The number of seconds of half life = 1580 × 365 × 24 × 60 × 60 (ii)

$$\text{Number of moles of } {}^{226}\text{Ra} = \frac{1\ \text{g}}{226\ \text{g mol}^{-1}}$$

Number of atoms of ^{226}Ra in 1 g of it (N) (iii)

$$= \text{number moles} \times 6.02 \times 10^{23}$$

Putting $t_{0.5}$ and N from equation (ii) and (iii) in equation (i).

$$A = \frac{0.693}{1580 \times 365 \times 24 \times 60 \times 60\ \text{s}} \times \frac{6.02 \times 10^{23}\ \text{mol}^{-1} \times 1\ \text{g}}{226\ \text{g mol}^{-1}}$$

$$A = \frac{0.693 \times 6.022 \times 10^{23} \times 1}{1580 \times 365 \times 24 \times 60 \times 60 \times 226}\ \text{dis. s}^{-1}$$

or $$A = 3.70 \times 10^{10}\ \text{dis. s}^{-1}$$

Sample Problem (13.7):

Calculate the weight of $^{14}_{6}$C atoms (half-life = 5720 yrs), which give 3.7 × 10^7 disintegrations per second.

Hint: If the weight of $^{14}_{6}$C be x gram, then and formula is $A = \lambda N = \frac{0.693}{t_{0.5}} N$.

$$3.70 \times 10^{7}\ \text{s}^{-1} = \frac{0.693}{5720 \times 365 \times 24 \times 60 \times 60\ \text{s}} \times \frac{(6.022 \times 10^{23}) \times x}{14}$$

or $$3.70 \times 10^{7}\ \text{s}^{-1} = 3.83 \times 10^{-12} \times 6.022 \times 10^{23} \times \frac{x}{14}\ \text{s}^{-1}$$

∴ $$x = \frac{14 \times 3.70 \times 10^{7}}{3.83 \times 10^{-12} \times 6.022 \times 10^{23}}\ \text{g} = \boxed{0.224 \times 10^{-3}\ \text{g}}$$

13.4 Factor Affecting the Nuclear Stability

Here are some important factors on which the nuclear stability depends,

(i) Packing fraction (f)

(ii) Binding energy per nucleon ($\overline{\text{B}}$)

(iii) Nuclear shell model : Magic numbers

(iv) Even and odd number of protons (p) and neutrons (n)

(v) Neutron - to - protons ratio (n/p ratio);

All these factors are discussed below:

13.4.1 Packing Fraction (F) and Stability of a Nucleus:

Mass spectroscopy (کمیت کی بنیاد پر علیحدگی) is one of important branches of spectroscopy. **Mass spectrograph** has enabled us to determine the **extact value of the mass of a given** isotope (کسی آئسوٹوپ کی صحیح کمیت). This is called **isotopic mass** or actual atomic mass of the isotope. It has been found that the **isotopic mass** (آئسوٹوپ کی کمیت) **is not the same as the mass number** (ماس نمبر) **of the isotope** which is a whole number, because it is the sum of the protons and neutrons. Isotopic mass is determined by **mass spectrograph.**

When the difference namely **(isotopic mass – mass number) × 10^4** is divided by mass number, we get what is known as **packing fraction (f)** of the isotope. Thus

Packing fraction (پیکنگ فریکشن) $(f) = \frac{\text{(Isotopic mass - Mass number)}}{\text{Mass number}} \times 10^4 = \frac{M - A}{A} \times 10^4$

Since the mass number is the sum of the protons (p) and neutrons (n) present in the nucleus of the isotope (called nucleons), the above equation can also be written as

(اصل میں ماس نمبر تمام آئسوٹوپس کی کمیتوں کے اوسط کے قریب ہوتا ہے۔)

$$\text{Packing fraction } (f) = \frac{\text{Isotopic mass - Mass number}}{\text{Nucleons}} \times 10^4$$

or

$$\boxed{f = \frac{M - A}{n + p} \times 10^4}$$

13.4.2 Variation of Stability of a Nucleus with its Packing Fraction:

The stability of a nuclide depends on the value of its packing fraction as given below.

