[redacted]

hand rule **Relation between linear acc and angular acceleration**

Linear velocity "v" and angular "ω" of a particle when it moves along a circle of radius "r" are related as: We know that.

$$V = r\omega$$

differentiate above Eq. on both sides with respect to time.

$$\frac{dv}{dt} = r \cdot \frac{d\omega}{dt}$$

In Limiting case $\left(\frac{dv}{dt}\right)$ is called tangential component of Linear accelration "a" and $\left(\frac{d\omega}{dt}\right)$ is called angular accelrat "α" of rotating body.

So above relation can be written as:

$$\boxed{a = r\alpha}$$

In vector form $\vec{a} = \vec{\alpha} \times \vec{r}$

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Rotational Kinetic Energy :-

The K.E possessed by a body due to its rotation about its own axis is called rotational K.E or angular K.E.

The translational K.E of a body having mass "m" moving with velocity "v" is given as:

$$(K.E)_t = \frac{1}{2} m v^2$$

For a circular motion we know that

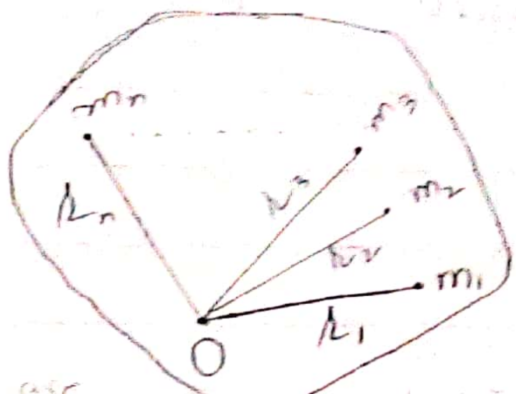
$$V = r\omega$$

putting value of "v" in above Eq.

$$(K.E)_t = \frac{1}{2} m (r\omega)^2$$

$$\boxed{(K.E)_t = \frac{1}{2} m r^2 \omega^2}$$

Rotational K.E of rigid body :-



Consider a rigid body which is collection of "n" mass particles. The distance of particles having masses $m_1, m_2, m_3, \dots, m_n$ from the axis of rotation "O" is $r_1, r_2, r_3, \dots, r_n$ respective.

The rotational K.E of mass m_1 :

$$(K.E)_1 = \frac{1}{2} m_1 r_1^2 \omega^2$$

The rotational K.E of mass m_2 :

$$(K.E)_2 = \frac{1}{2} m_2 r_2^2 \omega^2$$

ω : angular velocity is constant

The rotational K.E of mass m_3 :

$$(K.E)_3 = \frac{1}{2} m_3 r_3^2 \omega^2$$

and $(K.E)$ of mass m_n :

$$(K.E)_n = \frac{1}{2} m_n r_n^2 \omega^2$$

The total K.E of all particles is called rotational K.E of rigid body.

$$(K.E)_{rot} = (K.E)_1 + (K.E)_2 + (K.E)_3 + \cdots + (K.E)_n$$

$$(K.E)_{rot} = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + \cdots \cdots + \frac{1}{2} m_n r_n^2 \omega^2$$

$$= \frac{1}{2} \left(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots - m_n r_n \right) \omega^2$$

$$= \frac{1}{2} \sum_{i=1}^{n} m_i r_i^2 \omega^2$$

$$= \frac{1}{2} I \omega^2$$

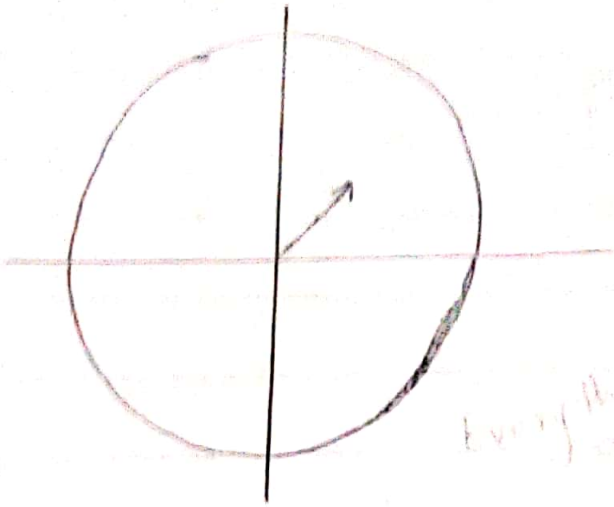
Where

$I = \sum_{i=1}^{n} m_i r_i^2$ is the moment of inertia of a rigid body.

