For Problems 85–100, find each of the following quotients, and express the answers in the standard form of a complex number. (Objective 6)

85. $\dfrac{3i}{2 + 4i}$

86. $\dfrac{4i}{5 + 2i}$

87. $\dfrac{-2i}{3 - 5i}$

88. $\dfrac{-5i}{2 - 4i}$

89. $\dfrac{-2 + 6i}{3i}$

90. $\dfrac{-4 - 7i}{6i}$

91. $\dfrac{2}{7i}$

92. $\dfrac{3}{10i}$

93. $\dfrac{2 + 6i}{1 + 7i}$

94. $\dfrac{5 + i}{2 + 9i}$

95. $\dfrac{3 + 6i}{4 - 5i}$

96. $\dfrac{7 - 3i}{4 - 3i}$

97. $\dfrac{-2 + 7i}{-1 + i}$

98. $\dfrac{-3 + 8i}{-2 + i}$

99. $\dfrac{-1 - 3i}{-2 - 10i}$

100. $\dfrac{-3 - 4i}{-4 - 11i}$

101. Some of the solution sets for quadratic equations in the next sections in this chapter will contain complex numbers such as $(-4 + \sqrt{-12})/2$ and $(-4 - \sqrt{-12})/2$. We can simplify the first number as follows.

$$\frac{-4 + \sqrt{-12}}{2} = \frac{-4 + i\sqrt{12}}{2}$$
$$= \frac{-4 + 2i\sqrt{3}}{2} = \frac{\cancel{2}(-2 + i\sqrt{3})}{\cancel{2}}$$
$$= -2 + i\sqrt{3}$$

Simplify each of the following complex numbers. (Objective 3)

(a) $\dfrac{-4 - \sqrt{-12}}{2}$

(b) $\dfrac{6 + \sqrt{-24}}{4}$

(c) $\dfrac{-1 - \sqrt{-18}}{2}$

(d) $\dfrac{-6 + \sqrt{-27}}{3}$

(e) $\dfrac{10 + \sqrt{-45}}{4}$

(f) $\dfrac{4 - \sqrt{-48}}{2}$

Thoughts Into Words

102. Why is the set of real numbers a subset of the set of complex numbers?

103. Can the sum of two nonreal complex numbers be a real number? Defend your answer.

104. Can the product of two nonreal complex numbers be a real number? Defend your answer.

Answers to the Concept Quiz

1. False **2.** True **3.** False **4.** True **5.** True **6.** True **7.** True **8.** False **9.** False **10.** True

6.2 Quadratic Equations

OBJECTIVES

1. Solve quadratic equations by factoring
2. Solve quadratic equations of the form $x^2 = a$
3. Solve problems pertaining to right triangles and 30°–60° triangles

A second-degree equation in one variable contains the variable with an exponent of 2, but no higher power. Such equations are also called *quadratic equations*. The following are examples of quadratic equations.

$$x^2 = 36 \qquad y^2 + 4y = 0 \qquad x^2 + 5x - 2 = 0$$
$$3n^2 + 2n - 1 = 0 \qquad 5x^2 + x + 2 = 3x^2 - 2x - 1$$

A **quadratic equation** in the variable x can also be defined as any equation that can be written in the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers and $a \neq 0$. The form $ax^2 + bx + c = 0$ is called the *standard form* of a quadratic equation.

In previous chapters you solved quadratic equations (the term *quadratic* was not used at that time) by factoring and applying the property, $ab = 0$ if and only if $a = 0$ or $b = 0$. Let's review a few such examples.

Classroom Example
Solve $4x^2 + 11x - 3 = 0$.

EXAMPLE 1 Solve $3n^2 + 14n - 5 = 0$.

Solution

$$3n^2 + 14n - 5 = 0$$

$$(3n - 1)(n + 5) = 0 \qquad \text{Factor the left side}$$

$$3n - 1 = 0 \quad \text{or} \quad n + 5 = 0 \qquad \text{Apply: } ab = 0 \text{ if and only if } a = 0 \text{ or } b = 0$$

$$3n = 1 \quad \text{or} \quad n = -5$$

$$n = \frac{1}{3} \quad \text{or} \quad n = -5$$

The solution set is $\left\{-5, \frac{1}{3}\right\}$.

Classroom Example
Solve $2\sqrt{y} = y - 3$.

EXAMPLE 2 Solve $2\sqrt{x} = x - 8$.

Solution

$$2\sqrt{x} = x - 8$$

$$(2\sqrt{x})^2 = (x - 8)^2 \qquad \text{Square both sides}$$

$$4x = x^2 - 16x + 64$$

$$0 = x^2 - 20x + 64$$

$$0 = (x - 16)(x - 4) \qquad \text{Factor the right side}$$

$$x - 16 = 0 \quad \text{or} \quad x - 4 = 0 \qquad \text{Apply: } ab = 0 \text{ if and only if } a = 0 \text{ or } b = 0$$

$$x = 16 \quad \text{or} \quad x = 4$$

✓ Check

If $x = 16$

$$2\sqrt{x} = x - 8$$

$$2\sqrt{16} \stackrel{?}{=} 16 - 8$$

$$2(4) \stackrel{?}{=} 8$$

$$8 = 8$$

If $x = 4$

$$2\sqrt{x} = x - 8$$

$$2\sqrt{4} \stackrel{?}{=} 4 - 8$$

$$2(2) \stackrel{?}{=} -4$$

$$4 \neq -4$$

The solution set is $\{16\}$.

We should make two comments about Example 2. First, remember that applying the property, if $a = b$, then $a^n = b^n$, might produce extraneous solutions. Therefore, we *must* check all potential solutions. Second, the equation $2\sqrt{x} = x - 8$ is said to be of *quadratic form* because it can be written as $2x^{\frac{1}{2}} = \left(x^{\frac{1}{2}}\right)^2 - 8$. More will be said about the phrase *quadratic form* later.

Solving Quadratic Equations of the Form $x^2 = a$

Let's consider quadratic equations of the form $x^2 = a$, where x is the variable and a is any real number. We can solve $x^2 = a$ as follows:

$$\begin{aligned}
x^2 &= a \\
x^2 - a &= 0 \\
x^2 - (\sqrt{a})^2 &= 0 && a = (\sqrt{a})^2 \\
(x - \sqrt{a})(x + \sqrt{a}) &= 0 && \text{Factor the left side} \\
x - \sqrt{a} = 0 \quad &\text{or} \quad x + \sqrt{a} = 0 && \text{Apply: } ab = 0 \text{ if and only if } a = 0 \text{ or } b = 0 \\
x = \sqrt{a} \quad &\text{or} \quad x = -\sqrt{a}
\end{aligned}$$

The solutions are $\sqrt{a}$ and $-\sqrt{a}$. We can state this result as a general property and use it to solve certain types of quadratic equations.

