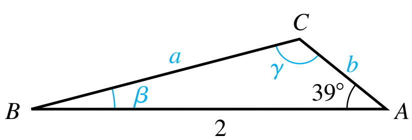


(a) If the boat travels at 20 mi/hr, approximate, to the nearest 0.1°, the direction it should head to intercept the school of fish.

(b) Find, to the nearest minute, the time it will take the boat to reach the fish.

SOLUTION

Figure 12



(a) The problem is illustrated by the triangle in Figure 12, with the school of fish at A, the boat at B, and the point of interception at C. Note that angle $\alpha = 90^\circ - 51^\circ = 39^\circ$. To obtain β, we begin as follows:

$$\frac{\sin \beta}{b} = \frac{\sin 39^\circ}{a} \qquad \text{law of sines}$$

$$\sin \beta = \frac{b}{a} \sin 39^\circ \qquad \text{solve for } \sin \beta \qquad (*)$$

We next find b/a, letting t denote the amount of time required for the boat and fish to meet at C:

$$a = 20t, \qquad b = 8t \qquad \text{(distance) = (rate)(time)}$$

$$\frac{b}{a} = \frac{8t}{20t} = \frac{2}{5} \qquad \text{divide } b \text{ by } a$$

$$\sin \beta = \tfrac{2}{5} \sin 39^\circ \qquad \text{substitute for } b/a \text{ in } (*)$$

$$\beta = \sin^{-1}\left(\tfrac{2}{5} \sin 39^\circ\right) \approx 14.6^\circ \qquad \text{approximate}$$

Since $90^\circ - 14.6^\circ = 75.4^\circ$, the boat should travel in the (approximate) direction N75.4°E.

(b) We can find t using the relationship $a = 20t$. Let us first find the distance a from B to C. Since the only known side is 2, we need to find the angle γ opposite the side of length 2 in order to use the law of sines. We begin by noting that

$$\gamma \approx 180^\circ - 39^\circ - 14.6^\circ = 126.4^\circ.$$

To find side a, we have

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad \text{law of sines}$$

$$a = \frac{c\sin\alpha}{\sin\gamma} \qquad \text{solve for } a$$

$$\approx \frac{2\sin 39^\circ}{\sin 126.4^\circ} \approx 1.56 \text{ mi.} \qquad \text{substitute and approximate}$$

Using $a = 20t$, we find the time t for the boat to reach C:

$$t = \frac{a}{20} \approx \frac{1.56}{20} \approx 0.08 \text{ hr} \approx 5 \text{ min}$$

8.1 Exercises

Exer. 1–16: Solve $\triangle ABC$.

1 $\alpha = 41^\circ$, $\gamma = 77^\circ$, $a = 10.5$

2 $\beta = 20^\circ$, $\gamma = 31^\circ$, $b = 210$

3 $\alpha = 27^\circ 40'$, $\beta = 52^\circ 10'$, $a = 32.4$

4 $\beta = 50^\circ 50'$, $\gamma = 70^\circ 30'$, $c = 537$

5 $\alpha = 42^\circ 10'$, $\gamma = 61^\circ 20'$, $b = 19.7$

6 $\alpha = 103.45^\circ$, $\gamma = 27.19^\circ$, $b = 38.84$

7 $\gamma = 81^\circ$, $c = 11$, $b = 12$

8 $\alpha = 32.32^\circ$, $c = 574.3$, $a = 263.6$

9 $\gamma = 53^\circ 20'$, $a = 140$, $c = 115$

10 $\alpha = 27^\circ 30'$, $c = 52.8$, $a = 28.1$

11 $\gamma = 47.74^\circ$, $a = 131.08$, $c = 97.84$

12 $\alpha = 42.17^\circ$, $a = 5.01$, $b = 6.12$

13 $\alpha = 65^\circ 10'$, $a = 21.3$, $b = 18.9$

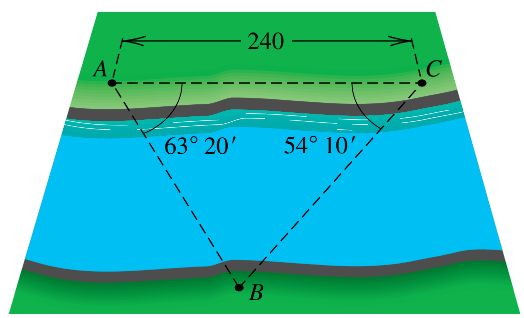
14 $\beta = 113^\circ 10'$, $b = 248$, $c = 195$

15 $\beta = 121.624^\circ$, $b = 0.283$, $c = 0.178$

16 $\gamma = 73.01^\circ$, $a = 17.31$, $c = 20.24$

17 **Surveying** To find the distance between two points A and B that lie on opposite banks of a river, a surveyor lays off a line segment AC of length 240 yards along one bank and determines that the measures of $\angle BAC$ and $\angle ACB$ are $63^\circ 20'$ and $54^\circ 10'$, respectively (see the figure). Approximate the distance between A and B.

Exercise 17



18 **Surveying** To determine the distance between two points A and B, a surveyor chooses a point C that is 375 yards from A and 530 yards from B. If $\angle BAC$ has measure $49^\circ 30'$, approximate the distance between A and B.

19 **Cable car route** As shown in the figure on the next page, a cable car carries passengers from a point A, which is 1.2 miles from a point B at the base of a mountain, to a point P at the top of the mountain. The angles of elevation of P from A and B are 21° and 65°, respectively.

(a) Approximate the distance between A and P.

(b) Approximate the height of the mountain.