under a constant applied shear stress are called *thixotropic*; many paints are thixotropic. Fluids that show an increase in η with time are termed *rheopectic*. In addition, some fluids after deformation partially return to their original shape when the applied stress is released; such fluids are called *viscoelastic*.

2-5 DESCRIPTION AND CLASSIFICATION OF FLUID MOTIONS

In Chapter 1 we listed a wide variety of typical problems encountered in fluid mechanics and outlined our method of approach to the subject. Before proceeding with our detailed study, we shall attempt a broad classification of fluid mechanics on the basis of observable physical characteristics of flow fields. Since there is much overlap in the types of flow fields encountered, there is no universally accepted classification scheme. One possible classification is shown in Fig. 2.10

2-5.1 Viscous and Inviscid Flows

The main subdivision indicated is between inviscid and viscous flows. In an inviscid flow the fluid viscosity, μ, is assumed to be zero. Fluids with zero viscosity do not exist; however, there are many problems where an assumption that $\mu = 0$ will simplify the analysis and, at the same time, lead to meaningful results. (While simplification of the analysis is always desirable, the results must be reasonably accurate if the solution is to be of value.)

All fluids possess viscosity and, consequently, viscous flows are of paramount importance in the study of continuum fluid mechanics. We shall study viscous flows in some detail later; here we consider a few examples of viscous flow phenomena.

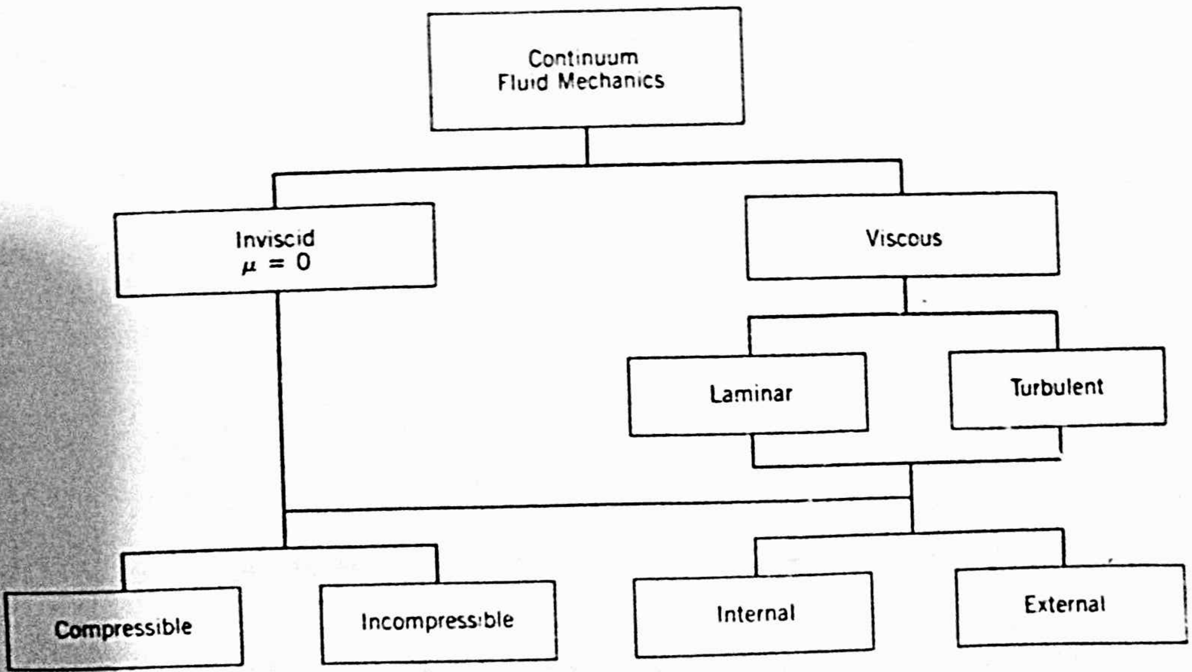


Fig. 2.10 Possible classification of continuum fluid mechanics.