Property 6.1

For any real number a,

$x^2 = a$ if and only if $x = \sqrt{a}$ or $x = -\sqrt{a}$

(The statement $x = \sqrt{a}$ or $x = -\sqrt{a}$ can be written as $x = \pm\sqrt{a}$.)

Property 6.1, along with our knowledge of square roots, makes it very easy to solve quadratic equations of the form $x^2 = a$.

Classroom Example
Solve $m^2 = 48$.

EXAMPLE 3 Solve $x^2 = 45$.

Solution

$$\begin{aligned}
x^2 &= 45 \\
x &= \pm\sqrt{45} \\
x &= \pm 3\sqrt{5} && \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}
\end{aligned}$$

The solution set is $\{\pm 3\sqrt{5}\}$.

Classroom Example
Solve $n^2 = -25$.

EXAMPLE 4 Solve $x^2 = -9$.

Solution

$$\begin{aligned}
x^2 &= -9 \\
x &= \pm\sqrt{-9} \\
x &= \pm 3i && \sqrt{-9} = i\sqrt{9} = 3i
\end{aligned}$$

Thus the solution set is $\{\pm 3i\}$.

Classroom Example
Solve $5x^2 = 16$.

EXAMPLE 5 Solve $7n^2 = 12$.

Solution

$$\begin{aligned}
7n^2 &= 12 \\
n^2 &= \frac{12}{7}
\end{aligned}$$

$$n = \pm\sqrt{\frac{12}{7}}$$

$$n = \pm\frac{2\sqrt{21}}{7} \qquad \sqrt{\frac{12}{7}} = \frac{\sqrt{12}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{84}}{7} = \frac{\sqrt{4}\sqrt{21}}{7} = \frac{2\sqrt{21}}{7}$$

The solution set is $\left\{\pm\frac{2\sqrt{21}}{7}\right\}$.

Classroom Example
Solve $(4x - 3)^2 = 49$.

EXAMPLE 6 Solve $(3n + 1)^2 = 25$.

Solution

$$(3n + 1)^2 = 25$$
$$(3n + 1) = \pm\sqrt{25}$$
$$3n + 1 = \pm 5$$
$$3n + 1 = 5 \quad \text{or} \quad 3n + 1 = -5$$
$$3n = 4 \quad \text{or} \quad 3n = -6$$
$$n = \frac{4}{3} \quad \text{or} \quad n = -2$$

The solution set is $\left\{-2, \frac{4}{3}\right\}$.

Classroom Example
Solve $(x + 4)^2 = -18$.

EXAMPLE 7 Solve $(x - 3)^2 = -10$.

Solution

$$(x - 3)^2 = -10$$
$$x - 3 = \pm\sqrt{-10}$$
$$x - 3 = \pm i\sqrt{10}$$
$$x = 3 \pm i\sqrt{10}$$

Thus the solution set is $\{3 \pm i\sqrt{10}\}$.

Remark: Take another look at the equations in Examples 4 and 7. We should immediately realize that the solution sets will consist only of nonreal complex numbers, because any nonzero real number squared is positive.

Sometimes it may be necessary to change the form before we can apply Property 6.1. Let's consider one example to illustrate this idea.

Classroom Example
Solve $2(5x - 1)^2 + 9 = 53$.

EXAMPLE 8 Solve $3(2x - 3)^2 + 8 = 44$.

Solution

$$3(2x - 3)^2 + 8 = 44$$
$$3(2x - 3)^2 = 36$$
$$(2x - 3)^2 = 12$$
$$2x - 3 = \pm\sqrt{12}$$
$$2x - 3 = \pm 2\sqrt{3}$$

$$2x = 3 \pm 2\sqrt{3}$$
$$x = \frac{3 \pm 2\sqrt{3}}{2}$$

The solution set is $\left\{\frac{3 \pm 2\sqrt{3}}{2}\right\}$.

Solving Problems Pertaining to Right Triangles and 30°–60° Triangles

Our work with radicals, Property 6.1, and the Pythagorean theorem form a basis for solving a variety of problems that pertain to right triangles.

Classroom Example
A 62-foot guy-wire hangs from the top of a tower. When pulled taut, the guy-wire reaches a point on the ground 25 feet from the base of the tower. Find the height of the tower to the nearest tenth of a foot.

EXAMPLE 9

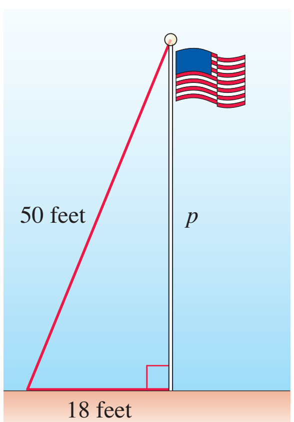
A 50-foot rope hangs from the top of a flagpole. When pulled taut to its full length, the rope reaches a point on the ground 18 feet from the base of the pole. Find the height of the pole to the nearest tenth of a foot.

Solution

Let's make a sketch (Figure 6.1) and record the given information. Use the Pythagorean theorem to solve for p as follows:

$$p^2 + 18^2 = 50^2$$
$$p^2 + 324 = 2500$$
$$p^2 = 2176$$
$$p = \sqrt{2176} = 46.6 \quad \text{to the nearest tenth}$$

The height of the flagpole is approximately 46.6 feet.



p represents the height of the flagpole.

Figure 6.1

There are two special kinds of right triangles that we use extensively in later mathematics courses. The first is the **isosceles right triangle**, which is a right triangle that has both legs of the same length. Let's consider a problem that involves an isosceles right triangle.

Classroom Example
Find the length of each leg of an isosceles right triangle that has a hypotenuse of length 16 inches.

EXAMPLE 10

Find the length of each leg of an isosceles right triangle that has a hypotenuse of length 5 meters.

Solution

Let's sketch an isosceles right triangle and let x represent the length of each leg (Figure 6.2). Then we can apply the Pythagorean theorem.

$$x^2 + x^2 = 5^2$$
$$2x^2 = 25$$
$$x^2 = \frac{25}{2}$$
$$x = \pm\sqrt{\frac{25}{2}} = \pm\frac{5}{\sqrt{2}} = \pm\frac{5\sqrt{2}}{2}$$

Each leg is $\frac{5\sqrt{2}}{2}$ meters long.

