4-8 THE FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics is a statement of conservation of energy. Recall that the system formulation of the first law was

$$\dot{Q} - \dot{W} = \left.\frac{dE}{dt}\right)_{\text{system}} \tag{4.4a}$$

where the total energy of the system is given by

$$E_{\text{system}} = \int_{M(\text{system})} e\,dm = \int_{V(\text{system})} e\rho\,dV \tag{4.4b}$$

and

$$e = u + \frac{V^2}{2} + gz$$

In Eq. 4.4a, the rate of heat transfer, $\dot{Q}$, is positive when heat is added to the system from the surroundings; the rate of work, $\dot{W}$, is positive when work is done by the system on its surroundings.

To derive the control volume formulation of the first law of thermodynamics, we set

$$N = E \qquad \text{and} \qquad \eta = e$$

in Eq. 4.11 and obtain

$$\left.\frac{dE}{dt}\right)_{\text{system}} = \frac{\partial}{\partial t}\int_{\text{CV}} e\rho\,dV + \int_{\text{CS}} e\rho\vec{V}\cdot d\vec{A} \tag{4.54}$$

Since the system and the control volume coincide at t_0,

$$[\dot{Q} - \dot{W}]_{\text{system}} = [\dot{Q} - \dot{W}]_{\text{control volume}}$$

In light of this, Eqs. 4.4a and 4.54 yield the control volume formulation of the first law of thermodynamics,

$$\dot{Q} - \dot{W} = \frac{\partial}{\partial t}\int_{\text{CV}} e\rho\,dV + \int_{\text{CS}} e\rho\vec{V}\cdot d\vec{A} \tag{4.55}$$

where

$$e = u + \frac{V^2}{2} + gz$$

Note that for steady flow the first term on the right side of Eq. 4.55 is zero.

Is Eq. 4.55 the form of the first law used in thermodynamics? Even for steady flow, Eq. 4.55 is not quite the same form used in applying the first law to control volume problems. To obtain a formulation suitable and convenient for problem solutions, let us take a closer look at the work term, $\dot{W}$.

4-8.1 Rate of Work Done by a Control Volume

The term $\dot{W}$ in Eq. 4.55 has a positive numerical value when work is done by the control volume on the surroundings. The rate of work done on the control volume is of opposite sign to the work done by the control volume.

The rate of work done by the control volume is conveniently subdivided into four classifications,

$$\dot{W} = \dot{W}_s + \dot{W}_{\text{normal}} + \dot{W}_{\text{shear}} + \dot{W}_{\text{other}}$$

Let us consider these separately:

1. Shaft Work

We shall designate shaft work W_s and hence the rate of work transferred out through the control surface by shaft work is designated $\dot{W}_s$.

2. Work Done by Normal Stresses at the Control Surface

Recall that work requires a force to act through a distance. Thus, when a force, $\vec{F}$, acts through an infinitesimal displacement, $d\vec{s}$, the work done is given by

$$\delta W = \vec{F} \cdot d\vec{s}$$

To obtain the rate at which work is done by the force, divide by the time increment, Δt, and take the limit as $\Delta t \to 0$. Thus the rate of work done by the force, $\vec{F}$, is

$$\dot{W} = \lim_{\Delta t \to 0} \frac{\delta W}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{F} \cdot d\vec{s}}{\Delta t} \qquad \text{or} \qquad \dot{W} = \vec{F} \cdot \vec{V}$$

The rate of work done on an element of area, $d\vec{A}$, of the control surface by normal stresses is

$$d\vec{F} \cdot \vec{V} = \sigma_{nn} d\vec{A} \cdot \vec{V}$$

Since the work out across the boundaries of the control volume is the negative of the work done on the control volume, the total rate of work out of the control volume due to normal stresses is

$$\dot{W}_{\text{normal}} = -\int_{CS} \sigma_{nn}\, d\vec{A} \cdot \vec{V} = -\int_{CS} \sigma_{nn} \vec{V} \cdot d\vec{A}$$

3. Work Done by Shear Stresses at the Control Surface

Just as work is done by the normal stresses at the boundaries of the control volume, so may work be done by the shear stresses.

The shear force acting on an element of area of the control surface is given by

$$d\vec{F} = \vec{\tau}\, dA$$

where the shear stress vector, $\vec{\tau}$, is the shear stress acting in the plane of dA.