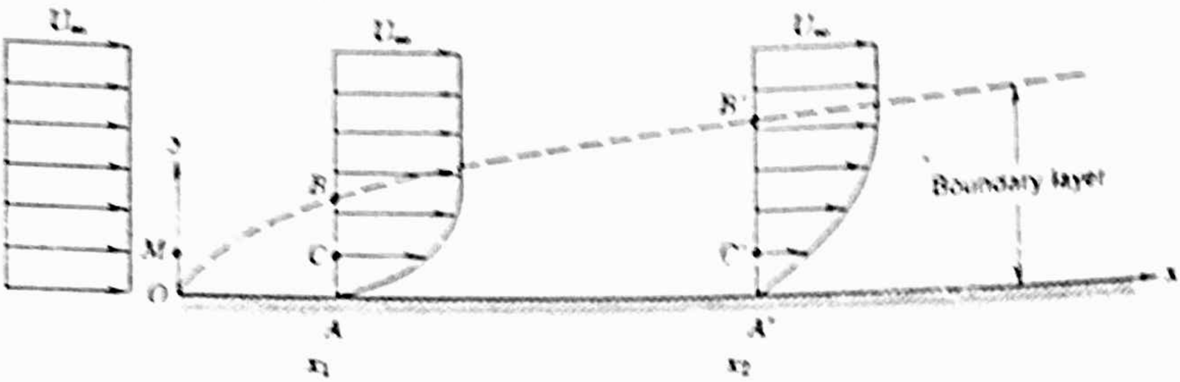


Fig. 2.11 Incompressible laminar viscous flow over a semi-infinite flat plate.

In our discussion following the definition of a fluid (Section 1-2), we noted that in any viscous flow, the fluid in direct contact with a solid boundary has the same velocity as the boundary itself; there is no slip at the boundary. For the one-dimensional viscous flow of Fig. 2.8, the shear stress* was given by Eq. 2.10,

$$\tau_{yx} = \mu \frac{du}{dy} \tag{2.10}$$

Since the fluid velocity at a stationary solid surface in a moving fluid is zero, but the bulk fluid is moving, velocity gradients and hence shear stresses must be present in the flow. These stresses in turn affect the motion.

As a practical case, consider the fluid motion around a thin wing or ship hull. Such a flow might be represented crudely by the flow over a flat plate, as shown in Fig. 2.11. The flow approaching the plate is of uniform velocity, U_∞. We are interested in providing a qualitative picture of the velocity distribution at various locations along the plate. Two such locations are denoted by x_1 and x_2. Consider first location, x_1. In order to arrive at a qualitative picture of the velocity distribution, we start by labeling the y coordinates at which the velocity is known. (For clarity, distances in the y direction have been exaggerated greatly in Fig. 2.11.)

From the no-slip condition, we know the velocity at point A must be zero; we have one point on the velocity profile. Can we locate any other points on the profile? Let us stop for a minute and ask ourselves, "What is the effect of the plate on the flow?" The plate is stationary and, therefore, exerts a retarding force on the flow; it slows the fluid in the neighborhood of the surface. At a y location sufficiently far from the plate, say point B, the flow will not be influenced by the presence of the plate. If the pressure does not vary in the x direction (as is the case for flow over a semi-infinite flat plate) the velocity at point B will be U_∞. It seems reasonable to expect the velocity to increase smoothly and monotonically from the value $u = 0$ at $y = 0$ to $u = U_\infty$ at $y = y_B$. The profile has been so drawn; thus at some point, C, intermediate between points A and B, the velocity has a value that lies between zero and U_∞. For $0 \le y \le y_B$, then $0 \le u \le U_\infty$. From these characteristics of the velocity profile and our definition of the

* In general, $\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$ for flows that are not one-dimensional, see Chapter 5.

shear stress,[9] we see that within the region $0 \le y \le y_B$, shear stresses are present; for $y > y_B$, the velocity gradient is zero and hence no shear stresses are present.

What about the velocity profile at location, x_2? Is it exactly the same as the profile at x_1? A look at Fig. 2.11 suggests that it is not. At least it has not been drawn that way! While it is qualitatively the same, why is it not exactly the same? We might guess that the plate would influence a greater region of the flow field as we move farther down the plate. Looking again at the profile at location x_1, we see that the slower moving fluid adjacent to the plate exerts a retarding force on the faster moving fluid above it. We can see this by considering the shear stress on the y plane through the point C. Since we are interested in the stress exerted on the faster moving fluid above the plane, we are looking for the direction of the shear stress on a negative y plane through the point C. Since $\partial u/\partial y > 0$, τ_{yx} on the plane through point C has a positive numerical value; consequently the shear stress must be in the negative x direction.

