✓ **Sufficient:-**

Any statistic is said to be sufficient statistic for estimating the population parameter θ, if it contains all the information in the sample about the parameter θ i.e

$\bar{x}$ is a sufficient estimator of 'μ'

To determine whether a given statistic is sufficient or Not we use the following Criterion

P.T.O

~~Where~~

$$g(\sum x_i ; \mu) = e^{-\frac{n}{2}\mu^2 + \mu \sum_{i=1}^{n} x_i}$$

&

$$h(x_1, x_2, \ldots, x_n) = \frac{1}{(2\pi)^{n/2}} e^{-\frac{1}{2}\sum_{i=1}^{n} x_i^2}$$

Neyman Fisher Factorization criterion is satisfied, Therefore $\sum x_i$ is a sufficient estimator of μ.

Note. $\bar{x}$ is also sufficient since any one-to-one function of a sufficient statistic is also sufficient.

Example :- Let $X_1, X_2, \ldots, X_n$ denote a random sample from the density function

$$f(x;p) = p x^{p-1}, \quad 0 < x < 1, \quad p > 0$$

Show that the product $X_1 X_2 \ldots X_n$ $\prod_{i=1}^{n} X_i$ is a sufficient estimator for p.

SOL :- ~~The joint density function is~~

Given that $f(x;p) = p x^{p-1}$

we want to show that the product

$$\prod_{i=1}^{n} x_i = x_1 \cdot x_2 \cdots x_n$$ is a sufficient estimator of p.

Joint Prob. function

$$\prod_{i=1}^{n} f(x_i;p) = \left[p x_1^{p-1}\right]\left[p x_2^{p-1}\right] \cdots \left[p x_n^{p-1}\right]$$

$$= p^n \cdot (x_1 \cdot x_2 \cdots x_n)^{p-1}$$

$$= p^n (x_1 \cdot x_2 \cdots x_n)^p \cdot (x_1 \cdot x_2 \cdots x_n)^{-1}$$

$$\prod_{i=1}^{n} f(x_i;p) = g(x_1, x_2, x_3 \ldots, x_n ; p)\, h(x_1, x_2, \ldots, x_n)$$

Where

$$g(x_1, x_2, \ldots, x_n ; p) = p^n \cdot (x_1 \cdot x_2 \cdots, x_n)^p$$

&

$$h(x_1, x_2, \ldots, x_n) = (x_1 \cdot x_2, \ldots \cdot x_n)^{-1}$$

Fisher Neyman Factorization Criterion is met Therefore $(x_1 \cdot x_2 \ldots x_n)$ is a sufficient estimator of p.

Example:-

Let $X_1, X_2, \ldots, X_n$ denote a random sample from the Bernoulli distribution $f(x;p) = p^x(1-p)^{1-x}$, $x = 0,1$

Show that $\sum X_i$ is a sufficient estimator of p.

SOL:-

Given that $f(x;p) = p^x(1-p)^{1-x}$ $\quad x = 0,1$

we want to show that $\sum x_i$ is a sufficient estimator of p.

Joint Probability Function

$$\prod_{i=1}^{n} f(x_i;p) = f(x_1;p)\,f(x_2;p)\cdots f(x_n;p)$$

$$= \left[p^{x_1}(1-p)^{1-x_1}\right]\left[p^{x_2}(1-p)^{1-x_2}\right]\cdots\left[p^{x_n}(1-p)^{1-x_n}\right]$$

$$= p^{x_1+x_2+\cdots+x_n}\,(1-p)^{n-(x_1+x_2+\cdots+x_n)}$$

$$= p^{\sum_{i=1}^{n} x_i}\,(1-p)^{n-\sum_{i=1}^{n} x_i}$$

$$\prod_{i=1}^{n} f(x_i;p) = g\left(\sum x_i;p\right) h(x_1, x_2, \ldots, x_n)$$

$\sum x.$

where $\quad g\left(\sum x_i;p\right) = p^{\sum^{n} x_i}(1-p)^{n-\sum^{n} x_i}$ $\qquad \left(\frac{p}{1-p}\right)$ (1-

&

$$h(x_1, x_2, \ldots, x_n) = 1$$

Neyman, Fisher Factorization Criterion is satisfied, therefore $\sum x_i$ is a sufficient estimator of p.