The rate of work done on the entire control surface by shear stresses is given by

$$\int_{CS} \vec{\tau}\, dA \cdot \vec{V} = \int_{CS} \vec{\tau} \cdot \vec{V}\, dA$$

Since the work out across the boundaries of the control volume is the negative of the work done on the control volume, the rate of work out of the control volume due to shear stresses is given by

$$\dot{W}_{\text{shear}} = -\int_{CS} \vec{\tau} \cdot \vec{V}\, dA$$

This integral is better expressed as three terms

$$\dot{W}_{\text{shear}} = -\int_{CS} \vec{\tau} \cdot \vec{V}\, dA$$

$$= -\int_{A(\text{shafts})} \vec{\tau} \cdot \vec{V}\, dA - \int_{A(\text{solid surface})} \vec{\tau} \cdot \vec{V}\, dA - \int_{A(\text{ports})} \vec{\tau} \cdot \vec{V}\, dA$$

We have already accounted for the first term, since we included $\dot{W}_s$ previously. At solid surfaces, $\vec{V} = 0$, so the second term is zero (for a fixed control volume). Thus

$$\dot{W}_{\text{shear}} = -\int_{A(\text{ports})} \vec{\tau} \cdot \vec{V}\, dA$$

This last term can be made zero by proper choice of control surfaces. If we choose a control surface that cuts across each port perpendicular to the flow, then $d\vec{A}$ is parallel to $\vec{V}$. Since $\vec{\tau}$ is in the plane of dA, $\vec{\tau}$ is perpendicular to $\vec{V}$. Thus, for a control surface perpendicular to $\vec{V}$,

$$\vec{\tau} \cdot \vec{V} = 0 \qquad \text{and} \qquad \dot{W}_{\text{shear}} = 0$$

4. Other Work

Electrical energy could be added to the control volume. Also electromagnetic energy, e.g., in radar or laser beams, could be absorbed. In most problems, such contributions will be absent, but we should note them in our general formulation.

With all of the terms in $\dot{W}$ evaluated, we obtain

$$\dot{W} = \dot{W}_s - \int_{CS} \sigma_{nn} \vec{V} \cdot d\vec{A} + \dot{W}_{\text{shear}} + \dot{W}_{\text{other}} \tag{4.56}$$

4-8.2 Control Volume Equation

Substituting the expression for $\dot{W}$ from Eq. 4.56 into Eq. 4.55 gives

$$\dot{Q} - \dot{W}_s + \int_{CS} \sigma_{nn} \vec{V} \cdot d\vec{A} - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t} \int_{CV} e\rho \, d\mathcal{V} + \int_{CS} e\rho \vec{V} \cdot d\vec{A}$$

Rearranging this equation, we obtain

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t} \int_{CV} e\rho \, d\mathcal{V} + \int_{CS} e\rho \vec{V} \cdot d\vec{A} - \int_{CS} \sigma_{nn} \vec{V} \cdot d\vec{A}$$

Since $\rho = 1/v$, where v is *specific volume*, then

$$\int_{CS} \sigma_{nn} \vec{V} \cdot d\vec{A} = \int_{CS} \sigma_{nn} v \rho \vec{V} \cdot d\vec{A}$$

Hence

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t} \int_{CV} e\rho \, d\mathcal{V} + \int_{CS} (e - \sigma_{nn} v) \rho \vec{V} \cdot d\vec{A}$$

Viscous effects can make the normal stress, σ_{nn}, different from the negative of the thermodynamic pressure, $-p$. However, for most flows of common engineering interest, $\sigma_{nn} \approx -p$. Then

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t} \int_{CV} e\rho \, d\mathcal{V} + \int_{CS} (e + pv) \rho \vec{V} \cdot d\vec{A}$$

Finally, substituting $e = u + V^2/2 + gz$ into the last term, we obtain the familiar form of the first law formulation for a control volume,

$$\dot{Q} - \dot{W}_s - \dot{W}_{\text{shear}} - \dot{W}_{\text{other}} = \frac{\partial}{\partial t} \int_{CV} e\rho \, d\mathcal{V} + \int_{CS} \left(u + pv + \frac{V^2}{2} + gz \right) \rho \vec{V} \cdot d\vec{A} \tag{4.57}$$

