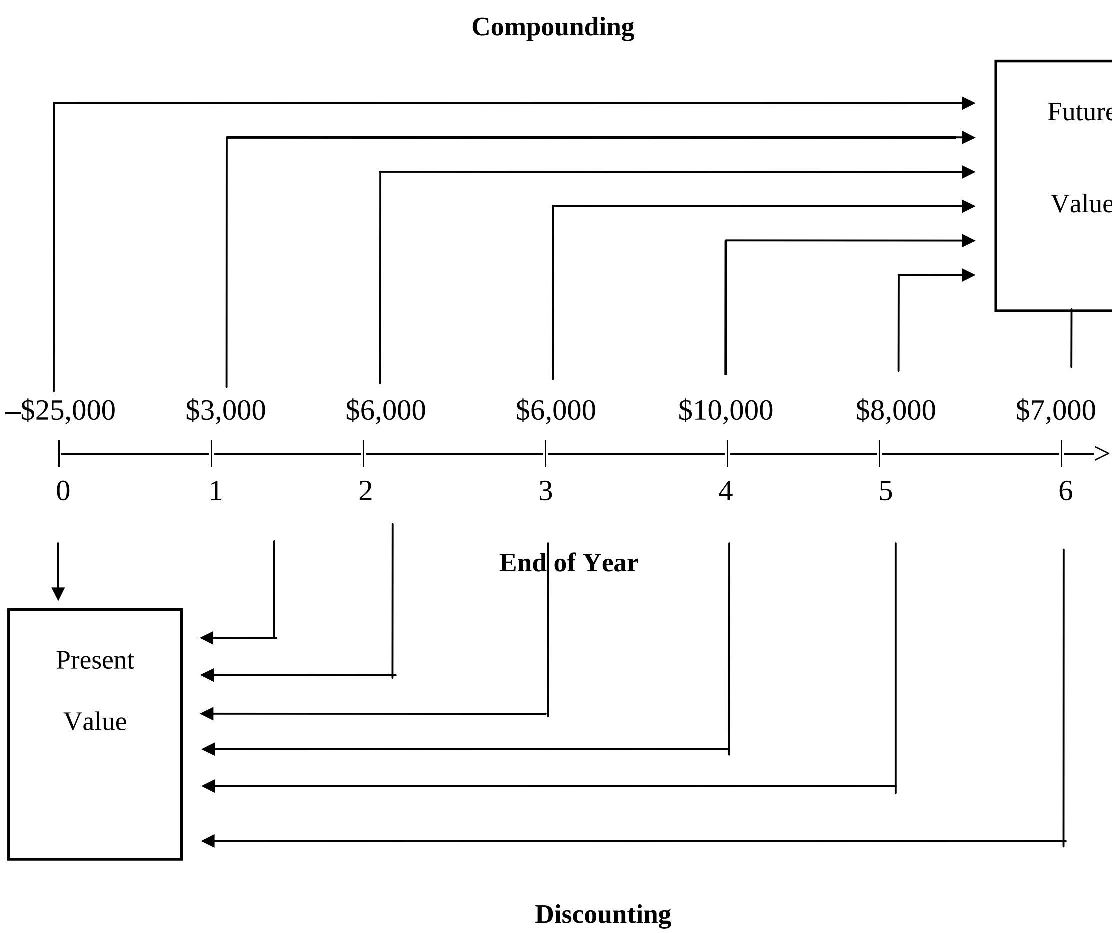
Chapter 4
Time Value of Money

■ Solutions to Problems

P4-1. LG 1: Using a Time Line

Basic

(a), (b), and (c)



(d) Financial managers rely more on present than future value because they typically make decisions before the start of a project, at time zero, as does the present value calculation.

