1-5 METHODS OF ANALYSIS

As we have indicated, the basic laws that are used to analyze problems in fluid mechanics are the same ones used in thermodynamics and basic mechanics. The first step in solving a problem is to define the system that you are attempting to analyze. In basic mechanics, extensive use was made of the free-body diagram. In thermodynamics closed or open systems were considered. In this text we use the terms *system* and *control volume*. The importance of defining the system or control volume before applying the basic equations in the analysis of a problem cannot be overemphasized. At this point we review the definitions of systems and control volumes.

1-5.1 System and Control Volume

A system is defined as a fixed, identifiable quantity of mass; the system boundaries separate the system from the surroundings. The boundaries of the system may be fixed or movable; however, there is no mass transfer across the system boundaries.

In the familiar piston-cylinder assembly from thermodynamics, Fig. 1.2, the gas in the cylinder is the system. If a high-temperature source is brought in contact with the

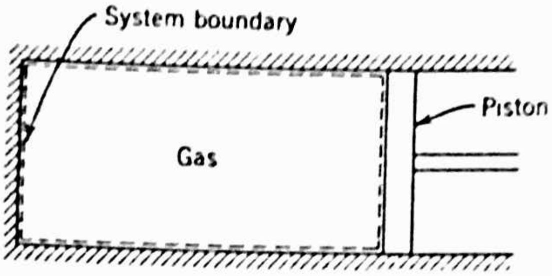


Fig. 1.2 Piston cylinder assembly.

left end of the cylinder, the piston will move to the right; the boundary of the system thus moves. Heat and work may cross the boundaries of the system, but the quantity of matter within the system boundaries remains fixed. There is no mass transfer across the system boundaries.

Example 1.1

A piston-cylinder device contains 0.95 kg of oxygen initially at a temperature of 27 C and a pressure of 150 kPa. Heat is added to the gas and it expands at constant pressure to a temperature of 627 C. Determine the amount of heat added during the process.

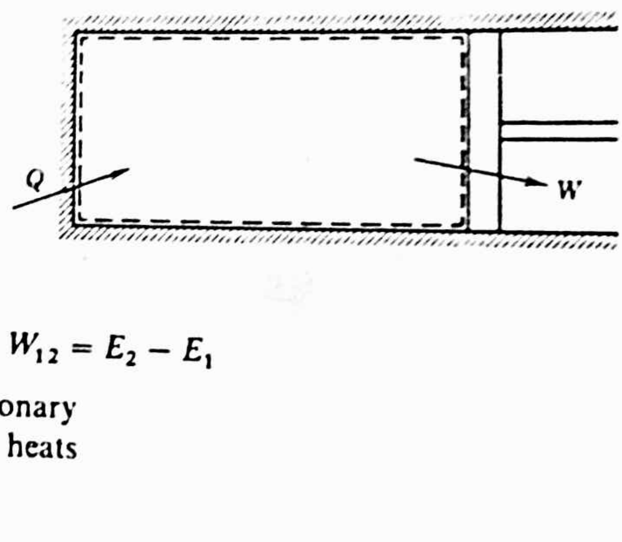
EXAMPLE PROBLEM 1.1

GIVEN: Piston-cylinder containing O_2, $m = 0.95$ kg.

$T_1 = 27$ C $\quad T_2 = 627$ C

p = constant = 150 kPa (abs)

FIND: Q_{1-2}.



SOLUTION:

We are dealing with a system, $m = 0.95$ kg.

Basic equation: First law for the system, $Q_{12} + W_{12} = E_2 - E_1$

Assumptions: $E = U$, since the system is stationary
Ideal gas with constant specific heats

Under the above assumptions,

$$E_2 - E_1 = U_2 - U_1 = m(u_2 - u_1) = mc_v(T_2 - T_1)$$

The work done during the process is moving boundary work

$$W_{12} = -\int_{\mathcal{V}_1}^{\mathcal{V}_2} p\,d\mathcal{V} = p(\mathcal{V}_1 - \mathcal{V}_2)$$