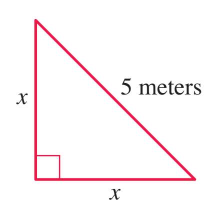


Figure 6.2

Remark: In Example 9 we made no attempt to express $\sqrt{2176}$ in simplest radical form because the answer was to be given as a rational approximation to the nearest tenth. However, in Example 10 we left the final answer in radical form and therefore expressed it in simplest radical form.

The second special kind of right triangle that we use frequently is one that contains acute angles of 30° and 60°. In such a right triangle, which we refer to as a **30°–60° right triangle**, the side opposite the 30° angle is equal in length to one-half of the length of the hypotenuse. This relationship, along with the Pythagorean theorem, provides us with another problem-solving technique.

Classroom Example
Suppose that a 30-foot ladder is leaning against a building and makes an angle of 60° with the ground. How far up the building does the top of the ladder reach? Express your answer to the nearest tenth of a foot.

EXAMPLE 11

Suppose that a 20-foot ladder is leaning against a building and makes an angle of 60° with the ground. How far up the building does the top of the ladder reach? Express your answer to the nearest tenth of a foot.

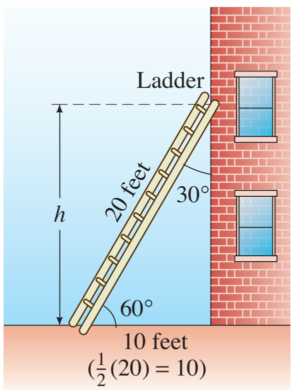


Figure 6.3

Solution

Figure 6.3 depicts this situation. The side opposite the 30° angle equals one-half of the hypotenuse, so it is of length $\frac{1}{2}(20) = 10$ feet. Now we can apply the Pythagorean theorem.

$$h^2 + 10^2 = 20^2$$
$$h^2 + 100 = 400$$
$$h^2 = 300$$
$$h = \sqrt{300} = 17.3 \quad \text{to the nearest tenth}$$

The ladder touches the building at a point approximately 17.3 feet from the ground.

Concept Quiz 6.2

For Problems 1–10, answer true or false.

1. The quadratic equation $-3x^2 + 5x - 8 = 0$ is in standard form.
2. The solution set of the equation $(x + 1)^2 = -25$ will consist only of nonreal complex numbers.
3. An isosceles right triangle is a right triangle that has a hypotenuse of the same length as one of the legs.
4. In a 30°–60° right triangle, the hypotenuse is equal in length to twice the length of the side opposite the 30° angle.
5. The equation $2x^2 + x^3 - x + 4 = 0$ is a quadratic equation.
6. The solution set for $4x^2 = 8x$ is $\{2\}$.
7. The solution set for $3x^2 = 8x$ is $\left\{0, \frac{8}{3}\right\}$.
8. The solution set for $x^2 - 8x - 48 = 0$ is $\{-12, 4\}$.
9. If the length of each leg of an isosceles right triangle is 4 inches, then the hypotenuse is of length $4\sqrt{2}$ inches.
10. If the length of the leg opposite the 30° angle in a right triangle is 6 centimeters, then the length of the other leg is 12 centimeters.

Problem Set 6.2

For Problems 1–20, solve each of the quadratic equations by factoring and applying the property, $ab = 0$ if and only if $a = 0$ or $b = 0$. If necessary, return to Chapter 3 and review the factoring techniques presented there. (Objective 1)

1. $x^2 - 9x = 0$
2. $x^2 + 5x = 0$
3. $x^2 = -3x$
4. $x^2 = 15x$
5. $3y^2 + 12y = 0$
6. $6y^2 - 24y = 0$

7. $5n^2 - 9n = 0$

8. $4n^2 + 13n = 0$

9. $x^2 + x - 30 = 0$

10. $x^2 - 8x - 48 = 0$

11. $x^2 - 19x + 84 = 0$

12. $x^2 - 21x + 104 = 0$

13. $2x^2 + 19x + 24 = 0$

14. $4x^2 + 29x + 30 = 0$

15. $15x^2 + 29x - 14 = 0$

16. $24x^2 + x - 10 = 0$

17. $25x^2 - 30x + 9 = 0$

18. $16x^2 - 8x + 1 = 0$

19. $6x^2 - 5x - 21 = 0$

20. $12x^2 - 4x - 5 = 0$

For Problems 21–26, solve each radical equation. Don't forget, you *must* check potential solutions. **(Objective 1)**

21. $3\sqrt{x} = x + 2$

22. $3\sqrt{2x} = x + 4$

23. $\sqrt{2x} = x - 4$

24. $\sqrt{x} = x - 2$

25. $\sqrt{3x} + 6 = x$

26. $\sqrt{5x} + 10 = x$

For Problems 27–62, use Property 6.1 to help solve each quadratic equation. **(Objective 2)**

27. $x^2 = 1$

28. $x^2 = 81$

29. $x^2 = -36$

30. $x^2 = -49$

31. $x^2 = 14$

32. $x^2 = 22$

33. $n^2 - 28 = 0$

34. $n^2 - 54 = 0$

35. $3t^2 = 54$

36. $4t^2 = 108$

37. $2t^2 = 7$

38. $3t^2 = 8$

39. $15y^2 = 20$

40. $14y^2 = 80$

41. $10x^2 + 48 = 0$

42. $12x^2 + 50 = 0$

43. $24x^2 = 36$

44. $12x^2 = 49$

45. $(x - 2)^2 = 9$

46. $(x + 1)^2 = 16$

47. $(x + 3)^2 = 25$

48. $(x - 2)^2 = 49$

49. $(x + 6)^2 = -4$

50. $(3x + 1)^2 = 9$

51. $(2x - 3)^2 = 1$

52. $(2x + 5)^2 = -4$

53. $(n - 4)^2 = 5$

54. $(n - 7)^2 = 6$

55. $(t + 5)^2 = 12$

56. $(t - 1)^2 = 18$

57. $(3y - 2)^2 = -27$

58. $(4y + 5)^2 = 80$

59. $3(x + 7)^2 + 4 = 79$

60. $2(x + 6)^2 - 9 = 63$

61. $2(5x - 2)^2 + 5 = 25$

62. $3(4x - 1)^2 + 1 = -17$

For Problems 63–68, a and b represent the lengths of the legs of a right triangle, and c represents the length of the hypotenuse. Express answers in simplest radical form. **(Objective 3)**