P4-2. LG 2: Future Value Calculation: $FV_n = PV \times (1 + i)^n$

Basic

Case		
A	$FVIF_{12\%,2\ periods}$	$= (1 + 0.12)^2 = 1.254$
B	$FVIF_{6\%,3\ periods}$	$= (1 + 0.06)^3 = 1.191$
C	$FVIF_{9\%,2\ periods}$	$= (1 + 0.09)^2 = 1.188$
D	$FVIF_{3\%,4\ periods}$	$= (1 + 0.03)^4 = 1.126$

P4-3. LG 2: Future Value Tables: $FV_n = PV \times (1 + i)^n$

Basic

Case A

(a) $2 = 1 \times (1 + 0.07)^n$
$2/1 = (1.07)^n$
$2 = FVIF_{7\%,n}$
10 years< n < 11 years
Nearest to 10 years

(b) $4 = 1 \times (1 + 0.07)^n$
$4/1 = (1.07)^n$
$4 = FVIF_{7\%,n}$
20 years < n < 21 years
Nearest to 20 years

Case B

(a) $2 = 1 \times (1 + 0.40)^n$
$2 = FVIF_{40\%,n}$
2 years < n < 3 years
Nearest to 2 years

(b) $4 = (1 + 0.40)^n$
$4 = FVIF_{40\%,n}$
4 years < n < 5 years
Nearest to 4 years

Case C

(a) $2 = 1 \times (1 + 0.20)^n$
$2 = FVIF_{20\%,n}$
3 years < n < 4 years
Nearest to 4 years

(b) $4 = (1 + 0.20)^n$
$4 = FVIF_{20\%,n}$
7 years < n < 8 years
Nearest to 8 years

Case D

(a) $2 = 1 \times (1 + 0.10)^n$
$2 = FVIF_{10\%,n}$
7 years < n < 8 years
Nearest to 7 years

(b) $4 = (1 + 0.10)^n$
$4 = FVIF_{40\%,n}$
14 years < n <15 years
Nearest to 15 years

P4-4. LG 2: Future Values: $FV_n = PV \times (1 + i)^n$ or $FV_n = PV \times (FVIF_{i\%,n})$

Intermediate

Case

A $FV_{20} = PV \times FVIF_{5\%,20\ yrs.}$
$FV_{20} = \$200 \times (2.653)$
$FV_{20} = \$530.60$
Calculator solution: $530.66

Case

B $FV_7 = PV \times FVIF_{8\%,7\ yrs.}$
$FV_7 = \$4,500 \times (1.714)$
$FV_7 = \$7,713$
Calculator solution; $7,712.21

C
$FV_{10} = PV \times FVIF_{9\%,10\text{ yrs.}}$
$FV_{10} = \$10{,}000 \times (2.367)$
$FV_{10} = \$23{,}670$
Calculator solution: $23,673.64

D
$FV_{12} = PV \times FVIF_{10\%,12\text{ yrs.}}$
$FV_{12} = \$25{,}000 \times (3.138)$
$FV_{12} = \$78{,}450$
Calculator solution: $78,460.71

E
$FV_5 = PV \times FVIF_{11\%,5\text{ yrs.}}$
$FV_5 = \$37{,}000 \times (1.685)$
$FV_5 = \$62{,}345$
Calculator solution: $62,347.15

F
$FV_9 = PV \times FVIF_{12\%,9\text{ yrs.}}$
$FV_9 = \$40{,}000 \times (2.773)$
$FV_9 = \$110{,}920$
Calculator solution: $110,923.15

P4-5. LG 2: Time Value: $FV_n = PV \times (1 + i)^n$ or $FV^n = PV \times (FVIF_{i\%,n})$
Intermediate

(a) (1) $FV_3 = PV \times (FVIF_{7\%,3})$
$FV_3 = \$1{,}500 \times (1.225)$
$FV_3 = \$1{,}837.50$
Calculator solution: $1,837.56

(2) $FV_6 = PV \times (FVIF_{7\%,6})$
$FV_6 = \$1{,}500 \times (1.501)$
$FV_6 = \$2{,}251.50$
Calculator solution: $2,251.10