To establish the qualitative picture of the velocity profile at x_2, we recognize that the no-slip condition requires the velocity at the wall to be zero; this fixes the velocity at A' as zero. Since, at location, x_1, the slower moving fluid exerts a retarding force on the fluid above it, we would expect the distance out to the point where the velocity is U_∞ to be increased at location, x_2; i.e. $y_{B'} > y_B$. Furthermore, it is reasonable to expect that $u_{C'} < u_C$.

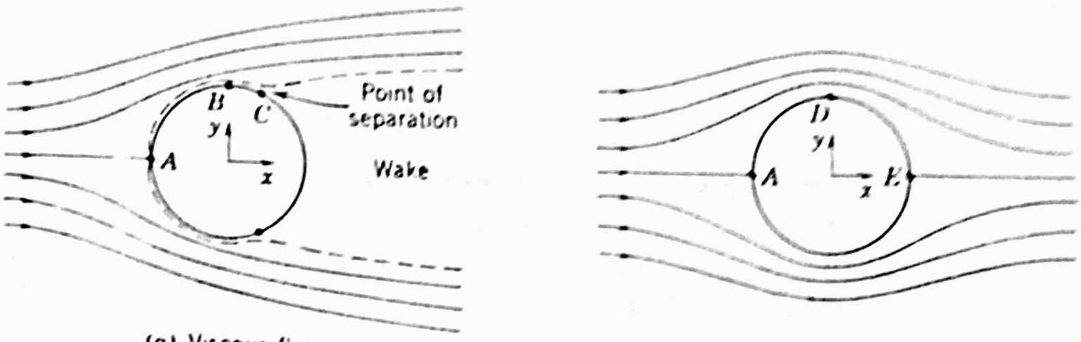
From our qualitative picture of the flow field, we see that we can divide the flow into two general regions. In the region adjacent to the boundary, shear stresses are present; this region is called the boundary layer.[10] Outside the boundary layer the velocity gradient is zero and hence the shear stresses are zero. In this region we may use inviscid flow theory to analyze the flow.

Before leaving our discussion of the viscous flow over a semi-infinite flat plate, we should stop and reflect on two points. In our qualitative description of the flow field, we were only concerned about the behavior of the x component of velocity, the component, u. What about the y component of velocity, the component, v? Is it zero throughout the flow field? We also might ask if the edge of the boundary layer is a streamline.

To answer these questions, consider the streamlines of the flow. Rather than consider all possible streamlines, let us consider the streamline through the point M. Recalling that a streamline is defined as a line drawn tangent to the velocity vector at every point in the flow, our first inclination might be to depict the streamline through M as a straight line parallel to the x axis. However, this would violate the requirement that there can be no flow across a streamline. Because there can be no flow across a streamline, the mass flow between adjacent streamlines (or between a streamline and a solid boundary) must be a constant. For the incompressible viscous flow of Fig. 2.11, we recognize that the streamline through the point M cannot be a straight line parallel to the x axis.

[9] For the two-dimensional boundary-layer flow of Fig. 2.11, the shear stress is given closely by $\tau_{yx} = \mu \dfrac{\partial u}{\partial y}$.

[10] The formation of a boundary layer is demonstrated in the film loop, S-FM006, *Boundary-Layer Formation*.



(a) Viscous flow (b) Inviscid flow

Fig. 2.12 Qualitative picture of incompressible flow over a cylinder.

The spacing between the streamline through the point M and the x axis must increase continuously as we move along the plate. Therefore, although small, the y component of velocity is not zero. The streamline through M crosses the dashed line we have used to denote the edge of the boundary layer. Consequently, we conclude that the edge of the boundary layer is not a streamline, and that there is flow into the boundary layer as we move down the plate. Indeed, if the boundary layer is to grow, there must be flow across the edge of the boundary layer.

For a given freestream velocity, U_∞, the size of the boundary layer will depend on the properties of the fluid. Since the shear stress is directly proportional to the viscosity, we expect the size of the boundary layer to depend on the viscosity of the fluid. In Chapter 9, we shall develop expressions for determining the rate of boundary-layer growth.

