Graphing Calculator Activities

51. Use a calculator to check the answers to all three parts of Example 1.

52. Use a calculator to check your answers for Problems 21–26.

53. Use the following matrices:

$$A = \begin{bmatrix} 7 & -4 \\ 6 & 9 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 8 \\ -5 & 7 \end{bmatrix} \quad C = \begin{bmatrix} 8 & -2 \\ 4 & -7 \end{bmatrix}$$

(a) Show that $(AB)C = A(BC)$.
(b) Show that $A(B + C) = AB + AC$.
(c) Show that $(B + C)A = BA + CA$.

Answers to the Concept Quiz

1. True **2.** True **3.** True **4.** False **5.** False **6.** False **7.** False **8.** False

12.2 Multiplicative Inverses

OBJECTIVES

1. Find the multiplicative inverse of a 2 × 2 matrix
2. Find the product of a 2 × 2 and a 2 × 1 matrix
3. Solve a system of two linear equations by using matrices

We know that 1 is a multiplicative identity element for the set of real numbers. That is, $a(1) = 1(a) = a$ for any real number a. Is there a multiplicative identity element for 2 × 2 matrices? Yes. The matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

is the **multiplicative identity element** because

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

and

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Therefore we can state that

$$AI = IA = A$$

for all 2 × 2 matrices.

Again, refer to the set of real numbers, in which every nonzero real number a has a multiplicative inverse $1/a$ such that $a(1/a) = (1/a)\,a = 1$. Does every 2 × 2 matrix have a multiplicative inverse? To help answer this question, let's think about finding the multiplicative inverse (if one exists) for a specific matrix. This should give us some clues about a general approach.

Classroom Example
Find the multiplicative inverse of
$A = \begin{bmatrix} 9 & 3 \\ 4 & 2 \end{bmatrix}$.

EXAMPLE 1 Find the multiplicative inverse of $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$.

Solution

We are looking for a matrix A^{-1} such that $AA^{-1} = A^{-1}A = I$. In other words, we want to solve the following matrix equation:

$$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We need to multiply the two matrices on the left side of this equation and then set the elements of the product matrix equal to the corresponding elements of the identity matrix. We obtain the following system of equations:

$$\begin{pmatrix} 3x + 5z = 1 \\ 3y + 5w = 0 \\ 2x + 4z = 0 \\ 2y + 4w = 1 \end{pmatrix} \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \end{matrix}$$

Solving equations (1) and (3) simultaneously produces values for x and z.

$$x = \frac{\begin{vmatrix} 1 & 5 \\ 0 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}} = \frac{1(4) - 5(0)}{3(4) - 5(2)} = \frac{4}{2} = 2$$

$$z = \frac{\begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}} = \frac{3(0) - 1(2)}{3(4) - 5(2)} = \frac{-2}{2} = -1$$

Likewise, solving equations (2) and (4) simultaneously produces values for y and w.

$$y = \frac{\begin{vmatrix} 0 & 5 \\ 1 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}} = \frac{0(4) - 5(1)}{3(4) - 5(2)} = \frac{-5}{2} = -\frac{5}{2}$$

$$w = \frac{\begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}} = \frac{3(1) - 0(2)}{3(4) - 5(2)} = \frac{3}{2}$$

Therefore

$$A^{-1} = \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{bmatrix}$$

To check this, we perform the following multiplication:

$$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now let's use the approach in Example 1 on the general matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

We want to find

$$A^{-1} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$