For an ideal gas, $p\mathcal{V} = mRT$. Hence $W_{12} = mR(T_1 - T_2)$. Then from the first law equation,

$$Q_{12} = E_2 - E_1 - W_{12} = mc_v(T_2 - T_1) + mR(T_2 - T_1)$$

$$Q_{12} = m(T_2 - T_1)(c_v + R)$$

$$Q_{12} = mc_p(T_2 - T_1) \qquad \{R = c_p - c_v\}$$

From the Appendix, Table A.6, for O_2, $c_p = 909.4$ J/kg · K. Solving for Q_{12}, we obtain

$$Q_{12} = 0.95\ \text{kg} \times 909.4\ \frac{\text{J}}{\text{kg}\cdot\text{K}} \times 600\ \text{K}$$

$$Q_{12} = 518\ \text{kJ} \qquad Q_{12}$$

The purpose of this problem was to review the use of:
the first law of thermodynamics for a system, and
(i) the first law of thermodynamics for a system, and
(ii) the equation of state for an ideal gas.

In mechanics courses you made extensive use of the free-body diagram (system approach). This was logical because you were dealing with an easily identifiable rigid body. However in fluid mechanics, we normally are concerned with the flow of fluids through devices such as compressors, turbines, pipelines, nozzles, and so on. In these cases it is difficult to focus attention on a fixed identifiable quantity of mass. It is much more convenient, for analysis, to focus attention on a volume in space through which the fluid flows. Therefore, we use the control volume approach.

A control volume is an arbitrary volume in space through which fluid flows. The geometric boundary of the control volume is called the control surface. The control surface may be real or imaginary; it may be at rest or in motion. Figure 1.3 shows a possible control surface for analysis of flow through a pipe. Here the inside surface of the pipe, a real physical boundary, comprises part of the control surface. However, the vertical portions of the control surface are imaginary. There is no corresponding physical surface; these imaginary boundaries are selected arbitrarily for accounting purposes. Since the location of the control surface has a direct effect on the accounting procedure in applying the basic laws, it is extremely important that the control surface be clearly defined before beginning any analysis.

1-5.2 Differential versus Integral Approach

The basic laws that we apply in our study of fluid mechanics can be formulated in terms of infinitesimal or finite systems and control volumes. As you might suspect, the equations will look different in each case. Both approaches are important in the study of fluid mechanics and both will be developed in the course of our work.

In the first case the resulting equations are differential equations. Solution of the differential equations of motion provides a means of determining the detailed (point by point) behavior of the flow.

Frequently, in the problems under study, the information sought does not require a detailed knowledge of the flow. We often are interested in the gross behavior of a

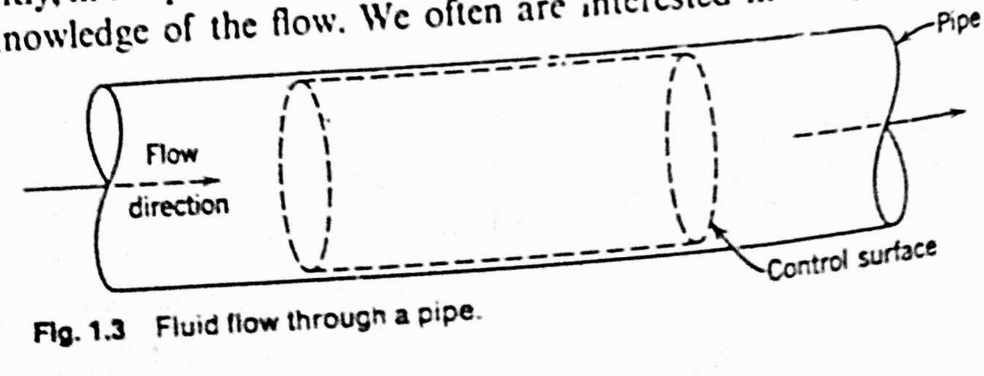


Fig. 1.3 Fluid flow through a pipe.

device; in such cases it is more appropriate to use the integral formulation of the basic laws. The integral formulation, using finite systems or control volumes, usually is easier to treat analytically. Since mechanics and thermodynamics deal with the formulation of the basic laws in terms of finite systems, these formulations are the basis for deriving the control volume equations in Chapter 4.