Example:- Let $X_1, X_2, \dots, X_n$ be a random sample from the normal distribution with mean zero and variance θ. Show that $\sum_{i=1}^{n} X_i^2$ is a sufficient estimator of θ.

SOL:-

Given that $f(x;\theta) = \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{1}{2}\cdot\frac{x^2}{\theta}}$, $-\infty < x < \infty$

we want to show that $\sum_{i=1}^{n} x_i^2$ is a sufficient estimator of θ.

Joint Prob. function

$$\prod_{i=1}^{n} f(x_i;\theta) = f(x_1;\theta)\cdot f(x_2;\theta)\dots f(x_n;\theta)$$

$$= \left[\frac{1}{\sqrt{2\pi\theta}} e^{-x_1^2/2\theta}\right]\left[\frac{1}{\sqrt{2\pi\theta}} e^{-x_2^2/2\theta}\right]\cdots\left[\frac{1}{\sqrt{2\pi\theta}} e^{-x_n^2/2\theta}\right]$$

$$\prod_{i=1}^{n} f(x_i;\theta) = \frac{1}{(2\pi\theta)^{n/2}} e^{-\frac{1}{2\theta}\left[x_1^2+x_2^2+\dots+x_n^2\right]}$$

P.T.O

Date 04-11-2020

⇒ Likelihood Function:-

The likelihood function of 'n' random variables $X_1, X_2, \ldots, X_n$ is defined to be the joint density of n random variables say $f_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n; \theta)$ which is considered to be function of θ. In particular if $X_1, X_2, \ldots, X_n$ is a random sample from a density $f(x; \theta)$ then the likelihood function is

$$f(\underline{x}; \theta) = f(x_1; \theta) f(x_2; \theta) \ldots\ldots f(x_n; \theta)$$

$$= \prod_{i=1}^{n} f(x_i; \theta)$$

$$f(\underline{x}; \theta) = L(\theta; \underline{x})$$

where $\underline{x}$ is a random vector of observations x_i. It is a joint pdf of $X_1, X_2, \ldots, X_n$ but viewed as a function of θ where θ is unknown.

$\boxed{\text{function} = f^n}$

⇒ Difference b/w likelihood f^n and joint density f^n:-

In likelihood f^n '$x_1, x_2, \ldots, x_n$' are simple observations and parameters are unknown but in joint pdf they are known or specified.

known as factorization criterion.
Let $x_1, x_2, \ldots, x_n$ be a random sample of size 'n' from the density $f(x;\theta)$ and let the joint density of these 'n' random variables be

$$L(x_1, x_2, \ldots, x_n; \theta) = L_1(t:\theta)\, L_2(x_1, x_2, x_3, \ldots, x_n)$$

where L_2 does not contain/involve θ thus 't' is a sufficient statistic for θ.

Example:- Suppose $x_1, x_2, \ldots, x_n$ is a random sample from the density function $f(x;\mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2}$, $-\infty < x < \infty$

Show that Σx_i is a sufficient estimator for 'μ'.

Sol:- Given that

$$f(x;\mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2}, \quad -\infty < x < +\infty$$

We want to show that Σx_i is a sufficient estimator of μ.

