Summation Notation

Sometimes a special notation is used to indicate the sum of a certain number of terms of a sequence. The capital Greek letter *sigma,* Σ, is used as a **summation symbol**. For example,

$$\sum_{i=1}^{5} a_i$$

represents the sum $a_1 + a_2 + a_3 + a_4 + a_5$. The letter i is frequently used as the **index of summation**; the letter i takes on all integer values from the lower limit to the upper limit, inclusive. Thus

$$\sum_{i=1}^{4} b_i = b_1 + b_2 + b_3 + b_4$$

$$\sum_{i=3}^{7} a_i = a_3 + a_4 + a_5 + a_6 + a_7$$

$$\sum_{i=1}^{15} i^2 = 1^2 + 2^2 + 3^2 + \cdots + 15^2$$

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

If $a_1, a_2, a_3, \ldots$ represents an arithmetic sequence, we can now write the sum formula

$$\sum_{i=1}^{n} a_i = \frac{n}{2}(a_1 + a_n)$$

Classroom Example
Find the sum $\sum_{i=1}^{28} (5i - 3)$.

EXAMPLE 8

Find the sum $\sum_{i=1}^{50} (3i + 4)$.

Solution

This indicated sum means

$$\sum_{i=1}^{50} (3i + 4) = [3(1) + 4] + [3(2) + 4] + [3(3) + 4] + \cdots + [3(50) + 4]$$

$$= 7 + 10 + 13 + \cdots + 154$$

Because this is an indicated sum of an arithmetic sequence, we can use our sum formula:

$$S_{50} = \frac{50}{2}(7 + 154) = 4025$$

Classroom Example
Find the sum $\sum_{i=4}^{9} 3i^2$.

EXAMPLE 9

Find the sum $\sum_{i=2}^{7} 2i^2$.

Solution

This indicated sum means

$$\sum_{i=2}^{7} 2i^2 = 2(2)^2 + 2(3)^2 + 2(4)^2 + 2(5)^2 + 2(6)^2 + 2(7)^2$$

$$= 8 + 18 + 32 + 50 + 72 + 98$$

This is not the indicated sum of an *arithmetic* sequence; therefore let's simply add the numbers in the usual way. The sum is 278.

Example 9 suggests a word of caution. Be sure to analyze the sequence of numbers that is represented by the summation symbol. You may or may not be able to use a formula for adding the numbers.

Concept Quiz 14.1

For Problems 1–8, answer true or false.

1. An infinite sequence is a function whose domain is the set of all real numbers.
2. An arithmetic sequence is a sequence that has a common difference between successive terms.
3. The sequence 2, 4, 8, 16, . . . is an arithmetic sequence.
4. The odd whole numbers form an arithmetic sequence.
5. The terms of an arithmetic sequence are always positive.
6. The 6th term of an arithmetic sequence is equal to the first term plus 6 times the common difference.
7. The sum formula for n terms of an arithmetic sequence is n times the average of the first and last terms.
8. The indicated sum $\sum_{i=1}^{4}(2i-7)^2$ is the sum of the first four terms of an arithmetic sequence.

Problem Set 14.1

For Problems 1–10, write the first five terms of the sequence that has the indicated general term. (Objective 1)

1. $a_n = 3n - 7$
2. $a_n = 5n - 2$
3. $a_n = -2n + 4$
4. $a_n = -4n + 7$
5. $a_n = 3n^2 - 1$
6. $a_n = 2n^2 - 6$
7. $a_n = n(n - 1)$
8. $a_n = (n + 1)(n + 2)$
9. $a_n = 2^{n+1}$
10. $a_n = 3^{n-1}$
11. Find the 15th and 30th terms of the sequence when $a_n = -5n - 4$.
12. Find the 20th and 50th terms of the sequence when $a_n = -n - 3$.
13. Find the 25th and 50th terms of the sequence when $a_n = (-1)^{n+1}$.
14. Find the 10th and 15th terms of the sequence when $a_n = -n^2 - 10$.

For Problems 15–24, find the general term (the nth term) for each arithmetic sequence. (Objective 2)

15. 11, 13, 15, 17, 19, . . .
16. 7, 10, 13, 16, 19, . . .
17. 2, −1, −4, −7, −10, . . .
18. 4, 2, 0, −2, −4, . . .
19. $\frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$
20. $0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$
21. 2, 6, 10, 14, 18, . . .
22. 2, 7, 12, 17, 22, . . .
23. −3, −6, −9, −12, −15, . . .
24. −4, −8, −12, −16, −20, . . .

For Problems 25–30, find the required term for each arithmetic sequence. (Objective 3)

25. The 15th term of 3, 8, 13, 18, . . .
26. The 20th term of 4, 11, 18, 25, . . .
27. The 30th term of 15, 26, 37, 48, . . .
28. The 35th term of 9, 17, 25, 33, . . .
29. The 52nd term of $1, \frac{5}{3}, \frac{7}{3}, 3, \ldots$
30. The 47th term of $\frac{1}{2}, \frac{5}{4}, 2, \frac{11}{4}, \ldots$

For Problems 31–42, solve each problem.

31. If the 6th term of an arithmetic sequence is 12 and the 10th term is 16, find the first term.
32. If the 5th term of an arithmetic sequence is 14 and the 12th term is 42, find the first term.
33. If the 3rd term of an arithmetic sequence is 20 and the 7th term is 32, find the 25th term.
34. If the 5th term of an arithmetic sequence is −5 and the 15th term is −25, find the 50th term.
35. Find the sum of the first 50 terms of the arithmetic sequence 5, 7, 9, 11, 13, ...