1-5.3 Methods of Description

Mechanics deals almost exclusively with systems; you have made extensive use of the basic equations applied to a fixed, identifiable quantity of mass. In attempting to analyze thermodynamic devices, you often found it necessary to use a control volume (open system) analysis. Clearly, the type of analysis depends on the problem. Where it is easy to keep track of identifiable elements of mass (e.g. in particle mechanics), we utilize a method of description that follows the particle. This sometimes is referred to as the *Lagrangian* method of description.

Consider, for example, the application of Newton's second law to a particle of fixed mass, m. Mathematically, we can write Newton's second law for a system of mass, m, as

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = m\frac{d^2\vec{r}}{dt^2} \tag{1.2}$$

In Eq. 1.2, $\sum \vec{F}$ is the sum of all external forces acting on the system, $\vec{a}$ is the acceleration of the center of mass of the system, $\vec{V}$ is the velocity of the center of mass of the system, and $\vec{r}$ is the position vector of the center of mass of the system relative to a fixed coordinate system.

Example 1.2

The air resistance on a 200 g ball in free flight is given by $f = 2 \times 10^{-4} v^2$, where f is in newtons and v is in meters per second. If the ball is dropped from rest 500 m above the ground, determine the speed at which it hits the ground. What percentage of the terminal speed is the result?

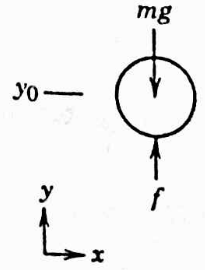
EXAMPLE PROBLEM 1.2

GIVEN: Ball, $m = 0.2$ kg, released from rest at $y_0 = 500$ m
Air resistance, $f = kv^2$, where $k = 2 \times 10^{-4}\ \text{N}\cdot\text{sec}^2/\text{m}^2$

Units: f(N), v(m/sec)

FIND: (a) The speed at which the ball hits the ground.
(b) Ratio of speed to terminal speed.

SOLUTION:



Basic equation: $\sum \vec{F} = m\vec{a}$

The motion of the ball is governed by the equation

$$\sum F_y = ma_y = m\frac{dv}{dt}$$

Since $v = v(y)$, $\sum F_y = m \dfrac{dv}{dy}\dfrac{dy}{dt} = mv \dfrac{dv}{dy}$

$$\sum F_y = f - mg = kv^2 - mg = mv \frac{dv}{dy}$$

Separating variables and integrating,

$$\int_{y_0}^{y} dy = \int_0^v \frac{mv\,dv}{kv^2 - mg}$$

$$y - y_0 = \left[\frac{m}{2k} \ln(kv^2 - mg)\right]_0^v = \frac{m}{2k} \ln \frac{kv^2 - mg}{-mg}$$

Taking antilogarithms, we obtain

$$kv^2 - mg = -mg\, e^{\left[\frac{2k}{m}(y - y_0)\right]}$$

Solving for v gives

$$v = \left[\frac{1}{k} mg\left(1 - e^{\left\{\frac{2k}{m}(y - y_0)\right\}}\right)\right]^{1/2}$$

Substituting numerical values with $y = 0$ yields

$$v = \left[0.2\ \text{kg} \times 9.81\ \frac{\text{m}}{\text{sec}^2} \times \frac{\text{m}^2}{2 \times 10^{-4}\ \text{N} \cdot \text{sec}^2} \times \frac{\text{N} \cdot \text{sec}^2}{\text{kg} \cdot \text{m}} \left(1 - e^{\left\{\frac{2 \times 2 \times 10^{-4}}{0.2}(-500)\right\}}\right)\right]^{1/2}$$

$v = 78.7$ m/sec

At terminal speed, $a_y = 0$ and $\sum F_y = 0 = kv_t^2 - mg$

Then $v_t = \left[\dfrac{mg}{k}\right]^{1/2} = \left[0.2\ \text{kg} \times 9.81\ \dfrac{\text{m}}{\text{sec}^2} \times \dfrac{\text{m}^2}{2 \times 10^{-4}\ \text{N} \cdot \text{sec}^2} \times \dfrac{\text{N} \cdot \text{sec}^2}{\text{kg} \cdot \text{m}}\right]^{1/2}$

$v_t = 99.0$ m/sec and $\dfrac{v}{v_t} = \dfrac{78.7}{99.0} = 79.5\%$ $\qquad \dfrac{v}{v_t}$

{This problem is included as a reminder of the method of description used in particle mechanics.}

We may consider a fluid to be composed of a very large number of particles whose motion must be described; keeping track of the motion of each fluid particle separately would become a horrendous bookkeeping problem. Consequently, a particle description becomes unmanageable. Often we find it convenient to use a different type of description. Particularly with control volume analyses, it is convenient to use the field, or *Eulerian*, method of description, which focuses attention on the properties of a flow at a given point in space as a function of time. In the Eulerian method of description, the properties of a flow field are described as functions of space coordinates and time. We shall see in Chapter 2 that this method of description is a logical outgrowth of the assumption that fluids may be treated as continuous media.