Joint Probability function $= \prod_{i=1}^{n} f(x_i;\mu)$

$f(x_1, x_2 \cdots x_n : \mu) =$

$$L(\mu) = f(x_1;\mu) \cdot f(x_2;\mu) \cdots f(x_n;\mu)$$

$$= \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x_1-\mu)^2}\right]\left[\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x_2-\mu)^2}\right] \cdots \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x_n-\mu)^2}\right]$$

$$\prod_{i=1}^{n} f(x_i;\mu) = \frac{1}{(2\pi)^{n/2}} e^{-\frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

p_i

$$= \frac{1}{(2\pi)^{n/2}} e^{-\frac{1}{2}\sum^{n}(x_i^2 + \mu^2 - 2\mu x_i)}$$

$$= \frac{1}{\sqrt[n]{2\pi}} e^{-\frac{1}{2}\sum_{i=1}^{n} x_i^2 - \frac{n}{2}\mu^2 + \mu\sum_{i=1}^{n} x_i}$$

$$= \left[e^{-\frac{n}{2}\mu^2 + \mu\sum_{i=1}^{n} x_i}\right]\left[\frac{1}{(2\pi)^{n/2}} e^{-\frac{1}{2}\sum_{i=1}^{n} x_i^2}\right]$$

$$= g(\Sigma x_i;\mu) \cdot h(x_1, x_2, \ldots, x_n)$$

Neyman Fisher Factorization criterion is satisfied, Therefore, $\prod_{i=1}^{n} x_i$ is a sufficient estimator of α.

Example:- Prove that sum of observations of a random sample of size 'n' from a Poisson distribution with parameter λ is a sufficient estimator.

SOL:- Given that Poisson distribution

$$f(x;\lambda) = e^{-\lambda}\lambda^{x}/x! \quad , \quad x = 0,1,2,\ldots$$

We want to show that $\sum_{i=1}^{n} x_i$ is a sufficient estimator.

Joint Probability function

$$\prod_{i=1}^{n} f(x_i;\lambda) = f(x_1;\lambda)\,f(x_2;\lambda)\cdots f(x_n;\lambda)$$

$$= \left[\frac{e^{-\lambda}\lambda^{x_1}}{\underline{|x_1}}\right]\cdot\left[\frac{e^{-\lambda}\lambda^{x_2}}{\underline{|x_2}}\right]\cdots\left[\frac{e^{-\lambda}\lambda^{x_n}}{\underline{|x_n}}\right]$$

$$= e^{-n\lambda}\,\lambda^{x_1+x_2+\cdots+x_n} \Big/ \underline{|x_1}\,\underline{|x_2}\cdots\underline{|x_n}$$

$$= e^{-n\lambda}\,\lambda^{\sum_{i=1}^{n} x_i}\cdot\frac{1}{\underline{|x_1}\,\underline{|x_2}\cdots\underline{|x_n}}$$

$$\prod_{i=1}^{n} f(x_i;\lambda) = g\left(\sum_{i=1}^{n} x_i;\lambda\right) h(x_1, x_2, \ldots, x_n)$$

Where

$$g\left(\sum_{i=1}^{n} x_i;\lambda\right) = e^{-n\lambda}\,\lambda^{\sum_{i=1}^{n} x_i}$$

&

$$h(x_1, x_2, \ldots, x_n) = \frac{1}{\underline{|x_1}\,\underline{|x_2}\cdots\underline{|x_n}} = \left[\prod_{i=1}^{n}\underline{|x_i}\right]^{-1}$$

Neyman Fisher Factorization Criterion is satisfied, Therefore, $\sum_{i=1}^{n} x_i$ is a sufficient estimator of λ.

$$\prod_{i=1}^{n} f(x_i;\theta) = \frac{1}{(2\pi\theta)^{n/2}} e^{-\frac{1}{2\theta}\sum_{i=1}^{n} x_i}$$

$$\prod_{i=1}^{n} f(x_i;\theta) = g\left(\sum_{i=1}^{n} x_i^2;\theta\right) h(x_1, x_2, \dots, x_n)$$

where $g\left(\sum_{i=1}^{n} x_i^2;\theta\right) = \frac{1}{(2\pi\theta)^{n/2}} e^{-\frac{1}{2\theta}\sum_{i=1}^{n} x_i^2}$

& $h(x_1, x_2, \dots, x_n) = 1$, Neyman Fisher Factorization criterion is satisfied; Therefore $\sum_{i=1}^{n} x_i^2$ is a sufficient estimator of θ.