63. Find c if $a = 4$ centimeters and $b = 6$ centimeters.

64. Find c if $a = 3$ meters and $b = 7$ meters.

65. Find a if $c = 12$ inches and $b = 8$ inches.

66. Find a if $c = 8$ feet and $b = 6$ feet.

67. Find b if $c = 17$ yards and $a = 15$ yards.

68. Find b if $c = 14$ meters and $a = 12$ meters.

For Problems 69–72, use the isosceles right triangle in Figure 6.4. Express your answers in simplest radical form. **(Objective 3)**

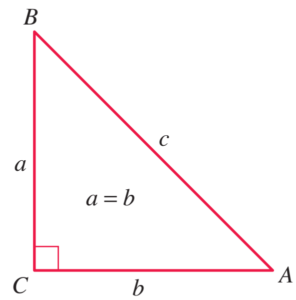


Figure 6.4

69. If $b = 6$ inches, find c.

70. If $a = 7$ centimeters, find c.

71. If $c = 8$ meters, find a and b.

72. If $c = 9$ feet, find a and b.

For Problems 73–78, use the triangle in Figure 6.5. Express your answers in simplest radical form. **(Objective 3)**

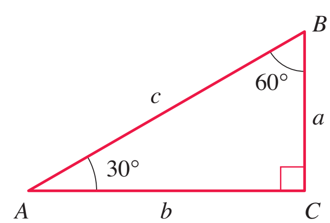


Figure 6.5

73. If $a = 3$ inches, find b and c.

74. If $a = 6$ feet, find b and c.

75. If $c = 14$ centimeters, find a and b.

76. If $c = 9$ centimeters, find a and b.

77. If $b = 10$ feet, find a and c.

78. If $b = 8$ meters, find a and c.

79. A 24-foot ladder resting against a house reaches a windowsill 16 feet above the ground. How far is the foot of the ladder from the foundation of the house? Express your answer to the nearest tenth of a foot.

80. A 62-foot guy-wire makes an angle of 60° with the ground and is attached to a telephone pole (see Figure 6.6). Find the distance from the base of the pole to the point on the

pole where the wire is attached. Express your answer to the nearest tenth of a foot.

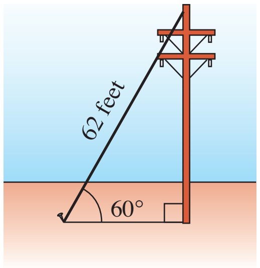


Figure 6.6

81. A rectangular plot measures 16 meters by 34 meters. Find, to the nearest meter, the distance from one corner of the plot to the corner diagonally opposite.

82. Consecutive bases of a square-shaped baseball diamond are 90 feet apart (see Figure 6.7). Find, to the nearest tenth of a foot, the distance from first base diagonally across the diamond to third base.

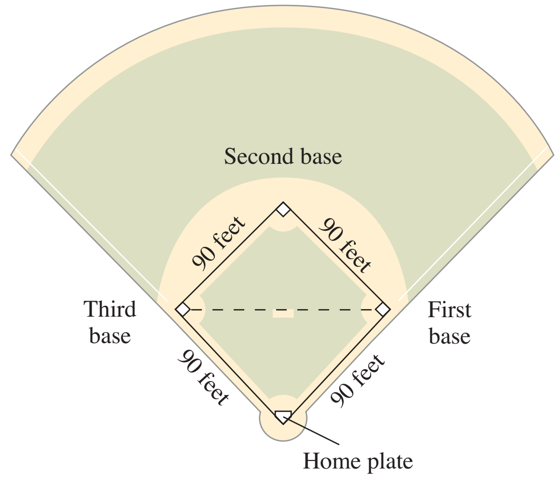


Figure 6.7

83. A diagonal of a square parking lot is 75 meters. Find, to the nearest meter, the length of a side of the lot.

Thoughts Into Words

84. Explain why the equation $(x + 2)^2 + 5 = 1$ has no real number solutions.

85. Suppose that your friend solved the equation $(x + 3)^2 = 25$ as follows:

$$(x + 3)^2 = 25$$
$$x^2 + 6x + 9 = 25$$
$$x^2 + 6x - 16 = 0$$
$$(x + 8)(x - 2) = 0$$
$$x + 8 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -8 \quad \text{or} \quad x = 2$$

Is this a correct approach to the problem? Would you offer any suggestion about an easier approach to the problem?

Further Investigations

86. Suppose that we are given a cube with edges 12 centimeters in length. Find the length of a diagonal from a lower corner to the diagonally opposite upper corner. Express your answer to the nearest tenth of a centimeter.

87. Suppose that we are given a rectangular box with a length of 8 centimeters, a width of 6 centimeters, and a height of 4 centimeters. Find the length of a diagonal from a lower corner to the upper corner diagonally opposite. Express your answer to the nearest tenth of a centimeter.

88. The converse of the Pythagorean theorem is also true. It states, "If the measures a, b, and c of the sides of a triangle are such that $a^2 + b^2 = c^2$, then the triangle is a right triangle with a and b the measures of the legs and c the measure of the hypotenuse." Use the converse of the Pythagorean theorem to determine which of the triangles with sides of the following measures are right triangles.

(a) 9, 40, 41 **(b)** 20, 48, 52

(c) 19, 21, 26 **(d)** 32, 37, 49

(e) 65, 156, 169 **(f)** 21, 72, 75

89. Find the length of the hypotenuse (h) of an isosceles right triangle if each leg is s units long. Then use this relationship to redo Problems 69–72.

90. Suppose that the side opposite the 30° angle in a 30°–60° right triangle is s units long. Express the length of the hypotenuse and the length of the other leg in terms of s. Then use these relationships and redo Problems 73–78.

Answers to the Concept Quiz

1. True **2.** True **3.** False **4.** True **5.** False **6.** False **7.** True **8.** False **9.** True **10.** False

6.3 Completing the Square

OBJECTIVE 1 Solve quadratic equations by completing the square

Thus far we have solved quadratic equations by factoring and applying the property, $ab = 0$ if and only if $a = 0$ or $b = 0$, or by applying the property, $x^2 = a$ if and only if $x = \pm\sqrt{a}$. In this section we examine another method called *completing the square*, which will give us the power to solve any quadratic equation.

