ces that define the shape of polygon.

13026. Dr. M. Faheem YM

Subdivison Scheme

MPhil - I.

Overview:-

The subdivison Schemes are defined recursively. The process start with a given polygonal mesh. A "refinement Scheme" is then applied to this mesh. This process takes that mesh and subdivides it, creating new vertices and new faces. The positions of the new vertices in the mesh are computed based on the positions of nearby old vertices. In some refinement schemes, the positions of old vertices might also be altered.

This process produces a denser mesh than the original one, containing more polygonal faces. This resulting mesh can be passed through the same refinement scheme again and so on.

"The limit subdivision curve" is a curve produced from this process being iteratively applied infinitely many times. In practical use however, this algorithm is only applied a limited number of times. This limit curve can also be calculated directly for most subdivision surfaces using the technique which eliminates the need for recursive refinement.

Bezier Curves, B spline Curves and Subdivison Curves are all based upon input of a control polygon and specification of algorithm method that construct a curve from the sequence of points. Fundamental of these elements is concept of a refinement.

Classification of Subdivision Schemes:-

Subdivision Schemes can be bradly classified in two categories.

(1) Aproximating Scheme (2) Interpolating Scheme

1) Approximating schemes:-

Approximating Subdivision Scheme means that the limit curve approximate the intial meshes and that the after Subdivision, the newly generated control points are in the limit curve.

For Example Uniform B-Spline, Chaiken Subdivisor

Interpolating Schemes:-

Interpolating Scheme means that after Subdivision, the control points of the original mesh and the new genera control points are on the limit curve.

** Smoothness of Interpolating Scheme is less than the Smoothness of Approximating Scheme.

Some Basic Definations:-

* Subdivision Scheme (Univariate Case / Curve Case)

A general form of univariate n-ary Subdivision Scheme which maps a control polygon $f^K = \{f_i^K\}_{i\in Z}$ a refined polygon $f^{K+1} = \{f_i^{K+1}\}_{i\in Z}$ is defined by

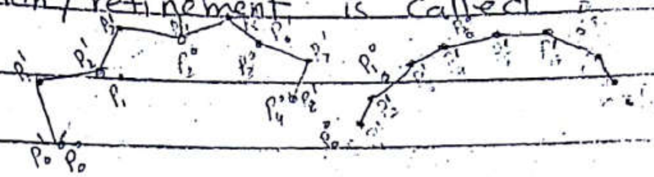
$$f_{ni+s}^{K+1} = \sum_{j\in Z} a_{nj+s} f_{i-j}^{K} \qquad s = 0,1,2,\ldots,n-1$$

where the set $A = \{a_i | i \in Z\}$ of coefficient is called " Mask / weight of Subdivision Scheme".

The necessary condition for convergence of n-ary Scheme is $\sum_{j\in Z} a_{nj+s} = 1 \qquad s = 0,1,2,\ldots$

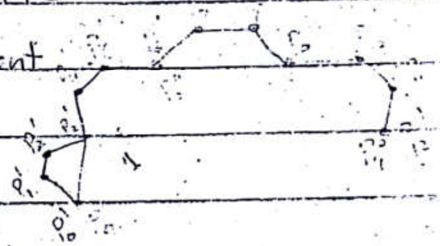
Binary Subdivision Scheme:-

If each line segment is divided into two line segments for each iteration/refinement is called binary. Subdivision Scheme.



Ternary Subdivision Scheme.

If each line segment is divided into three line segments for each refinement is called ternary Subdivison Scheme.



Stationary Subdivision Scheme:-

A Subdivision Scheme where the same set of Subdivision rule ~~is~~ are used to subdivide the control polygon at each level of Subdivison is ~~a~~ said to be stationary.

or

If mask of the scheme is independent of Subdivison level k then it is called stationary.