Example:- Let $X_1, X_2, \dots, X_n$ be a random sample of size n, from the density function

$$f(x;\alpha) = \alpha(\alpha+1) x^{\alpha-1}(1-x), \quad 0<x<1$$

Show that the product $X_1 \cdot X_2 \cdots X_n$ is sufficient estimator of α.

SOL:-

Given that $f(x;\alpha) = \alpha(\alpha+1)x^{\alpha-1}(1-x)$, $0<x<1$

we want to show that $\prod_{i=1}^{n} X_n$ is a sufficient estimator of α.

Joint Prob. function

$$\prod_{i=1}^{n} f(x_i;\alpha) = f(x_1;\alpha)\cdot f(x_2;\alpha)\cdots f(x_n;\alpha)$$

$$= \left[\alpha(\alpha+1)x_1^{\alpha-1}(1-x_1)\right]\left[\alpha(\alpha+1)x_2^{\alpha-1}(1-x_2)\right]\cdots\left[\alpha(\alpha+1)x_n^{\alpha-1}(1-x_n)\right]$$

$$= \alpha^n(\alpha+1)^n x_1^{\alpha}\cdot x_2^{\alpha}\cdots x_n^{\alpha}\cdot x_1^{-1}\cdot x_2^{-1}\cdots x_n^{-1}(1-x_1)(1-x_2)\cdots(1-x_n)$$

$$= \alpha^n(1+\alpha)^n\left[\prod_{i=1}^{n} x_i\right]^{\alpha}\cdot\left[\prod_{i=1}^{n} x_i\right]^{-1}(1-x_1)(1-x_2)\cdots(1-x_n)$$

$$\prod_{i=1}^{n} f(x_i;\alpha) = \underline{\underline{\left[\alpha(1+\alpha)\right]^n\left[\prod_{i=1}^{n} x_i\right]^{\alpha}}}\ \underline{\underline{\left[\prod_{i=1}^{n} x_i\right]^{-1}\left[\prod_{i=1}^{n}(1-x_i)\right]}}$$

$$\prod_{i=1}^{n} f(x_i;\alpha) = g\left(\prod_{i=1}^{n} x_i;\alpha\right) h(x_1, x_2, \dots, x_n)$$

where $g\left(\prod_{i=1}^{n} x_i;\alpha\right) = \left[\alpha(1+\alpha)\right]^n\left[\prod_{i=1}^{n} x_i\right]^{\alpha}$

&

$$h(x_1, x_2, \dots, x_n) = \left[\prod_{i=1}^{n} x_i\right]^{-1}\left[\prod_{i=1}^{n}(1-x_i)\right]$$

where $g\left(\sum_{i=1}^{n} x_i ; p\right) = p^{n}(1-p)^{\sum x_i}$

& $h(x_1, x_2, \dots, x_n) = 1$

Neyman Fisher Factorization criterion is satisfied. Therefore, $\sum_{i=1}^{n} x_i$ is a sufficient estimator of p.

Example:- Let $X_1, X_2, \dots, X_n$ be a random sample of size 'n' from the probability function

$$f(x;p) = (1-p)^{n-x}\, p^{x}, \quad x = 0, 1, 2, \dots$$

Show that $\sum_{i=1}^{n} X_i$ is a sufficient estimator for P.

SOL:-

Given that $f(x;p) = p^{x}(1-p)^{n-x}, \quad x = 0, 1, 2, \dots$

We want to show that $\sum_{i=1}^{n} X_i$ is a sufficient estimator for p.

Joint Prob. function.

$$\prod_{i=1}^{n} f(x_i;p) = f(x_1;p)\, f(x_2;p) \cdots f(x_n;p)$$

$$= \left[p^{x_1}(1-p)^{n-x_1}\right]\left[p^{x_2}(1-p)^{n-x_2}\right] \cdots \left[p^{x_n}(1-p)^{n-x_n}\right]$$

$$= p^{x_1+x_2+\dots+x_n}(1-p)^{n\cancel{N}-\sum^{n} x_i} = p^{\sum^{n} x_i}(1-p)^{n\cancel{N}-\sum^{n} x_i}$$

$$\prod_{i=1}^{n} f(x_i;p) = g\left(\sum^{n} x_i ; p\right) \cdot h(x_1, x_2, \dots, x_n)$$

where

$$g\left(\sum^{n} x_i ; p\right) = p^{\sum^{n} x_i}(1-p)^{n\cancel{N}-\sum^{n} x_i}$$

&

$$h(x_1, x_2, \dots, x_n) = 1$$

Neyman Fisher Factorization criterion satisfied.