(3) $FV_9 = PV \times (FVIF_{7\%,9})$
$FV_9 = \$1{,}500 \times (1.838)$
$FV_9 = \$2{,}757.00$
Calculator solution: $2,757.69

(b) (1) Interest earned = $FV_3 - PV$
Interest earned = $1,837.50
–$1,500.00
$337.50

(2) Interest earned = $FV_6 - FV_3$
Interest earned = $2,251.50
–$1,837.50
$414.00

(3) Interest earned = $FV_9 - FV_6$
Interest earned = $2,757.00
–$2,251.50
$505.50

(c) The fact that the longer the investment period is, the larger the total amount of interest collected will be, is not unexpected and is due to the greater length of time that the principal sum of $1,500 is invested. The most significant point is that the incremental interest earned per 3-year period increases with each subsequent 3 year period. The total interest for the first 3 years is $337.50; however, for the second 3 years (from year 3 to 6) the additional interest earned is $414.00. For the third 3-year period, the incremental interest is $505.50. This increasing change in interest earned is due to compounding, the earning of interest on previous interest earned. The greater the previous interest earned, the greater the impact of compounding.

P4-6. LG 2: Time Value

Challenge

(a) (1) $FV_5 = PV \times (FVIF_{2\%,5})$
$FV_5 = \$14,000 \times (1.104)$
$FV_5 = \$15,456.00$
Calculator solution: $15,457.13

(2) $FV_5 = PV \times (FVIF_{4\%,5})$
$FV_5 = \$14,000 \times (1.217)$
$FV_5 = \$17,038.00$
Calculator solution: $17,033.14

(b) The car will cost $1,582 more with a 4% inflation rate than an inflation rate of 2%. This increase is 10.2% more ($1,582 ÷ $15,456) than would be paid with only a 2% rate of inflation.

P4-7. LG 2: Time Value

Challenge

Deposit now:
$FV_{40} = PV \times FVIF_{9\%,40}$
$FV_{40} = \$10,000 \times (1.09)^{40}$
$FV_{40} = \$10,000 \times (31.409)$
$FV_{40} = \$314,090.00$
Calculator solution: $314,094.20

Deposit in 10 years:
$FV_{30} = PV_{10} \times (FVIF_{9\%,30})$
$FV_{30} = PV_{10} \times (1.09)^{30}$
$FV_{30} = \$10,000 \times (13.268)$
$FV_{30} = \$132,680.00$
Calculator solution: $132,676.79

You would be better off by $181,410 ($314,090 – $132,680) by investing the $10,000 now instead of waiting for 10 years to make the investment.

P4-8. LG 2: Time Value: $FV_n = PV \times FVIF_{i\%,n}$

Challenge

(a) $\$15,000 = \$10,200 \times FVIF_{i\%,5}$
$FVIF_{i\%,5} = \$15,000 \div \$10,200 = 1.471$
$8\% < i < 9\%$
Calculator Solution: 8.02%

(b) $\$15,000 = \$8,150 \times FVIF_{i\%,5}$
$FVIF_{i\%,5} = \$15,000 \div \$8,150 = 1.840$
$12\% < i < 13\%$
Calculator Solution: 12.98%

(c) $\$15,000 = \$7,150 \times FVIF_{i\%,5}$
$FVIF_{i\%,5} = \$15,000 \div \$7,150 = 2.098$
$15\% < i < 16\%$
Calculator Solution: 15.97%

P4-9. LG 2: Single-payment Loan Repayment: $FV_n = PV \times FVIF_{i\%,n}$
Intermediate

(a) $FV_1 = PV \times (FVIF_{14\%,1})$
$FV_1 = \$200 \times (1.14)$
$FV_1 = \$228$
Calculator Solution: \$228

(b) $FV_4 = PV \times (FVIF_{14\%,4})$
$FV_4 = \$200 \times (1.689)$
$FV_4 = \$337.80$
Calculator solution: \$337.79

(c) $FV_8 = PV \times (FVIF_{14\%,8})$
$FV_8 = \$200 \times (2.853)$
$FV_8 = \$570.60$
Calculator Solution: \$570.52