Non-stationary Subdivision Scheme:-

A subdivision Scheme is said to be non-stationary if the refinement rule changes from one subdivision level to another.

or

If the mask of the Scheme is depende upon the subdivision level k then it is called non-stationary

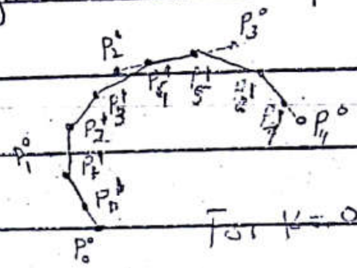
Schem

Chaiken Subdivison Scheme:- (Binary Approximating two point

It divides a line segment into two points

$$P_{2i}^{K+1} = \frac{3}{4} P_i^K + \frac{1}{4} P_{i+1}^K$$

$$P_{2i+1}^{K+1} = \frac{1}{4} P_i^K + \frac{3}{4} P_{i+1}^K$$



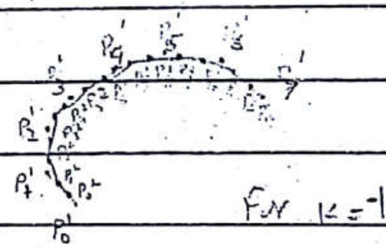
For $K=0$ $\quad i = 0, 1, 2, 3$

$i=0$: $P_0^1 = \frac{3}{4} P_0^0 + \frac{1}{4} P_1^0$, $\quad P_1^1 = \frac{1}{4} P_0^0 + \frac{3}{4} P_1^0$

$i=1$: $P_2^1 = \frac{3}{4} P_1^0 + \frac{1}{4} P_2^0$, $\quad P_3^1 = \frac{1}{4} P_1^0 + \frac{3}{4} P_2^0$

$i=2$: $P_4^1 = \frac{3}{4} P_2^0 + \frac{1}{4} P_3^0$, $\quad P_5^1 = 1/4\, P_2^0 + 3/4\, P_3^0$

$i=3$: $P_6^1 = 3/4\, P_3^0 + 1/4\, P_4^0$, $\quad P_7^1 = 1/4\, P_3^0 + 3/4\, P_4^0$



For $K=1$ $\quad i = 0, 1, 2, 3, 4, 5, 6$

$i=0$: $P_0^2 = \frac{3}{4} P_0^1 + \frac{1}{4} P_1^1$, $\quad P_1^2 = 1/4\, P_0^1 + 3/4\, P_1^1$

$i=1$: $P_2^2 = 3/4\, P_1^1 + 1/4\, P_2^1$, $\quad P_3^2 = 1/4\, P_1^1 + 3/4\, P_2^1$

$i=2$: $P_4^2 = 3/4\, P_2^1 + 1/4\, P_3^1$, $\quad P_5^2 = 1/4\, P_2^1 + 3/4\, P_3^1$

$i=3$: $P_6^2 = 3/4\, P_3^1 + 1/4\, P_4^1$, $\quad P_7^2 = 1/4\, P_3^1 + 3/4\, P_4^1$

$i=4$: $P_8^2 = 3/4\, P_4^1 + 1/4\, P_5^1$, $\quad P_9^2 = 1/4\, P_4^1 + 3/4\, P_5^1$

$i=5$: $P_{10}^2 = 3/4\, P_5^1 + 1/4\, P_6^1$, $\quad P_{11}^2 = 1/4\, P_5^1 + 3/4\, P_6^1$

$i=6$: $P_{12}^2 = 3/4\, P_6^1 + 1/4\, P_7^1$, $\quad P_{13}^2 = 1/4\, P_6^1 + 3/4\, P_7^1$

~~$i=7$~~ ~~$P_{14}^2 =$~~

Properties of Subdivision Scheme:-

An approximating Subdivision Scheme should have following properties

1.) Locality:- Means that rules which determine the position of new points should depend upon the position of distinct points in Input control polygon

) Compact Support:- Implies that control point in the control mesh should only effect a small and finite region of resultant limit curve/surface.

) Affine Invariance:- Implies that, If the control mesh is rotated, translated, sheared or Scaled then corresponding limit curve/surface under goes same rotation, translation, Shear or Scale.

) Continuity:- Refer to differentiability of limit curve/surface produced by Subdivision process. Subdivision Scheme should be continous of a certain order prior to construction

Continuity of Curve/Smoothness of curve

We will discuss now the type of continuity/smoothness of curve i.e

1) Parametric continuity 2) Geometric continuity

Parametric continuity:-

Parametric continuity is a concept applied to Parametric curve describing the smoothness of the Parameteric value with distance along the curve.

"A curve is said to have C^n continuity if $\frac{d^n s}{dt^n}$ is continuous of value throughout the curve"