A factoring technique we studied in Chapter 3 relied on recognizing *perfect-square trinomials*. In each of the following, the perfect-square trinomial on the right side is the result of squaring the binomial on the left side.

$$(x + 4)^2 = x^2 + 8x + 16 \qquad (x - 6)^2 = x^2 - 12x + 36$$
$$(x + 7)^2 = x^2 + 14x + 49 \qquad (x - 9)^2 = x^2 - 18x + 81$$
$$(x + a)^2 = x^2 + 2ax + a^2$$

Note that in each of the square trinomials, the constant term is equal to the square of one-half of the coefficient of the x term. This relationship enables us to form a perfect-square trinomial by adding a proper constant term. To find the constant term, take one-half of the coefficient of the x term and then square the result. For example, suppose that we want to form a perfect-square trinomial from $x^2 + 10x$. The coefficient of the x term is 10. Because $\frac{1}{2}(10) = 5$, and $5^2 = 25$, the constant term should be 25. The perfect-square trinomial that can be formed is $x^2 + 10x + 25$. This perfect-square trinomial can be factored and expressed as $(x + 5)^2$. Let's use the previous ideas to help solve some quadratic equations.

Classroom Example
Solve $x^2 + 8x - 5 = 0$.

EXAMPLE 1 Solve $x^2 + 10x - 2 = 0$.

Solution

$$x^2 + 10x - 2 = 0$$
$$x^2 + 10x = 2 \qquad \text{Isolate the } x^2 \text{ and } x \text{ terms}$$
$$\frac{1}{2}(10) = 5 \quad \text{and} \quad 5^2 = 25 \qquad \text{Take } \frac{1}{2} \text{ of the coefficient of the } x \text{ term and then square the result}$$
$$x^2 + 10x + 25 = 2 + 25 \qquad \text{Add 25 to } \textit{both} \text{ sides of the equation}$$
$$(x + 5)^2 = 27 \qquad \text{Factor the perfect-square trinomial}$$
$$x + 5 = \pm\sqrt{27} \qquad \text{Now solve by applying Property 6.1}$$
$$x + 5 = \pm 3\sqrt{3}$$
$$x = -5 \pm 3\sqrt{3}$$

The solution set is $\{-5 \pm 3\sqrt{3}\}$.

The method of completing the square to solve a quadratic equation is merely what the name implies. A perfect-square trinomial is formed, then the equation can be changed to the necessary form for applying the property "$x^2 = a$ if and only if $x = \pm\sqrt{a}$." Let's consider another example.

Classroom Example
Solve $x(x + 10) = -33$.

EXAMPLE 2 Solve $x(x + 8) = -23$.

Solution

$$x(x + 8) = -23$$
$$x^2 + 8x = -23 \qquad \text{Apply the distributive property}$$

$$\frac{1}{2}(8) = 4 \quad \text{and} \quad 4^2 = 16$$ Take $\frac{1}{2}$ of the coefficient of the x term and then square the result

$$x^2 + 8x + 16 = -23 + 16$$ Add 16 to *both* sides of the equation

$$(x + 4)^2 = -7$$ Factor the perfect-square trinomial

$$x + 4 = \pm\sqrt{-7}$$ Now solve by applying Property 6.1

$$x + 4 = \pm i\sqrt{7}$$

$$x = -4 \pm i\sqrt{7}$$

The solution set is $\{-4 \pm i\sqrt{7}\}$.

Classroom Example
Solve $m^2 - 3m - 5 = 0$.

EXAMPLE 3 Solve $x^2 - 3x + 1 = 0$.

Solution

$$x^2 - 3x + 1 = 0$$

$$x^2 - 3x = -1$$

$$x^2 - 3x + \frac{9}{4} = -1 + \frac{9}{4}$$ $\frac{1}{2}(3) = \frac{3}{2}$ and $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$$\left(x - \frac{3}{2}\right)^2 = \frac{5}{4}$$

$$x - \frac{3}{2} = \pm\sqrt{\frac{5}{4}}$$

$$x - \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

The solution set is $\left\{\frac{3 \pm \sqrt{5}}{2}\right\}$.

In Example 3 note that because the coefficient of the x term is odd, we are forced into the realm of fractions. Using common fractions rather than decimals enables us to apply our previous work with radicals.

The relationship for a perfect-square trinomial that states that the constant term is equal to the square of one-half of the coefficient of the x term holds only if the coefficient of x^2 is 1. Thus we must make an adjustment when solving quadratic equations that have a coefficient of x^2 other than 1. We will need to apply the multiplication property of equality so that the coefficient of the x^2 term becomes 1. The next example shows how to make this adjustment.

Classroom Example
Solve $3y^2 - 24y + 26 = 0$.

EXAMPLE 4 Solve $2x^2 + 12x - 5 = 0$.

Solution

$$2x^2 + 12x - 5 = 0$$

$$2x^2 + 12x = 5$$

$$x^2 + 6x = \frac{5}{2}$$ Multiply both sides by $\frac{1}{2}$

$$x^2 + 6x + 9 = \frac{5}{2} + 9$$ $\frac{1}{2}(6) = 3$, and $3^2 = 9$

$$x^2 + 6x + 9 = \frac{23}{2}$$
$$(x + 3)^2 = \frac{23}{2}$$
$$x + 3 = \pm\sqrt{\frac{23}{2}}$$
$$x + 3 = \pm\frac{\sqrt{46}}{2} \qquad \sqrt{\frac{23}{2}} = \frac{\sqrt{23}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{46}}{2}$$
$$x = -3 \pm \frac{\sqrt{46}}{2}$$
$$x = \frac{-6}{2} \pm \frac{\sqrt{46}}{2} \qquad \text{Common denominator of 2}$$
$$x = \frac{-6 \pm \sqrt{46}}{2}$$

The solution set is $\left\{\frac{-6 \pm \sqrt{46}}{2}\right\}$.

As we mentioned earlier, we can use the method of completing the square to solve *any* quadratic equation. To illustrate, let's use it to solve an equation that could also be solved by factoring.

Classroom Example
Solve $t^2 - 10t + 21 = 0$ by completing the square.

EXAMPLE 5 Solve $x^2 - 2x - 8 = 0$ by completing the square.

Solution

$$x^2 - 2x - 8 = 0$$
$$x^2 - 2x = 8$$
$$x^2 - 2x + 1 = 8 + 1 \qquad \frac{1}{2}(-2) = -1 \text{ and } (-1)^2 = 1$$
$$(x - 1)^2 = 9$$
$$x - 1 = \pm 3$$
$$x - 1 = 3 \quad \text{or} \quad x - 1 = -3$$
$$x = 4 \quad \text{or} \quad x = -2$$

The solution set is $\{-2, 4\}$.

Solving the equation in Example 5 by factoring would be easier than completing the square. Remember, however, that the method of completing the square will work with any quadratic equation.