(a) When value of packing fraction of a nuclide is a **negative quantity** (منفی مقدار), then the nuclide would be **stable.** For example, the values of packing fraction of ^{16}O and ^{40}Ar nuclides are negative i.e., $^{16}O = -3.6875$ and $^{40}Ar = -9.404$. Both these nuclei are very stable. Negative value of packing fraction is obtained **when isotopic mass is less than mass number.**

(b) When a nuclide has a low positive value (کم مثبت قیمت) for its packing fraction, then the nuclide would be stable. i.e., since ^{12}C and ^{4}He both have low positive values for their packing fraction $^{12}C = +3.1666$ and $^{4}He = +6.5$. Both these nuclides are simply stable. **Those nuclei which have very high positive values of packing fraction are unstable and hence such nuclei undergo spontaneous disintegration.**

(اگر P.F. زیادہ مثبت ہو تو نیوکلیس کافی غیر مستحکم ہو گا اور اپنے آپ ٹوٹ پھوٹ کا شکار ہو جائے گا۔)

Let us look at Fig. (13.4), which is a graph of packing fraction against the mass number of various isotopes.

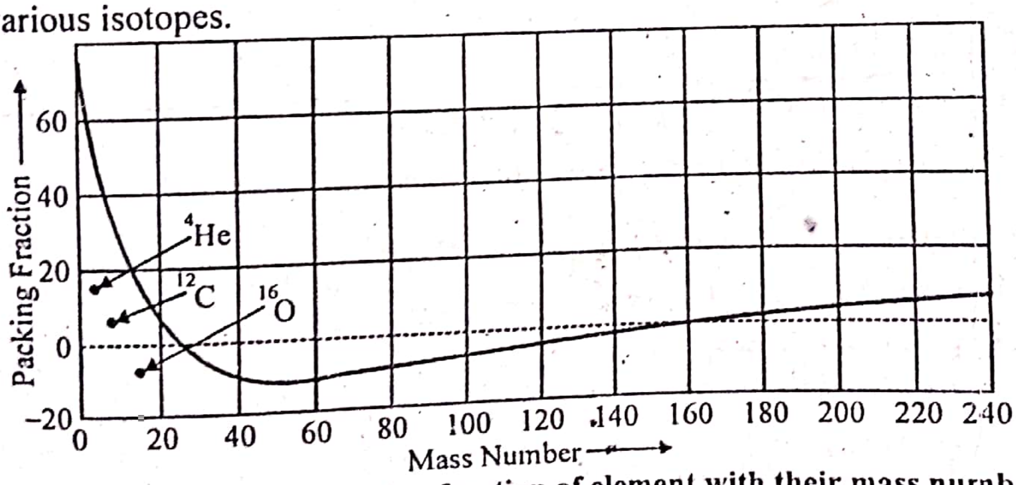

Fig. (13.4): Variation of packing fraction of element with their mass number.

From Fig (13.5), the following points are very clearly understood.

(i) $^{12}_{6}C$ and $^{4}_{2}He$ have low positive values for their packing fraction i.e., +3.1666 and +6.5 respectively, hence these are stable nuclei.

(ii) ^{16}O has negative value of its packing fraction i.e., −3.6875 and hence its nucleus is stable.

(iii) The transition elements like ${}_{42}$**Mo**, ${}_{43}$**Tc**, ${}_{44}$**Ru**, ${}_{45}$**Rh**, ${}_{46}$**Pd**, ${}_{47}$**Ag** which have mass numbers in the neighbourhood of 45 have the **lowest negative values** (بہت کم منفی قیمتیں) for their packing fraction and hence the nuclei of these elements are **highly stable.**

(iv) The value of packing fraction is positive (low value) for the elements having mass numbers **more than 190** (ماس نمبر 190 سے اوپر والے عناصر کے PF مثبت ہوتے ہیں). It becomes more positive as the mass number increases. *This shows that the stability of these isotopes goes on decreasing with the increase in their mass numbers* (ماس نمبر بڑھنے سے آئسوٹوپس کی استحکام پذیری کم ہوتی جاتی ہے).

(v) When the mass number is **more than 230**, they have very high positive values for their packing fraction and hence are highly unstable. Such elements are **radioactive** (تابکار) and hence undergo **disintegration spontaneously** (خود بخود، توڑ پھوڑ کا شکار ہوتے رہتے ہیں).

Sample Problem (13.8):

Calculate the packing fraction of argon isotope, ${}^{40}_{18}$Ar. Mass of the isotope of argon is 39.962384 amu. What does the value of packing fraction imply?