We have used incompressible flow over a semi-infinite flat plate to establish a qualitative picture of the viscous flow over a solid boundary. In that example, we had to consider only the effect of shear forces; the pressure was constant throughout the flow field. Now let us consider a steady flow field (the incompressible flow over a cylinder) where both pressure forces and viscous forces are important. For steady flow, pathlines, streaklines, and streamlines all are identical. If we were to use some means of flow visualization, we would find the flow field to be of the general character shown in Fig. 2.12*a*.[11]

We see that the streamlines are symmetric about the x axis. The fluid along the central streamline impinges on the cylinder at point A, divides, and flows around the cylinder. Point A on the cylinder is called a *stagnation point*. As in flow over a flat plate, a boundary layer develops in the neighborhood of the solid surface. The velocity distribution outside the boundary layer can be determined from the spacing of the streamlines. Since there can be no flow across a streamline, we would expect the flow velocity to increase in regions where the spacing between streamlines decreases. Conversely, an increase in streamline spacing implies a decrease in flow velocity.

Consider for a moment the incompressible flow field around a cylinder calculated assuming an inviscid flow, as shown in Fig. 2.12*b*; this flow is symmetric about both the

[11] The details of the flow will depend on the various flow properties. For all but very low-speed flows, the qualitative picture will be as shown.

x and y axes. The velocity around the cylinder increases to a maximum at point D and then decreases as we move further around the cylinder. For inviscid flow, an increase in velocity is accompanied by a decrease in pressure; conversely, a decrease in velocity is accompanied by an increase in pressure. Thus in the case of an incompressible inviscid flow, the pressure along the surface of the cylinder decreases as we move from point A to point D and then increases again from point D to point E. Since the flow is symmetric with respect to both the x and y axes, we would also expect the pressure distribution to be symmetric with respect to these axes. This is indeed the case for inviscid flow.

Since no shear stresses are present in an inviscid flow, the pressure forces are the only forces we need consider in determining the net force on the cylinder. The symmetry of the pressure distribution leads to the conclusion that for an inviscid flow, there is no net force on the cylinder in either the x or y directions. The net force in the x direction is termed the *drag*. Thus for an inviscid flow over a cylinder, we are led to the conclusion that the drag is zero; this conclusion is contrary to experience, for we know that all bodies experience some drag when placed in a real flow. In treating the inviscid flow over a body we have, by the definition of inviscid flow, neglected the presence of the boundary layer. Let us go back and look again at the real flow situation.

In the real flow, Fig. 2.12*a*, experiments show the boundary layer to be thin between points A and C. Since the boundary layer is thin, it is reasonable to assume that the pressure field is qualitatively the same as in the inviscid flow case. Since the pressure decreases continuously between points A and B, a fluid element inside the boundary layer experiences a net pressure force in the direction of flow. In the region between A and B, this net pressure force is sufficient to overcome the resisting shear force and motion of the element in the flow direction is maintained.

Now consider an element of fluid inside the boundary layer on the back of the cylinder beyond point B. Since the pressure increases in the direction of flow, the fluid element experiences a net pressure force opposite to its direction of motion. Finally the momentum of the fluid in the boundary layer is insufficient to carry the element further into the region of increasing pressure. The fluid layers adjacent to the solid surface will be brought to rest and the flow will *separate* from the surface.[12] The point at which this occurs is called the point of separation. Boundary-layer separation results in the formation of a relatively low pressure region behind a body; this region, which is deficient in momentum, is called the wake. Thus, for separated flow over a body, there is a net unbalance of pressure forces in the direction of flow; this results in a pressure drag on the body. The greater the size of the wake behind a body, the greater is the pressure drag.

It is logical to ask how one might reduce the size of the wake and thus reduce the pressure drag. Since a large wake results from boundary-layer separation, which in turn is related to the presence of an adverse pressure gradient (increase of pressure in the direction of flow), reducing the adverse pressure gradient should delay the onset of separation and, hence, reduce the drag.

[12] The flow over a variety of models, illustrating flow separation, is demonstrated in the NCFMF film loops: S-FM012, *Flow Separation and Vortex Shedding*; S-FM004, *Separated Flows—Part I*; and S-FM005, *Separated Flows—Part II*.

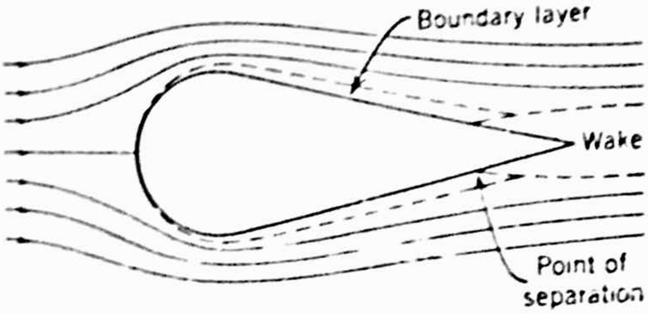


Fig. 2.13 Flow over a streamlined object.