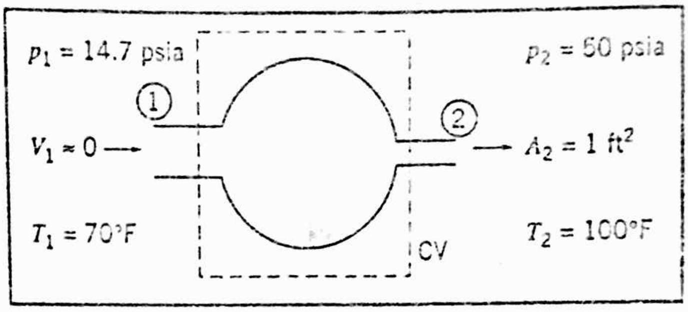
Each work term in Eq. 4.57 represents the rate of work done by the control volume on the surroundings.

EXAMPLE 4.16 Compressor: First Law Analysis

Air at 14.7 psia, 70°F, enters a compressor with negligible velocity and is discharged at 50 psia, 100°F through a pipe with 1 ft² area. The flow rate is 20 lbm/s. The power input to the compressor is 600 hp. Determine the rate of heat transfer.

EXAMPLE PROBLEM 4.16

GIVEN: Air enters a compressor at ① and leaves at ② with conditions as shown. The air flow rate is 20 lbm/s and the power input to the compressor is 600 hp.



FIND: Rate of heat transfer.

SOLUTION:

Basic equations:

$$0 = \overset{=0(1)}{\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall} + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$

$$\dot{Q} - \dot{W}_s - \overset{=0(4)}{\dot{W}_{shear}} = \overset{=0(1)}{\frac{\partial}{\partial t}\int_{CV} e\rho \, d\forall} + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

Assumptions:
(1) Steady flow
(2) Properties uniform over inlet and outlet sections
(3) Treat air as an ideal gas, $p = \rho RT$
(4) Area of CV at ① and ② perpendicular to velocity, thus $\dot{W}_{shear} = 0$
(5) $z_1 = z_2$
(6) Inlet kinetic energy is negligible

Under the assumptions listed, the first law becomes

$$\dot{Q} - \dot{W}_s = \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

$$\dot{Q} - \dot{W}_s = \int_{CS}\left(h + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A} \qquad \{h \equiv u + pv\}$$

or

$$\dot{Q} = \dot{W}_s + \int_{CS}\left(h + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A}$$

For uniform properties (assumption 2) we can write

$$\dot{Q} = \dot{W}_s + \left(h_1 + \overset{\approx 0(6)}{\frac{V_1^2}{2}} + gz_1\right)\{-|\rho_1 V_1 A_1|\} + \left(h_2 + \frac{V_2^2}{2} + gz_2\right)\{|\rho_2 V_2 A_2|\}$$

For steady flow, from conservation of mass,

$$\int_{CS} \rho\vec{V}\cdot d\vec{A} = 0$$

Therefore, $-|\rho_1 V_1 A_1| + |\rho_2 V_2 A_2| = 0$, or $|\rho_1 V_1 A_1| = |\rho_2 V_2 A_2| = \dot{m}$. Hence we can write

$$\dot{Q} = \dot{W}_s + \dot{m}\left[(h_2 - h_1) + \frac{V_2^2}{2} + g(z_2 - z_1)\right] \quad = 0(5)$$

Assume that air behaves as an ideal gas with constant c_p. Then $h_2 - h_1 = c_p(T_2 - T_1)$, and

$$\dot{Q} = \dot{W}_s + \dot{m}\left[c_p(T_2 - T_1) + \frac{V_2^2}{2}\right]$$

From continuity $|V_2| = \dot{m}/\rho_2 A_2$. Since $p_2 = \rho_2 R T_2$,

$$|V_2| = \frac{\dot{m}}{A_2}\frac{RT_2}{p_2} = \frac{20\ \text{lbm}}{\text{s}} \times \frac{1}{1\ \text{ft}^2} \times \frac{53.3\ \text{ft}\cdot\text{lbf}}{\text{lbm}\cdot{}^\circ\text{R}} \times 560^\circ\text{R} \times \frac{\text{in.}^2}{50\ \text{lbf}} \times \frac{\text{ft}^2}{144\ \text{in.}^2}$$