Concept Quiz 6.3

For Problems 1–10, answer true or false.

1. In a perfect-square trinomial, the constant term is equal to one-half the coefficient of the x term.
2. The method of completing the square will solve any quadratic equation.
3. Every quadratic equation solved by completing the square will have real number solutions.
4. The completing-the-square method cannot be used if factoring could solve the quadratic equation.
5. To use the completing-the-square method for solving the equation $3x^2 + 2x = 5$, we would first divide both sides of the equation by 3.

6. The equation $x^2 + 2x = 0$ cannot be solved by using the method of completing the square.
7. To solve the equation $x^2 - 5x = 1$ by completing the square, we would start by adding $\frac{25}{4}$ to both sides of the equation.
8. To solve the equation $x^2 - 2x = 14$ by completing the square, we must first change the form of the equation to $x^2 - 2x - 14 = 0$.
9. The solution set of the equation $x^2 - 2x = 14$ is $\{1 \pm \sqrt{15}\}$.
10. The solution set of the equation $x^2 - 5x - 1 = 0$ is $\left\{\frac{5 \pm \sqrt{29}}{2}\right\}$.

Problem Set 6.3

For Problems 1–14, solve each quadratic equation by using (a) the factoring method and (b) the method of completing the square. **(Objective 1)**

1. $x^2 - 4x - 60 = 0$
2. $x^2 + 6x - 16 = 0$
3. $x^2 - 14x = -40$
4. $x^2 - 18x = -72$
5. $x^2 - 5x - 50 = 0$
6. $x^2 + 3x - 18 = 0$
7. $x(x + 7) = 8$
8. $x(x - 1) = 30$
9. $2n^2 - n - 15 = 0$
10. $3n^2 + n - 14 = 0$
11. $3n^2 + 7n - 6 = 0$
12. $2n^2 + 7n - 4 = 0$
13. $n(n + 6) = 160$
14. $n(n - 6) = 216$

For Problems 15–38, use the method of completing the square to solve each quadratic equation. **(Objective 1)**

15. $x^2 + 4x - 2 = 0$
16. $x^2 + 2x - 1 = 0$
17. $x^2 + 6x - 3 = 0$
18. $x^2 + 8x - 4 = 0$
19. $y^2 - 10y = 1$
20. $y^2 - 6y = -10$
21. $n^2 - 8n + 17 = 0$
22. $n^2 - 4n + 2 = 0$
23. $n(n + 12) = -9$
24. $n(n + 14) = -4$
25. $n^2 + 2n + 6 = 0$
26. $n^2 + n - 1 = 0$
27. $x^2 + 3x - 2 = 0$
28. $x^2 + 5x - 3 = 0$
29. $x^2 + 5x + 1 = 0$
30. $x^2 + 7x + 2 = 0$
31. $y^2 - 7y + 3 = 0$
32. $y^2 - 9y + 30 = 0$
33. $2x^2 + 4x - 3 = 0$
34. $2t^2 - 4t + 1 = 0$
35. $3n^2 - 6n + 5 = 0$
36. $3x^2 + 12x - 2 = 0$
37. $3x^2 + 5x - 1 = 0$
38. $2x^2 + 7x - 3 = 0$

For Problems 39–60, solve each quadratic equation using the method that seems most appropriate.

39. $x^2 + 8x - 48 = 0$
40. $x^2 + 5x - 14 = 0$
41. $2n^2 - 8n = -3$
42. $3x^2 + 6x = 1$
43. $(3x - 1)(2x + 9) = 0$
44. $(5x + 2)(x - 4) = 0$
45. $(x + 2)(x - 7) = 10$
46. $(x - 3)(x + 5) = -7$
47. $(x - 3)^2 = 12$
48. $x^2 = 16x$
49. $3n^2 - 6n + 4 = 0$
50. $2n^2 - 2n - 1 = 0$
51. $n(n + 8) = 240$
52. $t(t - 26) = -160$
53. $3x^2 + 5x = -2$
54. $2x^2 - 7x = -5$
55. $4x^2 - 8x + 3 = 0$
56. $9x^2 + 18x + 5 = 0$
57. $x^2 + 12x = 4$
58. $x^2 + 6x = -11$
59. $4(2x + 1)^2 - 1 = 11$
60. $5(x + 2)^2 + 1 = 16$
61. Use the method of completing the square to solve $ax^2 + bx + c = 0$ for x, where a, b, and c are real numbers and $a \neq 0$.

Thoughts Into Words

62. Explain the process of completing the square to solve a quadratic equation.
63. Give a step-by-step description of how to solve $3x^2 + 9x - 4 = 0$ by completing the square.

Further Investigations

Solve Problems 64–67 for the indicated variable. Assume that all letters represent positive numbers.

64. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ for y

65. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ for x

66. $s = \dfrac{1}{2}gt^2$ for t

67. $A = \pi r^2$ for r

Solve each of the following equations for x.

68. $x^2 + 8ax + 15a^2 = 0$

69. $x^2 - 5ax + 6a^2 = 0$

70. $10x^2 - 31ax - 14a^2 = 0$

71. $6x^2 + ax - 2a^2 = 0$

72. $4x^2 + 4bx + b^2 = 0$

73. $9x^2 - 12bx + 4b^2 = 0$

Answers to the Concept Quiz

1. False **2.** True **3.** False **4.** False **5.** True **6.** False **7.** True **8.** False **9.** True **10.** True

6.4 Quadratic Formula

OBJECTIVES

1. Use the quadratic formula to solve quadratic equations
2. Determine the nature of roots to quadratic equations

As we saw in the last section, the method of completing the square can be used to solve any quadratic equation. Thus if we apply the method of completing the square to the equation $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, we can produce a formula for solving quadratic equations. This formula can then be used to solve any quadratic equation. Let's solve $ax^2 + bx + c = 0$ by completing the square.

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c \qquad \text{Isolate the } x^2 \text{ and } x \text{ terms}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \qquad \text{Multiply both sides by } \frac{1}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \qquad \frac{1}{2}\left(\frac{b}{a}\right) = \frac{b}{2a} \text{ and } \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

Complete the square by adding $\dfrac{b^2}{4a^2}$ to both sides

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \qquad \text{Common denominator of } 4a^2 \text{ on right side}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} \qquad \text{Commutative property}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \qquad \text{The right side is combined into a single fraction}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \qquad \sqrt{4a^2} = |2a| \text{ but } 2a \text{ can be used because of the use of } \pm$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The quadratic formula is usually stated as follows:

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad a \neq 0$$

We can use the quadratic formula to solve *any* quadratic equation by expressing the equation in the standard form $ax^2 + bx + c = 0$ and substituting the values for a, b, and c into the formula. Let's consider some examples.