Streamlining a body reduces the adverse pressure gradient by spreading a given pressure rise over a larger distance. For example, if a gradually tapered rear section were added to the cylinder of Fig. 2.12, the flow field would appear qualitatively as shown in Fig. 2.13. Streamlining the body delays the onset of separation; although the surface area of the body and, hence, the total shear force acting on the body is increased, the drag is reduced significantly.[13]

Flow separation also may occur in internal flows (flows through ducts) as a result of rapid or abrupt changes in duct geometry.[14]

2-5.2 Laminar and Turbulent Flows

Viscous flow regimes are classified as laminar or turbulent on the basis of internal flow structure. In the laminar regime, flow structure is characterized by smooth motion in laminae or layers. Flow structure in the turbulent regime is characterized by random, three-dimensional motions of fluid particles superimposed on the mean motion.

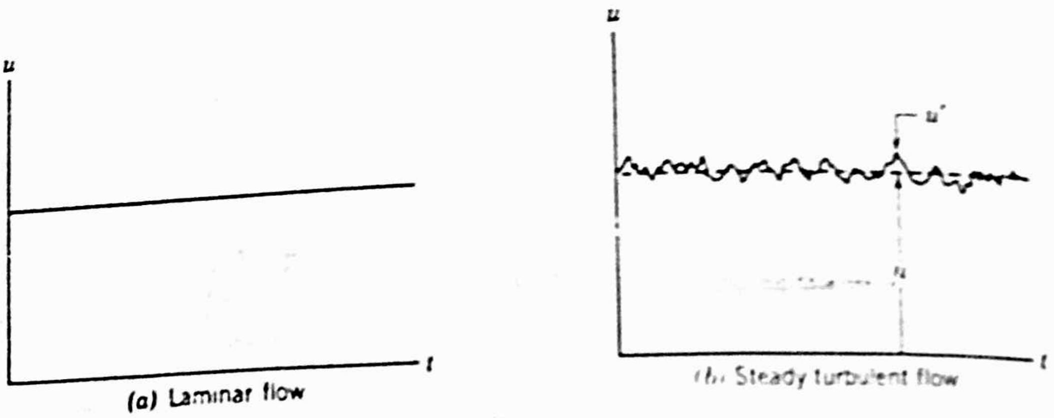
In laminar flow there is no macroscopic mixing of adjacent fluid layers. A thin filament of dye injected into a laminar flow appears as a single line; there is no dispersion of dye throughout the flow, except the slow dispersion due to molecular motion. On the other hand, a dye filament injected into a turbulent flow disperses quickly throughout the flow field; the line of dye breaks up into myriad entangled threads of dye. This behavior of turbulent flow is due to small velocity fluctuations superimposed on the mean motion; the macroscopic mixing of fluid particles from adjacent layers of fluid results in rapid dispersion of the dye. The straight filament of smoke rising from a cigarette in still surroundings gives a clear picture of laminar flow. As the smoke continues to rise, it breaks up into random, haphazard motions; this is an example of turbulent flow.[15]

One can obtain a more quantitative picture of the difference between laminar and turbulent flow by examining the output from a sensitive velocity-measuring device immersed in the flow. If one measures the x component of velocity at a fixed location in a pipe for both laminar and turbulent steady flow, the traces of velocity versus time

[13] The effect of streamlining a body is demonstrated in the film loop, S-FM004, *Separated Flows—Part I.*

[14] Examples of separation in internal flows are shown in the film loop, S-FM015, *Incompressible Flow through Area Contractions and Expansions.*

[15] Several examples illustrating the nature of laminar and turbulent flows are shown in the film loop S-FM008, *The Occurrence of Turbulence.*



(a) Laminar flow

(b) Steady turbulent flow

Fig. 2.14 Variation of axial velocity with time

appear as shown in Fig. 2.14. For steady laminar flow, the velocity at a point remains constant with time. In turbulent flow the velocity trace indicates random fluctuations of the instantaneous velocity, u, about the time mean velocity, $\bar{u}$. We can consider the instantaneous velocity, u, as the sum of the time mean velocity, $\bar{u}$, and the fluctuating component, u',

$$u = \bar{u} + u'$$

Because the flow is steady, the mean velocity, $\bar{u}$, does not vary with time.

