1.1 Sets, Real Numbers, and Numerical Expressions

OBJECTIVES

1. Identify certain sets of numbers
2. Apply the properties of equality
3. Simplify numerical expressions

In arithmetic, we use symbols such as 6, $\frac{2}{3}$, 0.27, and π to represent numbers. The symbols $+$, $-$, $\cdot$, and $\div$ commonly indicate the basic operations of addition, subtraction, multiplication, and division, respectively. Thus we can form specific **numerical expressions**. For example, we can write the indicated sum of six and eight as $6 + 8$.

In algebra, the concept of a variable provides the basis for generalizing arithmetic ideas. For example, by using x and y to represent any numbers, we can use the expression $x + y$ to represent the indicated sum of any two numbers. The x and y in such an expression are called **variables**, and the phrase $x + y$ is called an **algebraic expression**.

We can extend to algebra many of the notational agreements we make in arithmetic, with a few modifications. The following chart summarizes the notational agreements that pertain to the four basic operations.

Operation	Arithmetic	Algebra	Vocabulary
Addition	$4 + 6$	$x + y$	The *sum* of x and y
Subtraction	$14 - 10$	$a - b$	The *difference* of a and b
Multiplication	$7 \cdot 5$ or 7×5	$a \cdot b$, $a(b)$, $(a)b$, $(a)(b)$, or ab	The *product* of a and b
Division	$8 \div 4$, $\frac{8}{4}$, or $4\overline{)8}$	$x \div y$, $\frac{x}{y}$, or $y\overline{)x}$	The *quotient* of x and y

Note the different ways to indicate a product, including the use of parentheses. The ab form is the simplest and probably the most widely used form. Expressions such as abc, $6xy$, and $14xyz$ all indicate multiplication. We also call your attention to the various forms that indicate division; in algebra, we usually use the fractional form $\frac{x}{y}$ although the other forms do serve a purpose at times.

Use of Sets

We can use some of the basic vocabulary and symbolism associated with the concept of sets in the study of algebra. A **set** is a collection of objects, and the objects are called **elements** or **members** of the set. In arithmetic and algebra the elements of a set are usually numbers.

The use of set braces, { }, to enclose the elements (or a description of the elements) and the use of capital letters to name sets provide a convenient way to communicate about sets. For example, we can represent a set A, which consists of the vowels of the alphabet, in any of the following ways:

$A = \{\text{vowels of the alphabet}\}$ Word description

$A = \{a, e, i, o, u\}$ List or roster description

$A = \{x \mid x \text{ is a vowel}\}$ Set builder notation

We can modify the listing approach if the number of elements is quite large. For example, all of the letters of the alphabet can be listed as

$\{a, b, c, \ldots, z\}$

We simply begin by writing enough elements to establish a pattern; then the three dots indicate that the set continues in that pattern. The final entry indicates the last element of the pattern. If we write

$\{1, 2, 3, \ldots\}$

the set begins with the counting numbers 1, 2, and 3. The three dots indicate that it continues in a like manner forever; there is no last element. A set that consists of no elements is called the **null set** (written $\varnothing$).

Set builder notation combines the use of braces and the concept of a variable. For example, $\{x|x$ is a vowel$\}$ is read "the set of all x such that x is a vowel." Note that the vertical line is read "such that." We can use set builder notation to describe the set $\{1, 2, 3, \ldots\}$ as $\{x|x > 0$ and x is a whole number$\}$.

We use the symbol $\in$ to denote set membership. Thus if $A = \{a, e, i, o, u\}$, we can write $e \in A$, which we read as "e is an element of A." The slash symbol, /, is commonly used in mathematics as a negation symbol. For example, $m \notin A$ is read as "m is not an element of A."

Two sets are said to be *equal* if they contain exactly the same elements. For example,

$\{1, 2, 3\} = \{2, 1, 3\}$

because both sets contain the same elements; the order in which the elements are written doesn't matter. The slash mark through the equality symbol denotes "is not equal to." Thus if $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$, we can write $A \neq B$, which we read as "set A is not equal to set B."

Real Numbers

We refer to most of the algebra that we will study in this text as the **algebra of real numbers**. This simply means that the variables represent real numbers. Therefore, it is necessary for us to be familiar with the various terms that are used to classify different types of real numbers.

$\{1, 2, 3, 4, \ldots\}$	Natural numbers, counting numbers, positive integers
$\{0, 1, 2, 3, \ldots\}$	Whole numbers, nonnegative integers
$\{\ldots -3, -2, -1\}$	Negative integers
$\{\ldots -3, -2, -1, 0\}$	Nonpositive integers
$\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$	Integers

We define a **rational number** as follows:

Definition 1.1 Rational Numbers

A rational number is any number that can be written in the form $\frac{a}{b}$, where a and b are integers, and b does not equal zero.

We can easily recognize that each of the following numbers fits the definition of a rational number.

$$\frac{-3}{4} \quad \frac{2}{3} \quad \frac{15}{4} \quad \text{and} \quad \frac{1}{-5}$$