1-6 DIMENSIONS AND UNITS

Engineering problems are solved to answer specific questions. It goes without saying that the answer must include units. (It makes a difference whether a pipe diameter required is 1 meter or 1 foot!) Consequently, it is appropriate to present a brief review of dimensions and units. We say "review" because the topic is familiar from your earlier work in mechanics.

We refer to physical quantities such as length, time, mass, and temperature as *dimensions*. In terms of a particular system of dimensions all measurable quantities can be subdivided into two groups—primary quantities and secondary quantities. We refer to a small group of dimensions from which all others can be formed as primary quantities. Primary quantities are those for which we set up arbitrary scales of measure; secondary quantities are those quantities whose dimensions are expressible in terms of the dimensions of the primary quantities.

Units are the arbitrary names (and magnitudes) assigned to the primary dimensions adopted as standards for measurement. For example, the primary dimension of length may be measured in units of meters, feet, yards, or miles. These units of length are related to each other through unit conversion factors (1 mile = 5280 feet = 1609 meters).

1-6.1 Systems of Dimensions

Any valid equation that relates physical quantities must be dimensionally homogeneous; each term in the equation must have the same dimensions. We recognize that Newton's second law ($\vec{F} \propto m\vec{a}$) relates the four dimensions, F, M, L, and t. Thus force and mass cannot both be selected as primary dimensions without introducing a constant of proportionality that has dimensions (and units).

Length and time are primary dimensions in all dimensional systems in common use. In some systems, mass is taken as a primary dimension. In others, force is selected as a primary dimension; a third system chooses both force and mass as primary dimensions. Thus we have three basic systems of dimensions, corresponding to the different ways of specifying the primary dimensions.

a. Mass $[M]$, length $[L]$, time $[t]$, temperature $[T]$.
b. Force $[F]$, length $[L]$, time $[t]$, temperature $[T]$.
c. Force $[F]$, mass $[M]$, length $[L]$, time $[t]$, temperature $[T]$.

In system *a*, force $[F]$ is a secondary dimension and the constant of proportionality in Newton's second law is dimensionless. In system *b*, mass $[M]$ is a secondary dimension, and again the constant of proportionality in Newton's second law is dimensionless. In system *c*, both force $[F]$ and mass $[M]$ have been selected as primary dimensions. In this case the constant of proportionality, g_c, in Newton's second law (written $\vec{F} = m\vec{a}/g_c$) is not dimensionless. The dimensions of g_c must in fact be $[ML/Ft^2]$ for the equation to be dimensionally homogeneous. The numerical value of the constant of proportionality depends on the units of measure chosen for each of the primary quantities.

$F = \frac{ma}{g_c} = \frac{ML}{T^2}$ $\quad F = \frac{ma}{g_c}$

$F = ma$

Newton's second law we see that (to three significant figures)

$$1\ \text{lbf} \equiv \frac{1\ \text{lbm} \times 32.2\ \text{ft/sec}^2}{g_c}$$

or

$$g_c \equiv 32.2\ \text{ft} \cdot \text{lbm/lbf} \cdot \text{sec}^2$$

The constant of proportionality, g_c, has both dimensions and units. The dimensions arose because we selected both force and mass as primary dimensions; the units (and the numerical value) are a consequence of our choices for the standards of measurement.

Since a force of 1 lbf accelerates 1 lbm at 32.2 ft/sec^2, it would accelerate 32.2 lbm at 1 ft/sec^2. A slug also is accelerated at 1 ft/sec^2 by a force of 1 lbf. Therefore,

$$1\ \text{slug} \equiv 32.2\ \text{lbm}$$