Although many turbulent flows of interest are steady in the mean ($\bar{u}$ is not a function of time), the presence of the random, high-frequency velocity fluctuations makes the analysis of turbulent flows extremely difficult. In a one-dimensional laminar flow, the shear stress is related to the velocity gradient by the simple relation

$$\tau_{yx} = \mu \frac{du}{dy} \tag{2.10}$$

For a turbulent flow in which the mean velocity field is one-dimensional, no such simple relation is valid. Random, three-dimensional velocity fluctuations (u', v', and w') transport momentum across the mean flow streamlines, increasing the effective shear stress. Consequently, in turbulent flow there is no universal relationship between the stress field and the mean-velocity field. Thus in turbulent flows we must rely heavily on semi-empirical theories and on experimental data.

2-5.3 Compressible and Incompressible Flows

Flows in which variations in density are negligible are termed *incompressible*; when density variations within a flow are not negligible, the flow is called *compressible*. If one considers the two states of matter, liquid and gas, included within the definition of a fluid, one is tempted to make the general statement that all liquid flows are incompressible flows and all gas flows are compressible flows. For many practical cases the first portion of the statement is correct; most liquid flows are essentially incompressible. However, water hammer and cavitation are examples of the importance of compressibility effects in liquid flows. Gas flows also may be considered incompressible provided the flow speeds are small relative to the speed of sound; the ratio of the flow speed, V, to the local speed of sound, c, in the gas is defined as the Mach number.

$$M \equiv \frac{V}{c}$$

For values of $M < 0.3$, the maximum density variation is less than 5 percent. Thus gas flows with $M < 0.3$ can be treated as incompressible; a value of $M = 0.3$ in air at standard conditions corresponds to a speed of approximately 100 m/sec.

Compressible flows occur frequently in engineering applications. Common examples include compressed air systems used to power shop tools and dental drills, transmission of gases in pipelines at high pressure, and pneumatic or fluidic control and sensing systems. Compressibility effects are very important in the design of modern high-speed aircraft and missiles, power plants, fans, and compressors.

2-5.4 Internal and External Flows

Flows completely bounded by solid surfaces are called internal or duct flows. Internal flows may be laminar or turbulent, compressible or incompressible.

In the case of incompressible flow through a pipe, the nature of the flow (laminar or turbulent) is determined by the value of a dimensionless parameter, the Reynolds number, $Re = \rho \bar{V} D/\mu$, where ρ is the density of the fluid, $\bar{V}$ the average flow velocity, D the pipe diameter, and μ the viscosity of the fluid. Pipe flow is laminar when $Re \leq 2300$; it may be turbulent for larger values. (The Reynolds number and other important dimensionless parameters encountered in fluid mechanics will be discussed in Chapter 7.) Chapter 8 will be devoted to a study of internal incompressible flow.

In the case of internal compressible flows, proper duct design is necessary to attain supersonic flow. The variation of fluid properties within a variable-area flow passage is not the same for supersonic flow ($M > 1$) as it is for subsonic flow ($M < 1$). Likewise the boundary conditions on the flow at the exit of an internal flow (e.g. the discharge from a nozzle) are different in the two cases. For subsonic flow discharge, the pressure in the exit plane of the nozzle is ambient pressure. For sonic flow, the nozzle exit pressure may be greater than ambient. For a supersonic jet, the pressure in the exit plane of the nozzle may be greater than, equal to, or less than ambient pressure. One-dimensional, steady compressible flow will be treated in Chapters 11 and 12.

External flows occur over bodies immersed in an unbounded fluid. The flow over a semi-infinite flat plate (Fig. 2.11) and the flow over a cylinder (Fig. 2.12*a*) are examples of external flows.

Boundary-layer flows also may be laminar or turbulent; the definitions of laminar and turbulent flows given earlier also apply to boundary-layer flows; the details of a flow field may be significantly different depending on whether the boundary layer is laminar or turbulent. In Chapter 9, boundary-layer flows and flow over immersed bodies will be discussed in detail.

Flows of liquids in which the duct does not flow full—where there is a free surface subject to a constant pressure—are termed *open-channel* flows. Common examples of open-channel flow include flow in rivers, irrigation ditches, and aqueducts. Open-channel flow will be treated in Chapter 10.