$$|V_2| = 82.9\ \text{ft/s}$$

$$\dot{Q} = \dot{W}_s + \dot{m}c_p(T_2 - T_1) + \dot{m}\frac{V_2^2}{2}$$

Note that power input is *to* the CV, so $\dot{W}_s = -600$ hp, and

$$\dot{Q} = -600\ \text{hp} \times \frac{550\ \text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} \times \frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}} + \frac{20\ \text{lbm}}{\text{s}} \times \frac{0.24\ \text{Btu}}{\text{lbm}\cdot{}^\circ\text{R}} \times 30^\circ\text{R}$$

$$+ \frac{20\ \text{lbm}}{\text{s}} \times \frac{(82.9)^2}{2}\frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{slug}}{32.2\ \text{lbm}} \times \frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$\dot{Q} = -277\ \text{Btu/s}$$

{heat rejection} $\dot{Q}$

In addition to demonstrating a straightforward application of the first law, this problem illustrates the need for keeping units straight.

EXAMPLE 4.17 Tank Filling: First Law Analysis

A tank of 0.1 m³ volume is connected to a high-pressure air line; both line and tank are initially at a uniform temperature of 20°C. The initial tank gage pressure is 100 kPa. The absolute line pressure is 2.0 MPa; the line is large enough so that its temperature and pressure may be assumed constant. The tank temperature is monitored by a fast-response thermocouple. At the instant after the valve is opened, the tank temperature rises at the rate of 0.05°C/s. Determine the instantaneous flow rate of air into the tank if heat transfer is neglected.

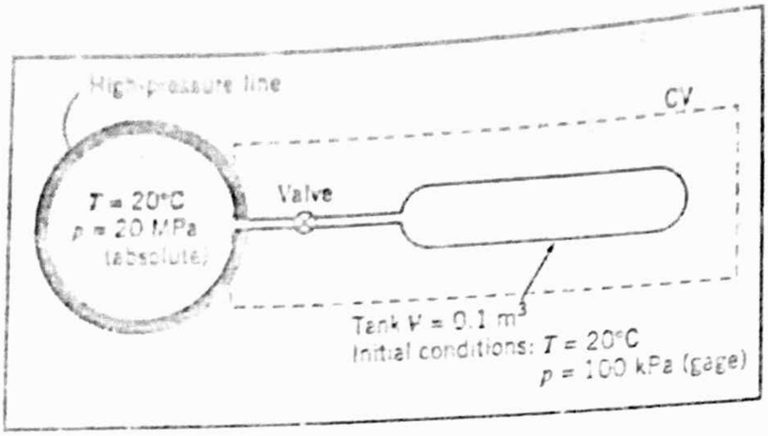
EXAMPLE PROBLEM 4.17

GIVEN: Air supply pipe and tank as shown. At $t = 0^+$, $\partial T/\partial t = 0.05$°C/s.

FIND: $\dot{m}$ at $t = 0^+$.

SOLUTION:

Choose CV shown, apply energy equation.



Basic equation:

$$\overset{=0(1)}{\dot{Q}} - \overset{=0(2)}{\dot{W}_s} - \overset{=0(3)}{\dot{W}_{shear}} - \overset{=0(4)}{\dot{W}_{other}} = \frac{\partial}{\partial t}\int_{CV} e\rho\, dV + \int_{CS}(e + pv)\rho\vec{V}\cdot d\vec{A}$$

$$e = u + \overset{\approx 0(5)}{\frac{V^2}{2}} + \overset{\approx 0(6)}{gz}$$

Assumptions:
(1) $\dot{Q} = 0$ (given)
(2) $\dot{W}_s = 0$
(3) $\dot{W}_{shear} = 0$
(4) $\dot{W}_{other} = 0$
(5) Velocities in line and tank are small
(6) Neglect potential energy
(7) Uniform flow at tank inlet
(8) Properties uniform in tank
(9) Ideal gas, $p = \rho RT$, $du = c_v dT$

Then

$$0 = \frac{\partial}{\partial t}\int_{CV} u_{tank}\rho\, dV + (u_{line} + pv)\{-|\rho VA|\}$$