Classroom Example
Solve $n^2 - 5n - 9 = 0$.

EXAMPLE 1 Solve $x^2 + 5x + 2 = 0$.

Solution

$$x^2 + 5x + 2 = 0$$

The given equation is in standard form with $a = 1$, $b = 5$, and $c = 2$. Let's substitute these values into the formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 8}}{2}$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

The solution set is $\left\{\frac{-5 \pm \sqrt{17}}{2}\right\}$.

Classroom Example
Solve $a^2 + 8a + 5 = 0$.

EXAMPLE 2 Solve $x^2 - 2x - 4 = 0$.

Solution

$$x^2 - 2x - 4 = 0$$

We need to think of $x^2 - 2x - 4 = 0$ as $x^2 + (-2)x + (-4) = 0$ to determine the values $a = 1$, $b = -2$, and $c = -4$. Let's substitute these values into the quadratic formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 16}}{2}$$

$$x = \frac{2 \pm \sqrt{20}}{2}$$

$$x = \frac{2 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2(1 \pm \sqrt{5})}{2} \qquad \text{Factor out a 2 in the numerator}$$

$$x = \frac{\cancel{2}(1 \pm \sqrt{5})}{\cancel{2}} = 1 \pm \sqrt{5}$$

The solution set is $\{1 \pm \sqrt{5}\}$.

Classroom Example
Solve $f^2 - 8f + 18 = 0$.

EXAMPLE 3 Solve $x^2 - 2x + 19 = 0$.

Solution

$$x^2 - 2x + 19 = 0$$

We can substitute $a = 1$, $b = -2$, and $c = 19$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(19)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - 76}}{2}$$

$$x = \frac{2 \pm \sqrt{-72}}{2}$$

$$x = \frac{2 \pm 6i\sqrt{2}}{2} \qquad \sqrt{-72} = i\sqrt{72} = i\sqrt{36}\sqrt{2} = 6i\sqrt{2}$$

$$x = \frac{2(1 \pm 3i)}{2} \qquad \text{Factor out a 2 in the numerator}$$

$$x = \frac{\cancel{2}(1 \pm 3i\sqrt{2})}{\cancel{2}} = 1 \pm 3i\sqrt{2}$$

The solution set is $\{1 \pm 3i\sqrt{2}\}$.

Classroom Example
Solve $2b^2 + 6b - 5 = 0$.

EXAMPLE 4 Solve $2x^2 + 4x - 3 = 0$.

Solution

$$2x^2 + 4x - 3 = 0$$

Here $a = 2$, $b = 4$, and $c = -3$. Solving by using the quadratic formula is unlike solving by completing the square in that there is no need to make the coefficient of x^2 equal to 1.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{16 + 24}}{4}$$

$$x = \frac{-4 \pm \sqrt{40}}{4}$$

$$x = \frac{-4 \pm 2\sqrt{10}}{4}$$

$$x = \frac{2(-2 \pm \sqrt{10})}{4} \qquad \text{Factor out a 2 in the numerator}$$

$$x = \frac{\cancel{2}(-2 \pm \sqrt{10})}{\cancelto{2}{4}} = \frac{-2 \pm \sqrt{10}}{2}$$

The solution set is $\left\{\frac{-2 \pm \sqrt{10}}{2}\right\}$.

Classroom Example
Solve $x(5x - 7) = 6$.

EXAMPLE 5 Solve $n(3n - 10) = 25$.

Solution

$$n(3n - 10) = 25$$

First, we need to change the equation to the standard form $an^2 + bn + c = 0$.

$$n(3n - 10) = 25$$
$$3n^2 - 10n = 25$$
$$3n^2 - 10n - 25 = 0$$

Now we can substitute $a = 3$, $b = -10$, and $c = -25$ into the quadratic formula.

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(-25)}}{2(3)}$$

$$n = \frac{10 \pm \sqrt{100 + 300}}{2(3)}$$

$$n = \frac{10 \pm \sqrt{400}}{6}$$

$$n = \frac{10 \pm 20}{6}$$

$$n = \frac{10 + 20}{6} \quad \text{or} \quad n = \frac{10 - 20}{6}$$

$$n = 5 \quad \text{or} \quad n = -\frac{5}{3}$$

The solution set is $\left\{-\frac{5}{3}, 5\right\}$.

In Example 5, note that we used the variable n. The quadratic formula is usually stated in terms of x, but it certainly can be applied to quadratic equations in other variables. Also note in Example 5 that the polynomial $3n^2 - 10n - 25$ can be factored as $(3n + 5)(n - 5)$. Therefore, we could also solve the equation $3n^2 - 10n - 25 = 0$ by using the factoring approach. Section 6.5 will offer some guidance about which approach to use for a particular equation.

Determining the Nature of Roots of Quadratic Equations

The quadratic formula makes it easy to determine the nature of the roots of a quadratic equation without completely solving the equation. The number

$$b^2 - 4ac$$

which appears under the radical sign in the quadratic formula, is called the **discriminant** of the quadratic equation. The discriminant is the indicator of the kind of roots the equation has. For example, suppose that you start to solve the equation $x^2 - 4x + 7 = 0$ as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 28}}{2}$$

$$x = \frac{4 \pm \sqrt{-12}}{2}$$

At this stage you should be able to look ahead and realize that you will obtain two nonreal complex solutions for the equation. (Note, by the way, that these solutions are complex conjugates.) In other words, the discriminant (-12) indicates what type of roots you will obtain.

We make the following general statements relative to the roots of a quadratic equation of the form $ax^2 + bx + c = 0$.

1. If $b^2 - 4ac < 0$, then the equation has two nonreal complex solutions.
2. If $b^2 - 4ac = 0$, then the equation has one real solution.
3. If $b^2 - 4ac > 0$, then the equation has two real solutions.

The following examples illustrate each of these situations. (You may want to solve the equations completely to verify the conclusions.)

Equation	Discriminant	Nature of roots
$x^2 - 3x + 7 = 0$	$b^2 - 4ac = (-3)^2 - 4(1)(7)$ $= 9 - 28$ $= -19$	Two nonreal complex solutions
$9x^2 - 12x + 4 = 0$	$b^2 - 4ac = (-12)^2 - 4(9)(4)$ $= 144 - 144$ $= 0$	One real solution
$2x^2 + 5x - 3 = 0$	$b^2 - 4ac = (5)^2 - 4(2)(-3)$ $= 25 + 24$ $= 49$	Two real solutions

Remark: A clarification is called for at this time. Previously, we made the statement that if $b^2 - 4ac = 0$, then the equation has one real solution. Technically, such an equation has two solutions, but they are equal. For example, each factor of $(x - 7)(x - 7) = 0$ produces a solution, but both solutions are the number 7. We sometimes refer to this as one real solution with a *multiplicity of two*. Using the idea of multiplicity of roots, we can say that every quadratic equation has two roots.