Therefore, $\sum_{i=1}^{n} x_i$ is a sufficient estimator of p.

Example:- Show that for a random sample of size n from the density function $f(x;\theta) = \frac{1}{\theta} e^{-x/\theta}$, $0 < x < \infty$. $\sum_{i=1}^{n} x_i$ is a sufficient estimator for θ.

SOL:- Given that $f(x;\theta) = \frac{1}{\theta} e^{-x/\theta}$, $0 < x < \infty$.
we want to show that $\sum_{i=1}^{n} x_i$ is a sufficient estimator for θ.
Joint p.d.function

$$\prod_{i=1}^{n} f(x_i;\theta) = f(x_1;\theta)\, f(x_2;\theta) \ldots f(x_n;\theta)$$

$$= \left[\frac{1}{\theta} e^{-x_1/\theta}\right]\left[\frac{1}{\theta} e^{-x_2/\theta}\right] \ldots \left[\frac{1}{\theta} e^{-x_n/\theta}\right]$$

$$= \left(\frac{1}{\theta}\right)^{n} e^{-\sum_{i=1}^{n} x_i/\theta} = g\left(\sum_{i=1}^{n} x_i;\theta\right) \cdot h(x_1, x_2, \ldots, x_n)$$

Where $g\left(\sum_{i=1}^{n} x_i;\theta\right) = \left(\frac{1}{\theta}\right)^{n} e^{-\sum_{i=1}^{n} x_i/\theta}$

& $h(x_1, x_2, \ldots, x_n) = 1$

Neyman Fisher Factorization criterion is satisfied,
Therefore, $\sum_{i=1}^{n} x_i$ is a sufficient estimator for θ.

Imp. Example:- Let $X_1, X_2, \ldots, X_n$ be a random sample of size 'n' from the probability function

$$f(x;p) = (1-p)^{x} p, \quad x = 0, 1, 2, \ldots$$

Show that $\sum_{i=1}^{n} X_i$ is a sufficient estimator for p.

SOL:-

Given that $f(x;p) = (1-p)^{x} p$, $x = 0, 1, 2, \ldots$
We want to show that $\sum_{i=1}^{n} X_i$ is a sufficient estimator for p.
Joint Prob. function

$$\prod_{i=1}^{n} f(x_i;p) = f(x_1;p) \cdot f(x_2;p) \ldots f(x_n;p)$$

$$= \left[(1-p)^{x_1} p\right]\left[(1-p)^{x_2} p\right] \ldots \left[(1-p)^{x_n} p\right]$$

$$= (1-p)^{x_1 + x_2 + \cdots + x_n} p^{n} = p^{n} (1-p)^{\sum_{i=1}^{n} x_i}$$

$$\prod_{i=1}^{n} f(x_i;p) = g\left(\sum_{i=1}^{n} x_i;p\right) h(x_1, x_2, \ldots, x_n)$$