P4-10. LG 2: Present Value Calculation: $PVIF = \frac{1}{(1+i)^n}$

Basic

Case	
A	$PVIF = 1 \div (1 + 0.02)^4 = 0.9238$
B	$PVIF = 1 \div (1 + 0.10)^2 = 0.8264$
C	$PVIF = 1 \div (1 + 0.05)^3 = 0.8638$
D	$PVIF = 1 \div (1 + 0.13)^2 = 0.7831$

P4-11. LG 2: Present Values: $PV = FV_n \times (PVIF_{i\%,n})$
Basic

Case		**Calculator Solution**
A	$PV_{12\%,4yrs} = \$7,000 \times 0.636 = \$4,452$	\$4,448.63
B	$PV_{8\%,20yrs} = \$28,000 \times 0.215 = \$6,020$	\$6,007.35
C	$PV_{14\%,12yrs} = \$10,000 \times 0.208 = \$2,080$	\$2,075.59
D	$PV_{11\%,6yrs} = \$150,000 \times 0.535 = \$80,250$	\$80,196.13
E	$PV_{20\%,8yrs} = \$45,000 \times 0.233 = \$10,485$	\$10,465.56

P4-12. LG 2: Present Value Concept: $PV_n = FV_n \times (PVIF_{i\%,n})$
Intermediate

(a) $PV = FV_6 \times (PVIF_{12\%,6})$
$PV = \$6,000 \times (.507)$
$PV = \$3,042.00$
Calculator solution: \$3,039.79

(b) $PV = FV_6 \times (PVIF_{12\%,6})$
$PV = \$6,000 \times (0.507)$
$PV = \$3,042.00$
Calculator solution: \$3,039.79

(c) $PV = FV_6 \times (PVIF_{12\%,6})$
$PV = \$6,000 \times (0.507)$
$PV = \$3,042.00$
Calculator solution: \$3,039.79

(d) The answer to all three parts are the same. In each case the same questions is being asked but in a different way.

P4-13. LG 2: Time Value: $PV = FV_n \times (PVIF_{i\%,n})$

Basic

Jim should be willing to pay no more than \$408.00 for this future sum given that his opportunity cost is 7%.

$PV = \$500 \times (PVIF_{7\%,3})$

$PV = \$500 \times (0.816)$

$PV = \$408.00$

Calculator solution: \$408.15

P4-14. LG 2: Time Value: $PV = FV_n \times (PVIF_{i\%,n})$

Intermediate

$PV = \$100 \times (PVIF_{8\%,6})$

$PV = \$100 \times (0.630)$

$PV = \$63.00$

Calculator solution: \$63.02

P4-15. LG 2: Time Value and Discount Rates: $PV = FV_n \times (PVIF_{i\%,n})$

Intermediate

(a) (1) $PV = \$1{,}000{,}000 \times (PVIF_{6\%,10})$
$PV = \$1{,}000{,}000 \times (0.558)$
$PV = \$558{,}000.00$
Calculator solution: \$558,394.78

(2) $PV = \$1{,}000{,}000 \times (PVIF_{9\%,10})$
$PV = \$1{,}000{,}000 \times (0.422)$
$PV = \$422{,}000.00$
Calculator solution: \$422,410.81

(3) $PV = \$1{,}000{,}000 \times (PVIF_{12\%,10})$
$PV = \$1{,}000{,}000 \times (0.322)$
$PV = \$322{,}000.00$
Calculator solution: \$321,973.24

(b) (1) $PV = \$1{,}000{,}000 \times (PVIF_{6\%,15})$
$PV = \$1{,}000{,}000 \times (0.417)$
$PV = \$417{,}000.00$
Calculator solution: \$417,265.06

(2) $PV = \$1{,}000{,}000 \times (PVIF_{9\%,15})$
$PV = \$1{,}000{,}000 \times (0.275)$
$PV = \$275{,}000.00$
Calculator solution: \$274,538.04

(3) $PV = \$1{,}000{,}000 \times (PVIF_{12\%,15})$
$PV = \$1{,}000{,}000 \times (0.183)$
$PV = \$183{,}000.00$
Calculator solution: \$182,696.26

(c) As the discount rate increases, the present value becomes smaller. This decrease is due to the higher opportunity cost associated with the higher rate. Also, the longer the time until the lottery payment is collected, the less the present value due to the greater time over which the opportunity cost applies. In other words, the larger the discount rate and the longer the time until the money is received, the smaller will be the present value of a future payment.