Classroom Example
Use the discriminant to determine whether the equation $3x^2 - 7x + 2 = 0$ has two nonreal complex solutions, one real solution with a multiplicity of 2, or two real solutions.

EXAMPLE 6

Use the discriminant to determine if the equation $5x^2 + 2x + 7 = 0$ has two nonreal complex solutions, one real solution with a multiplicity of two, or two real solutions.

Solution

For the equation $5x^2 + 2x + 7 = 0$, $a = 5$, $b = 2$, and $c = 7$.

$$\begin{aligned} b^2 - 4ac &= (2)^2 - 4(5)(7) \\ &= 4 - 140 \\ &= -136 \end{aligned}$$

Because the discriminant is negative, the solutions will be two nonreal complex numbers.

Most students become very adept at applying the quadratic formula to solve quadratic equations but make errors when reducing the answers. The next example shows two different methods for simplifying the answers.

Classroom Example
Solve $7m^2 + 4m - 2 = 0$.

EXAMPLE 7 Solve $3x^2 - 8x + 2 = 0$.

Solution

Here $a = 3$, $b = -8$, and $c = 2$. Let's substitute these values into the quadratic formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{8 \pm \sqrt{64 - 24}}{6}$$

$$x = \frac{8 \pm \sqrt{40}}{6} = \frac{8 \pm 2\sqrt{10}}{6} \qquad \sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$$

Now to simplify, one method is to factor 2 out of the numerator and reduce.

$$x = \frac{8 \pm 2\sqrt{10}}{6} = \frac{2(4 \pm \sqrt{10})}{6} = \frac{\cancel{2}(4 \pm \sqrt{10})}{\underset{3}{\cancel{6}}} = \frac{4 \pm \sqrt{10}}{3}$$

Another method for simplifying the answer is to write the result as two separate fractions and reduce each fraction.

$$x = \frac{8 \pm 2\sqrt{10}}{6} = \frac{8}{6} \pm \frac{2\sqrt{10}}{6} = \frac{4}{3} \pm \frac{\sqrt{10}}{3} = \frac{4 \pm \sqrt{10}}{3}$$

Be very careful when simplifying your result because that is a common source of incorrect answers.

Concept Quiz 6.4

For Problems 1–10, answer true or false.

1. The quadratic formula can be used to solve any quadratic equation.
2. The number $\sqrt{b^2 - 4ac}$ is called the discriminant of the quadratic equation.
3. Every quadratic equation will have two solutions.
4. The quadratic formula cannot be used if the quadratic equation can be solved by factoring.
5. To use the quadratic formula for solving the equation $3x^2 + 2x - 5 = 0$, you must first divide both sides of the equation by 3.
6. The equation $9x^2 + 30x + 25 = 0$ has one real solution with a multiplicity of 2.
7. The equation $2x^2 + 3x + 4 = 0$ has two nonreal complex solutions.
8. The equation $x^2 + 9 = 0$ has two real solutions.
9. Because the quadratic formula has a denominator, it could be simplified and written as $x = -b \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$.
10. Rachel reduced the result $x = \dfrac{6 \pm 5\sqrt{7}}{2}$ to obtain $x = 3 \pm \dfrac{5\sqrt{7}}{2}$. Her result is correct.

Problem Set 6.4

For Problems 1–10, simplify and reduce each expression.

1. $\dfrac{2 \pm \sqrt{20}}{4}$

2. $\dfrac{4 \pm \sqrt{20}}{6}$

3. $\dfrac{-6 \pm \sqrt{27}}{3}$

4. $\dfrac{-9 \pm \sqrt{54}}{3}$

5. $\dfrac{6 \pm \sqrt{18}}{9}$

6. $\dfrac{12 \pm \sqrt{32}}{8}$

7. $\dfrac{-10 \pm \sqrt{75}}{10}$

8. $\dfrac{-4 \pm \sqrt{8}}{4}$

9. $\dfrac{-6 \pm \sqrt{48}}{4}$

10. $\dfrac{-8 \pm \sqrt{72}}{4}$

For Problems 11–50, use the quadratic formula to solve each of the quadratic equations. **(Objective 1)**

11. $x^2 + 2x - 1 = 0$

12. $x^2 + 4x - 1 = 0$

13. $n^2 + 5n - 3 = 0$

14. $n^2 + 3n - 2 = 0$

15. $a^2 - 8a = 4$

16. $a^2 - 6a = 2$

17. $n^2 + 5n + 8 = 0$

18. $2n^2 - 3n + 5 = 0$

19. $x^2 - 18x + 80 = 0$

20. $x^2 + 19x + 70 = 0$

21. $-y^2 = -9y + 5$

22. $-y^2 + 7y = 4$

23. $2x^2 + x - 4 = 0$

24. $2x^2 + 5x - 2 = 0$

25. $4x^2 + 2x + 1 = 0$

26. $3x^2 - 2x + 5 = 0$

27. $3a^2 - 8a + 2 = 0$

28. $2a^2 - 6a + 1 = 0$

29. $-2n^2 + 3n + 5 = 0$

30. $-3n^2 - 11n + 4 = 0$

31. $3x^2 + 19x + 20 = 0$

32. $2x^2 - 17x + 30 = 0$

33. $36n^2 - 60n + 25 = 0$

34. $9n^2 + 42n + 49 = 0$

35. $4x^2 - 2x = 3$

36. $6x^2 - 4x = 3$

37. $5x^2 - 13x = 0$

38. $7x^2 + 12x = 0$

39. $3x^2 = 5$

40. $4x^2 = 3$

41. $6t^2 + t - 3 = 0$

42. $2t^2 + 6t - 3 = 0$

43. $n^2 + 32n + 252 = 0$

44. $n^2 - 4n - 192 = 0$

45. $12x^2 - 73x + 110 = 0$

46. $6x^2 + 11x - 255 = 0$

47. $-2x^2 + 4x - 3 = 0$

48. $-2x^2 + 6x - 5 = 0$

49. $-6x^2 + 2x + 1 = 0$

50. $-2x^2 + 4x + 1 = 0$