~~Let~~ $E_1 = \frac{V}{Var(t_1)} \Rightarrow Var(t_1) = \frac{V}{E_1}$

Similarly, $E_2 = \frac{V}{Var(t_2)} \Rightarrow Var(t_2) = \frac{V}{E_2}$

We know that correlation coefficient between t_1 & t_2

$$\rho = \frac{cov(t_1, t_2)}{\sqrt{var(t_1)\, var(t_2)}} \Rightarrow cov(t_1, t_2) = \rho \cdot \frac{V}{\sqrt{E_1 E_2}}$$

where V is the minimum variance

Let $t = \lambda_1 t_1 + \lambda_2 t_2$

where λ_1 & λ_2 are constant

Such that $\lambda_1 + \lambda_2 = 1$

Given that t_1 & t_2 are unbiased estimators

$$Var(t) = Var(\lambda_1 t_1 + \lambda_2 t_2)$$

$$Var(t) = \lambda_1^2 Var(t_1) + \lambda_2^2 Var(t_2) + 2\lambda_1\lambda_2 Cov(t_1, t_2)$$

$$Var(t) = \frac{\lambda_1^2 V}{E_1} + \frac{\lambda_2^2 V}{E_2} + 2\lambda_1\lambda_2 \frac{\rho V}{\sqrt{E_1 E_2}}$$

$$Var(t) = V\left[\frac{\lambda_1^2}{E_1} + \frac{\lambda_2^2}{E_2} + \frac{2\rho\lambda_1\lambda_2}{\sqrt{E_1 E_2}}\right]$$

Since $Var(t) \geq V$, where V is the minimum value of variance.

$$\Rightarrow \frac{\lambda_1^2}{E_1} + \frac{\lambda_2^2}{E_2} + \frac{2\rho\lambda_1\lambda_2}{\sqrt{E_1 E_2}} \geq 1$$

$$\frac{\lambda_1^2}{E_1} + \frac{\lambda_2^2}{E_2} + \frac{2\rho\lambda_1\lambda_2}{\sqrt{E_1 E_2}} \geq \lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2$$

Hint

$\because \lambda_1 + \lambda_2 = 1$

$(\lambda_1 + \lambda_2)^2 = 1$

$\lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2 = 1$

$a \qquad c \qquad b$

$$\lambda_1^2\left[\frac{1}{E_1} - 1\right] + \lambda_2^2\left[\frac{1}{E_2} - 1\right] + 2\lambda_1\lambda_2\left[\frac{\rho}{\sqrt{E_1 E_2}} - 1\right] \geq 0$$

Which is Quadratic in λ_1

So

$$4\lambda_2^2\left[\frac{\rho}{\sqrt{E_1 E_2}} - 1\right]^2 - 4\lambda_2^2\left[\frac{1}{E_1} - 1\right]\left[\frac{1}{E_2} - 1\right] \geq 0$$

Hint

$b^2 - 4ac \geq 0$

$$4\lambda_2^2\left[\frac{\rho^2}{E_1 E_2} - \frac{2\rho}{\sqrt{E_1 E_2}} + 1\right] - 4\lambda_2^2\left[\frac{1}{E_1 E_2} - \frac{1}{E_1} - \frac{1}{E_2} + 1\right] \geq 0$$

$$\frac{\rho^2}{E_1E_2} - \frac{2\rho}{\sqrt{E_1E_2}} + \gamma - \frac{1}{E_1E_2} + \frac{1}{E_1} + \frac{1}{E_2} - \gamma \geq 0$$

$$\frac{\rho^2 - 2\rho\sqrt{E_1E_2} - 1 + E_2 + E_1}{E_1E_2} \geq 0$$

$$\rho^2 - 2\sqrt{E_1E_2}\,\rho + (E_1 + E_2 - 1) \geq 0 \quad (1)$$

Which is Quadratic in ρ

$$\rho = \frac{2\sqrt{E_1E_2} \pm \sqrt{4E_1E_2 - 4(E_1+E_2-1)}}{2} \qquad \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\rho = \sqrt{E_1E_2} \pm \sqrt{E_1E_2 - E_1 - E_2 + 1}$$

$$\rho = \sqrt{E_1E_2} \pm \sqrt{-E_1(1-E_2) + 1(1-E_2)}$$

$$\rho = \sqrt{E_1E_2} \pm \sqrt{(1-E_1)(1-E_2)}$$

$$\sqrt{E_1E_2} - \sqrt{(1-E_1)(1-E_2)} < \rho < \sqrt{E_1E_2} + \sqrt{(1-E_1)(1-E_2)} \quad \text{//Proved.}$$

Example:- Let $X_1, X_2, \ldots, X_n$ be a random sample from the normal distribution with mean zero and variance θ. Show that $\sum_{i=1}^{n} X_i^2$ is a sufficient estimator of θ.

