Numerical Analysis

Introduction:-

Since the early 1940's the speed of computation has increased a millionfold due to the invention of the electronic digital computer. Numerical analysis has becomes one of the most important and frequently used tools of scientists and engineers faced with the necessity of having to solve complicated mathematical problems.

There is no general agreement how to term Numerical analysis should be interpreted. Even today, there are widely diverging views of the subject. It did not become generally used until the instead of Numerical analysis was founded at the University of California in 1947.

In almost all areas of science mathematical problems arise which do not possess either exact information or cannot be solved by excact mathematical formulas. (Ordinary analytical methods fail to give solutions, i.e the existance of the solutions can be proved but they are unknown.)

Numerical analysis deals with such type of mathematical problems and may be called primarily. A mathematical discipline which deals with finding the numerical approximate solution of a mathematical problem. This does not mean that when all the analytical method have failed only then numerical analysis is applicable.

ERROR

There are two ways in determing the size of the error

1- Absolute error
2- Relative error.

Some useful definitions in this regard

Approximate Number. By an approximate number of exact number "X", we mean a number "x" which differ from "X" very slightly and can be used in place of X.

Example The number 1.41 is an approximate number of $\sqrt{2} = 1.41421356\ldots$

If $x < X$, then x is called minor approximation of X.

If $X < x$ then x is called major approximation of X.

for example $1.41 < \sqrt{2} < 1.42$

here 1.41 is minor approximation of $\sqrt{2}$
and 1.42 is major approximation of $\sqrt{2}$.

Error: By the error of an approximation number "x". we mean the quantity $X - x$ denoted by Δ

$$\Delta = X - x.$$

Thus an exact number may be regarded as an approximate number with error zero.

Absolute Error

By the absolute error of an approximation number "x", we mean the absolute value of its error

i.e $$|\Delta| = |X - x| = |x - X|$$

Relative Error

By the relative error of an approximate number x corresponding to the exact number X, we mean the number

$$\frac{|\Delta|}{|X|} = \frac{\text{Absolute value of error}}{\text{Absolute value of exact number}}$$

$$= \frac{|X - x|}{|X|}$$

denoted by $$\delta = \frac{|\Delta|}{|X|}$$

Example: 27 may be used as an approximation to the number 26.76. In this case

$X = 26.76$ and $x = 27$

The error (due to round-off in this case) is -0.24

The absolute error is $\Delta = |X - x| = 0.24$

Relative error is $\delta = \frac{|\Delta|}{|X|} = \frac{0.24}{26.76} = 0.009$

NOTE: Each of the concepts of absolute error and relative error is useful in different circumstances. In general, we adding and subtracting, the absolute errors are more useful, whereas when multiplying and dividing the relative errors are more useful.

(*) If the number being estimated is itself zero, any error at all (however small) would give an infinite relative error. i.e relative error is undefined. In such a case the absolute error would be more meaningful.

Linear equation:-

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and the first power of a single variable.

A linear equation can involve more than two variables. The general linear equation in n variables is:

$$K_1 x_1 + K_2 x_2 + K_3 x_3 + \cdots + K_n x_n = b \quad \text{——— (A)}$$

$K_1, K_2, K_3, \cdots, K_n$ are the coefficients and $x_1, x_2, \cdots, x_n$ are the variables.

Non-linear equation:-

An equation whose solution set does not form a straight line when plotted on a co-ordinate graph.

There are two forms:-

— Transcendental equation:- An equation containing the exponential function or logarithmic functions or trigonometric functions is called the transcedental equation.

— Algebraic Equation: Polynomial equation of nth degree may be regarded as algebraic equation i.e

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 = 0, \quad a_n \neq 0$$

If $a_n, a_{n-1}, \cdots, a_0$ are integers then the roots of the equation are called algebraic numbers.

Def A real number "ξ" for $f(\xi) = 0$ is called the real root or Zero of the equation $f(x) = 0$.