Solution. We know that the

$$\text{Packing fraction of } {}^{48}_{18}\text{Ar isotopes} = \frac{\text{Mass of } {}^{48}_{18}\text{Ar} - \text{Mass number of } {}^{40}_{18}\text{Ar}}{\text{Mass number of } {}^{40}_{18}\text{Ar}} \times 10^4$$

$$= \frac{39.962384 - 40}{40} \times 10^4 = -9.404$$

Negative value indicates that ${}^{40}_{18}$Ar is a stable isotope.

13.4.3 Mass-Energy Relationship of amu and Energy in MeV

First of all calculate that 1amu = 1.66×10^{-27} kg

AMU = Atomic Mass Units
MeV = Million Electron volts

Prove that 1 amu = 1.66×10^{-27} kg? کمیت کا کمیت میں تبادلہ

1 atomic mass unit (1 amu) = 1/12th of the mass of a ${}^{12}_{6}$C atom

$$\text{Now since mass of a } {}^{12}_{6}\text{C atom} = \frac{12 \text{ g mol}^{-1}}{\text{Avogadro's number}} = \frac{12 \text{ g mol}^{-1}}{6.022 \times 10^{23} \text{ mol}^{-1}} = 12 \text{ amu}$$

We also know that mass of one C-atom = 12 amu.

$$\text{So,} \quad 12 \text{ amu} = \frac{12 \text{ g}}{6.022 \times 10^{23}}$$

$$\text{or} \quad 1 \text{ amu} = \frac{1}{12} \times \frac{12 \text{ g}}{6.022 \times 10^{23}} = \frac{1}{6.022 \times 10^{23}} \text{ g}$$

$$= 0.166 \times 10^{-23} \text{ g} = \frac{0.166 \times 10^{-23}}{10^3} \text{ kg}$$

$$\text{that} \quad 1 \text{ amu} = 0.166 \times 10^{-26} \text{ kg} = \boxed{1.66 \times 10^{-27} \text{ kg}}$$

Prove that $1\ \text{amu} = 931.3\ \text{MeV}$? (کمیت کا انرجی میں تبادلہ)

The relation between mass and energy corresponding to this mass is given by Einstein's mass-energy relationship *viz.* $E = mc^2$

Here E = energy in joules, m = mass in kg and c = velocity of light in ms^{-1} which is $2.998 \times 10^8\ ms^{-1}$.

Energy (E) corresponding to 1 amu $= (0.166 \times 10^{-26}\ \text{kg}) \times (2.998 \times 10^8\ \text{m.s}^{-1})^2$

$$= 0.166 \times 10^{-26} \times 8.9880 \times 10^{16}\ \text{kg.m}^2.\text{s}^{-2}$$

or

$$1\ \text{amu} = 1.4920 \times 10^{-10}\text{J} \qquad \boxed{(\because\ \text{kg. m}^2.\text{s}^{-2} = \text{J})}$$

We know that $1\text{eV} = 1.602 \times 10^{-19}\text{J}$

(It means eV is a very very small unit as compared to joul).

$$\therefore \quad 1\ \text{amu} = \frac{1.4920 \times 10^{-10}}{1.602 \times 10^{-19}}\ \text{eV} = \boxed{0.9313 \times 10^9\ \text{eV}}$$

Here M stands for million (10^6)

$$1\ \text{amu} = \frac{0.9313 \times 10^9}{10^6}\ \text{MeV} \qquad \boxed{(\because\ 1\ \text{MeV} = 10^6\ \text{eV})}$$

$$= 0.9313 \times 10^3\ \text{MeV} = 931.3\ \text{MeV}$$

i.e., $1\ \text{amu} = 931.3\ \text{MeV}$

For the sake of convenience, in our calculations we shall be using the relation:

$$\boxed{1\ \text{amu} = 931.50\ \text{MeV}}$$

13.4.4 Mass Defect (ΔM) (کمیت میں ناکافی) and Binding Energy:

The nucleus of an atom contains protons and neutrons. The mass of electron is negligible (نظر انداز کرنے کے قابل) because almost the entire mass (ساری کمیت) of the atom is concentrated (مرکوز ہے) in the nucleus. If the mass of the electron is also considered, then it has been found that **the actual mass (M) of an atom as determined by mass spectrograph is always less than the sum of the masses (M') of all protons, neutrons and electrons present in the atom.** In other words, $M < M'$. **The difference in masses *viz* (M' – M) is called mas defect, mass loss (کمیت کا نقصان), or mass deficit (ΔM).** Thus:

Mass defect = (Masses of all protons + masses of all neutrons + masses of all electrons) – (Actual atomic mass of the atom)

$$\Delta M = M' - M$$

Actually when nucleons *i.e.*, neutrons and proton, combine to form the nucleus of an atom, mass equal to (M' – M) is lost.