SOL:-

Given that $f(x;\theta) = \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{1}{2}\frac{x^2}{\theta}}, \quad -\infty < x < \infty$

We want to show that $\sum_{i=1}^{n} x_i^2$ is a sufficient estimator of θ.

Joint Prob. function

$$\prod_{i=1}^{n} f(x_i;\theta) = f(x_1;\theta)\cdot f(x_2;\theta) \cdots f(x_n;\theta)$$

$$= \left[\frac{1}{\sqrt{2\pi\theta}} e^{-x_1^2/2\theta}\right]\left[\frac{1}{\sqrt{2\pi\theta}} e^{-x_2^2/2\theta}\right] \cdots \left[\frac{1}{\sqrt{2\pi\theta}} e^{-x_n^2/2\theta}\right]$$

$$\prod_{i=1}^{n} f(x_i;\theta) = \frac{1}{(2\pi\theta)^{n/2}} e^{-\frac{1}{2\theta}\left[x_1^2 + x_2^2 + \cdots + x_n^2\right]}$$

P.T.O

$$P\left(\underline{X} = \underline{x} \mid T(x) = \tau(x)\right) = \frac{P(\underline{X} = \underline{x})}{P\left(T(\underline{x}) = \tau(\underline{x})\right)}$$

OR

$$P\left(\underline{X} = \underline{x} \mid S = s(x)\right) = \frac{P(\underline{X} = \underline{x})}{P(S = s(\underline{x}))}$$

Day Date

$$f(0,0,0 \mid s=0) = \frac{f(0,0,0)}{P(s=0)}$$

$$= \frac{P(X_1=0)\,P(X_2=0)\,P(X_3=0)}{P(X_1=0, X_2=0, X_3=0)}$$

$$= \frac{(1-p)(1-p)(1-p)}{(1-p)^3}$$

$$\cancel{\therefore\ p^x(1-p)^{1-x} = p^0(1-p)^{1-0} = (1-p)}$$

$$\cancel{\therefore\ p^{0+0+0}(1-p)^{3-(0+0+0)} = p^0(1-p)^{3-0} = (1-p)^3}$$

$$P(X_1=0) = p^x(1-p)^{1-x} = p^0(1-p)^{1-0} = (1-p)$$

$P(X_2=0) =$ " " "

$P(X_3=0) =$ " " "

$$P(X_1, X_2, X_3=0) = p^{0+0+0}(1-p)^{3-(0+0+0)}$$

$$= p^0(1-p)^{3-0} = (1-p)^3$$

$$P(\underline{X}=\underline{x} / S = s(\underline{x})) = \frac{P(\underline{X}=\underline{x})}{P(S = s(\underline{x}))} \neq 0.$$

$$f(x) = P(\underline{X}=x) = P^{x}(1-P)^{1-x} \qquad x = 0, 1.$$

$$f(x_1, x_2, x_3; P) = P^{x_1+x_2+x_3}(1-P)^{1-(x_1+x_2+x_3)}$$

$$= P^{s}(1-P)^{1-s}$$

$$P(\underline{X} = \underline{x} \mid T(x) = \tau(x)) = \frac{P(\underline{X} = \underline{x})}{P(T(\underline{x}) = \tau(\underline{x}))} \neq \theta$$

OR

$$P(\underline{X} = \underline{x} \mid S = s(\underline{x})) = \frac{P(\underline{X} = \underline{x})}{P(S = s(\underline{x}))} \neq \theta.$$

$$f(x) = P(\underline{X} = x) = P^{x}(1-P)^{1-x} \qquad x = 0, 1.$$

$$= P^{x_1 + x_2 + x_3}(1-P)^{1-(x_1 + x_2 + x_3)}.$$

$$= P^{s}(1-P)^{1-s}$$