P4-16. LG 2: Time Value Comparisons of Lump Sums: $PV = FV_n \times (PVIF_{i\%,n})$

Intermediate

(a) **A**

$PV = \$28{,}500 \times (PVIF_{11\%,3})$
$PV = \$28{,}500 \times (0.731)$
$PV = \$20{,}833.50$
Calculator solution: $20,838.95

B

$PV = \$54{,}000 \times (PVIF_{11\%,9})$
$PV = \$54{,}000 \times (0.391)$
$PV = \$21{,}114.00$
Calculator solution: $21,109.94

C

$PV = \$160{,}000 \times (PVIF_{11\%,20})$
$PV = \$160{,}000 \times (0.124)$
$PV = \$19{,}840.00$
Calculator solution: $19,845.43

(b) Alternatives A and B are both worth greater than $20,000 in term of the present value.

(c) The best alternative is B because the present value of B is larger than either A or C and is also greater than the $20,000 offer.

P4-17. LG 2: Cash Flow Investment Decision: $PV = FV_n \times (PVIF_{i\%,n})$

Intermediate

A

$PV = \$30{,}000 \times (PVIF_{10\%,5})$
$PV = \$30{,}000 \times (0.621)$
$PV = \$18{,}630.00$
Calculator solution: $18,627.64

B

$PV = \$3{,}000 \times (PVIF_{10\%,20})$
$PV = \$3{,}000 \times (0.149)$
$PV = \$447.00$
Calculator solution: $445.93

C

$PV = \$10{,}000 \times (PVIF_{10\%,10})$
$PV = \$10{,}000 \times (0.386)$
$PV = \$3{,}860.00$
Calculator solution: $3,855.43

D

$PV = \$15{,}000 \times (PVIF_{10\%,40})$
$PV = \$15{,}000 \times (0.022)$
$PV = \$330.00$
Calculator solution: $331.42

Purchase	**Do Not Purchase**
A	B
C	D

P4-18. LG 3: Future Value of an Annuity

Intermediate

(a) Future Value of an Ordinary Annuity vs. Annuity Due

	(1) **Ordinary Annuity**	(2) **Annuity Due**
	$FVA_{k\%,n} = PMT \times (FVIFA_{k\%,n})$	$FVA_{due} = PMT \times [(FVIFA_{k\%,n} \times (1 + k)]$
A	$FVA_{8\%,10} = \$2,500 \times 14.487$ $FVA_{8\%,10} = \$36,217.50$ Calculator solution: $36,216.41	$FVA_{due} = \$2,500 \times (14.487 \times 1.08)$ $FVA_{due} = \$39,114.90$ Calculator solution: $39,113.72
B	$FVA_{12\%,6} = \$500 \times 8.115$ $FVA_{12\%,6} = \$4,057.50$ Calculator solution: $4,057.59	$FVA_{due} = \$500 \times(8.115 \times 1.12)$ $FVA_{due} = \$4,544.40$ Calculator solution: $4,544.51
C	$FVA_{20\%,5} = \$30,000 \times 7.442$ $FVA_{20\%,5} = \$223,260$ Calculator solution: $223,248	$FVA_{due} = \$30,000 \times (7.442 \times 1.20)$ $FVA_{due} = \$267,912$ Calculator solution: $267,897.60
D	$FVA_{9\%,8} = \$11,500 \times 11.028$ $FVA_{9\%,8} = \$126,822$ Calculator solution: $126,827.45	$FVA_{due} = \$11,500 \times (11.028 \times 1.09)$ $FVA_{due} = \$138,235.98$ Calculator solution: $138,241.92
E	$FVA_{14\%,30} = \$6,000 \times 356.787$ $FVA_{14\%,30} = \$2,140,722$ Calculator solution: $2,140,721.10	$FVA_{due} = \$6,000 \times (356.787 \times 1.14)$ $FVA_{due} = \$2,440,422.00$ Calculator solution: $2,440,422.03