(جب ایٹم کے نیوکلیس میں نیوٹرونز اور پروٹونز باہم اکٹھے ہوتے ہیں تو ان کی بائنڈنگ انرجی کے برابر کمیت میں کمی آجاتی ہے۔ یعنی ماس انرجی میں تبدیل ہو جاتا ہے۔)

(A) Calculations of Mass Defect of Helium Atom

${}^4_2\text{He}$ contains 2 protons, 2 electrons and 2 neutrons.

M' = mass of 2 protons + mass of 2 electrons + mass of 2 neutrons

$= 2 \times$ (mass of one proton + mass of one electron) + mass of 2 neutrons

$= 2 \times$ mass of a H-atom + mass of 2 neutrons.

$= (2 \times 1.007825 + 2 \times 1.008665)$ amu $= (2.01560 + 2.017330)$ amu $= 4.032980$ amu

M = 4.002600 amu (Determined by mass spectroscopy)

$\therefore$ Mass defect (ΔM) of He atom $= (4.032980 - 4.002600)$ amu $= \boxed{0.030380 \text{ amu}}$

(B) Calculation of Mass Defect of O-atom

${}^{16}_{8}O$ contains 8 protons, 8 neutrons and 8 electrons. Thus

mass of 8 protons $= 8 \times 1.007277 = 8.058216$ amu

mass of 8 neutrons $= 8 \times 1.008665 = 8.069320$ amu

mass of 8 electrons $= 8 \times 0.0005486 = 0.004388$ amu

$\therefore$ $M' = 8.058216 + 8.069320 + 0.004388 = 16.131924$ amu

M = 15.994910 amu (Determined by mass spectroscopy)

$\therefore$ Mass defect (ΔM) of O-atom $= 16.131924 - 15.994910 = \boxed{0.137014 \text{ amu}}$

Some Important Values and Relations (وہ چند اہم قیمتیں اور اُن کے آپس میں تعلقات)

(i) Mass of one proton (m_p) = 1.007277 amu

(ii) Mass of one neutron (m_n) = 1.008665 amu

(iii) Mass of one electron (m_e) = 0.0005486 amu

(iv) Mass of a helium nucleus, ${}^{4}_{2}H_2 = 4.0026$ amu

(v) Mass of a deuterium nucleus, ${}^{2}_{1}H = 2.01471$ amu

(vi) Mass of one H-atom ${}^{1}_{1}H$ (m_H) = Mass of one proton + Mass of one electron

$= 1.0072770 + 0.0005486 = 1.0078256$ amu

(vii) Avogadro's number $= 6.022 \times 10^{23}$

(viii) 1 amu $= 0.166 \times 10^{-26}$ kg $= 1.66 \times 10^{-27}$ kg

(ix) Velocity of light (c) $= 2.997 \times 10^{10}$ cms^{-1} $= \dfrac{2.997 \times 10^{10}}{100}$ ms^{-1} $= 2.997 \times 10^{8}$ ms^{-1}

(x) $J = kg.m^2.s^{-2}$

(xi) 1 amu $= 1.4920 \times 10^{-10}$ J $= 931.5$ MeV

(xii) 1 eV $= 10^{-6}$ MeV $= 23.06$ k cal $= 1.602 \times 10^{-19}$ J $= 1.60 \times 10^{-21}$ erg

(xiii) 1 MeV $= 1.602 \times 10^{-16}$ kJ

Nuclear Binding Energy (B) (نیوکلیس کو اکٹھا رکھنے کی انرجی):

As discussed earlier that the mass is lost in the process of combination of nucleons to form the nucleus. This mass is converted into energy and is released in the process. The energy released in this way is called **binding energy** of the nucleus or **nuclear binding energy (B)** (نیوکلیان کو اکٹھے رکھنے میں انرجی خرچ ہوتی ہے وہی بائنڈنگ انرجی کہلاتی ہے).

It should be evident (صاف ظاہر ہوتا) from the above discussion that when a nucleus is broken up into its constituents neutrons and protons, **energy is needed** (انرجی کی ضرورت پڑتی ہے) **and this energy is equal to the binding energy of the nucleus** (یہ انرجی بائنڈنگ انرجی کے برابر ہوتی ہے).