Therefore above equation becomes

$$(K.E)_{rot} = \frac{1}{2} I \omega^2$$

Rotational K.E of Disc :-



The energy possessed by a disc due to its spin motion about its own axis of rotation is called rotational K.E of disc.

$$(K.E)_{rot} = \frac{1}{2} I\omega^2$$

We know that

$$\boxed{V = r\omega} \quad \text{or} \quad \omega = \frac{V}{r}$$

putting this value of "ω" and "I" in above Eq:

$$(K.E)_{rot} = \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} . I \left(\frac{V}{r}\right)^2$$

$$= \frac{1}{2}\left(\frac{1}{2} m r^2\right)\frac{V^2}{r^2} \qquad \because I = \frac{1}{2} m r^2 \text{ (disc)}$$

$$(K.E)_{rot} = \frac{1}{4} mv^2$$

Rotational K.E of Hoop :-

The energy possessed by a body due to its spin motion about its own axis of rotation is called rotational K.E of hoop.

$$(K.E)_{rot} = \frac{1}{2} I\omega^2$$

We know that

$V = r\omega$ or $\omega = \frac{V}{r}$

putting value of "ω" and "I" in above Eq:

$$(K.E)_{rot} = \frac{1}{2} I\omega^2$$

$\because I = mr^2$ (hoop)

$$(K.E)_{rot} = \frac{1}{2} mr^2 \cdot \left(\frac{V}{r}\right)^2$$

$$(K.E)_{rot} = \frac{1}{2} m\cancel{r^2} \frac{V^2}{\cancel{r^2}}$$

$$\boxed{(K.E)_{rot} = \frac{1}{2} mv^2}$$

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Rotational K.E of a Sphere:-

(1) Hollow Sphere
(2) Solid Sphere

The energy possessed by a body (sphere) due to its spin motion about its own axis of rotation are called rotational K.E of Sphere.

$$(K.E)_{rot} = \frac{1}{2} I \omega^2$$

We know that

$$V = r\omega \quad \text{or} \quad \omega = \frac{V}{r}$$

putting values of "ω" and "I" in above Eq:

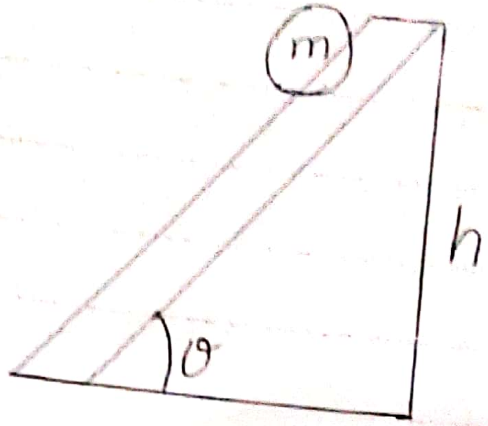
$$(K.E)_{rot} = \frac{1}{2} I \omega^2$$

$\because I = \frac{2}{5} mr^2$ (sphere)

$$(K.E)_{rot} = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{5} m\cancel{r^2}\right)\frac{V^2}{\cancel{r^2}}$$

$$(K.E)_{rot} = \frac{1}{5} mV^2$$

Velocity of disc, hoop, sphere w... at the bottom of inclined plane (Rolling without slipping) :-



Velocity of disc :-

Consider a disc of mass "m" is placed at the top of inclined plane having height "h" as shown in figure.

The total energy is P.E at the top of inclined plane. The total energy is the sum of translational and rotational K.E when it rolls down the inclined plane.

The velocity of the disc

when it Reaches at the bottom of nclined plane.

By the law of conservation of energy

Loss in P.E = Gain in $[(K.E)_t + (K.E)_{rot}]$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$\cancel{m}gh = \cancel{m}\left(\frac{1}{2}v^2 + \frac{1}{4}v^2\right)$$

$$gh = \frac{3}{4}v^2$$

$$v^2 = \frac{4}{3}gh$$

$$v = \sqrt{\frac{4}{3}gh}$$

Velocity of hoop :-

Consider a hoop of mass "m" is placed at the top of inclined plane having height "h" as shown in figure

The total energy is the P.E at the top of inclined plane. The total energy is the sum of translational and rotational K.E when it down the inclined plane.

The velocity of hoop when it reaches at the bottom of inclined plane. By the Law of conservation of energy

Loss in P.E = Gain in $[(K.E)_t + (K.E)_{rot}]$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\cancel{m}gh = \cancel{m}\left(\frac{1}{2}v^2 + \frac{1}{2}v^2\right)$$

$$gh = v^2$$

$$v = \sqrt{gh}$$

Velocity of Sphere :-

Consider a sphere of mass "m" is placed at the top of inclined plane having height "h" as shown in figure. The total energy is the P.E at the top of inclined plane. The total energy is the sum of translation and rotational K E when it rolls down the inclined plane.

The velocity of the sphere when it reaches at the bottom of inclined plane.

By law of conservation of Energy :

Loss in P.E = Gain in $[(K.E)_t + (K.E)_{rot}]$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$\cancel{m}gh = \cancel{m}\left[\frac{1}{2}v^2 + \frac{1}{5}v^2\right]$$

$$gh = \frac{7}{10}v^2$$

$$v = \sqrt{\frac{10}{7}gh}$$

velocity of sphere $= 1.195\sqrt{gh}$

velocity of disc $= 1.154\sqrt{gh}$

velocity of hoop $= \sqrt{gh}$

$V_{sphere} > V_{disc} > V_{hoop}$