But initially, T is uniform, so $u_{tank} = u_{line} = u$, and

$$0 = \frac{\partial}{\partial t}\int_{CV} u\rho\, dV + (u + pv)\{-|\rho VA|\}$$

Since tank properties are uniform, $\partial/\partial t$ may be replaced by d/dt, and

$$0 = \frac{d}{dt}[uM] - (u + pv)\dot{m}$$

or

$$0 = u\frac{dM}{dt} + M\frac{du}{dt} - u\dot{m} - pv\dot{m} \qquad (1)$$

The term dM/dt may be evaluated from continuity:

Basic equation: $$0 = \frac{\partial}{\partial t}\int_{CV}\rho\, dV + \int_{CS}\rho\vec{V}\cdot d\vec{A}$$

$$0 = \frac{dM}{dt} + \{-|\rho VA|\} \qquad \text{or} \qquad \frac{dM}{dt} = \dot{m}$$

Substituting into Eq. 1 gives

$$0 = \cancel{u\dot{m}} + M\frac{du}{dt} - \cancel{u\dot{m}} - pv\dot{m} = Mc_v\frac{dT}{dt} - pv\dot{m}$$

or

$$\dot{m} = \frac{Mc_v(dT/dt)}{pv} = \frac{\rho \forall c_v(dT/dt)}{pv} = \frac{\rho \forall c_v(dT/dt)}{RT} \tag{2}$$

But at $t = 0$, $p_{tank} = 100$ kPa (gage), and

$$\rho = \rho_{tank} = \frac{p_{tank}}{RT} = (1.00 + 1.01)10^5 \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg} \cdot \text{K}}{287 \text{ N} \cdot \text{m}} \times \frac{1}{293 \text{ K}} = 2.39 \text{ kg/m}^3$$

Substituting into Eq. 2, we obtain

$$\dot{m} = \frac{2.39 \text{ kg}}{\text{m}^3} \times 0.1 \text{ m}^3 \times \frac{717 \text{ N} \cdot \text{m}}{\text{kg} \cdot \text{K}} \times \frac{0.05 \text{ K}}{\text{s}} \times \frac{\text{kg} \cdot \text{K}}{287 \text{ N} \cdot \text{m}} \times \frac{1}{293 \text{ K}} \times \frac{1000 \text{ g}}{\text{kg}}$$

$\dot{m} = 0.102$ g/s ← $\dot{m}$

{This problem illustrates the application of the energy equation to an unsteady flow situation.}

4-9 THE SECOND LAW OF THERMODYNAMICS

Recall that the system formulation of the second law is

$$\left.\frac{dS}{dt}\right)_{\text{system}} \geq \frac{1}{T}\dot{Q} \tag{4.5a}$$

where the total entropy of the system is given by

$$S_{\text{system}} = \int_{M(\text{system})} s\,dm = \int_{V(\text{system})} s\rho\,dV \tag{4.5b}$$

To derive the control volume formulation of the second law of thermodynamics, we set

$$N = S \qquad \text{and} \qquad \eta = s$$

in Eq. 4.11 and obtain

$$\left.\frac{dS}{dt}\right)_{\text{system}} = \frac{\partial}{\partial t}\int_{\text{CV}} s\rho\,dV + \int_{\text{CS}} s\rho\vec{V}\cdot d\vec{A} \tag{4.58}$$

The system and the control volume coincide at t_0; thus in Eq. 4.5a,

$$\left.\frac{1}{T}\dot{Q}\right)_{\text{system}} = \left.\frac{1}{T}\dot{Q}\right)_{\text{CV}} = \int_{\text{CS}} \frac{1}{T}\left(\frac{\dot{Q}}{A}\right)dA$$

In light of this, Eqs. 4.5a and 4.58 yield the control volume formulation of the second law of thermodynamics

$$\frac{\partial}{\partial t}\int_{\text{CV}} s\rho\,dV + \int_{\text{CS}} s\rho\vec{V}\cdot d\vec{A} \geq \int_{\text{CS}} \frac{1}{T}\left(\frac{\dot{Q}}{A}\right)dA \tag{4.59}$$

the surroundings.

In Eq. 4.59, the factor $(\dot{Q}/A)$ represents the heat flux per unit area into the control volume through the area element dA. To evaluate the term

$$\int_{CS} \frac{1}{T}\left(\frac{\dot{Q}}{A}\right) dA$$

both the local heat flux, $(\dot{Q}/A)$, and local temperature, T, must be known for each area element of the control surface.