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A Generalization of the Jenkins-Traub Method

By J. A. Ford

Abstract. A class of methods for finding zeros of polynomials is derived which depends upon an arbitrary parameter ρ. The Jenkins-Traub algorithm is a special case, corresponding to the choice $\rho = \infty$. Global convergence is proved for large and small values of ρ and a duality between pairs of members is exhibited. Finally, we show that many members of the class (including the Jenkins-Traub method) converge with R-order at least 2.618 . . . , which improves upon the result obtained by Jenkins and Traub [3].

1. Introduction. A well-known and effective method for computing the zeros of a polynomial has been presented by Jenkins and Traub [3]. Their algorithm consists of three stages, the first of which simply serves to accentuate the smaller zeros. The second stage implements a fixed-shift process which isolates one of the zeros, while the third stage may be viewed as a variable-shift process which converges very quickly to the zero. The third stage may also be viewed as a Newton-Raphson iteration performed on a sequence of rational functions. Another interpretation of the third stage arises from showing that it is equivalent to a generalized Rayleigh iteration applied to the companion matrix of the polynomial. Jenkins and Traub [3] have, in a detailed analysis of their method, shown it to be globally convergent and, furthermore, that the convergence to a zero is faster than second-order.

In this paper, we shall define and investigate a class of methods for the solution of polynomials which contains the Jenkins-Traub method as a special case. We shall analyze both the global and local convergence of members of the class and, in particular, obtain an improved result concerning the local convergence of the Jenkins-Traub method.

We consider a polynomial $P(z)$, where

$$P(z) \equiv \sum_{i=0}^{n} a_i z^i, \qquad a_0 a_n \neq 0. \tag{1.1}$$

(Note the reversal of the order of the coefficients from that used by Jenkins and Traub.) We denote the *distinct* zeros of P by $\{\rho_i\}_{i=1}^{j}$ and their respective multiplicities by $\{m_i\}_{i=1}^{j}$, so that

$$P(z) \equiv a_n \prod_{i=1}^{j} (z - \rho_i)^{m_i}, \qquad \sum_{i=1}^{j} m_i = n. \tag{1.2}$$

Finally, we denote the deflated polynomials obtained from P by

$$P_i(z) \equiv P(z)/(z - \rho_i), \qquad i = 1(1)j. \tag{1.3}$$

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