(b) The annuity due results in a greater future value in each case. By depositing the payment at the beginning rather than at the end of the year, it has one additional year of compounding.

P4-19. LG 3: Present Value of an Annuity: $PV_n = PMT \times (PVIFA_{i\%,n})$

Intermediate

(a) Present Value of an Ordinary Annuity vs. Annuity Due

	(1) **Ordinary Annuity**	(2) **Annuity Due**
	$PVA_{k\%,n} = PMT \times (PVIFA_{i\%,n})$	$PVA_{due} = PMT \times [(PVIFA_{i\%,n} \times (1 + k)]$
A	$PVA_{7\%,3} = \$12,000 \times 2.624$ $PVA_{7\%,3} = \$31,488$ Calculator solution: $31,491.79	$PVA_{due} = \$12,000 \times (2.624 \times 1.07)$ $PVA_{due} = \$33,692$ Calculator solution: $33,692.16
B	$PVA_{12\%15} = \$55,000 \times 6.811$ $PVA_{12\%,15} = \$374,605$ Calculator solution: $374,597.55	$PVA_{due} = \$55,000 \times (6.811 \times 1.12)$ $PVA_{due} = \$419,557.60$ Calculator solution: $419,549.25

C	$PVA_{20\%,9}$ = \$700 × 4.031 $PVA_{20\%,9}$ = \$2,821.70 Calculator solution: \$2,821.68	PVA_{due} = \$700 × (4.031 × 1.20) PVA_{due} = \$3,386.04 Calculator solution: \$3,386.01
D	$PVA_{5\%,7}$ = \$140,000 × 5.786 $PVA_{5\%,7}$ = \$810,040 Calculator solution: \$810,092.28	PVA_{due} = \$140,000 × (5.786 × 1.05) PVA_{due} = \$850,542 Calculator solution: \$850,596.89
E	$PVA_{10\%,5}$ = \$22,500 × 3.791 $PVA_{10\%,5}$ = \$85,297.50 Calculator solution: \$85,292.70	PVA_{due} = \$22,500 × (2.791 × 1.10) PVA_{due} = \$93,827.25 Calculator solution: \$93,821.97

(b) The annuity due results in a greater present value in each case. By depositing the payment at the beginning rather than at the end of the year, it has one less year to discount back.

P4-20. LG 3: Time Value–Annuities
Challenge

(a) **Annuity C (Ordinary)** / **Annuity D (Due)**

Annuity C (Ordinary)	Annuity D (Due)
$FVA_{i\%,n} = PMT \times (FVIFA_{i\%,n})$	$FVA_{due} = PMT \times [FVIFA_{i\%,n} \times (1 + i)]$
(1) $FVA_{10\%,10}$ = \$2,500 × 15.937 $FVA_{10\%,10}$ = \$39,842.50 Calculator solution: \$39,843.56	FVA_{due} = \$2,200 × (15.937 × 1.10) FVA_{due} = \$38,567.54 Calculator solution: \$38,568.57
(2) $FVA_{20\%,10}$ = \$2,500 × 25.959 $FVA_{20\%,10}$ = \$64,897.50 Calculator solution: \$64,896.71	FVA_{due} = \$2,200 × (25.959 × 1.20) FVA_{due} = \$68,531.76 Calculator solution: \$68,530.92

(b) (1) At the end of year 10, at a rate of 10%, Annuity C has a greater value (\$39,842.50 vs. \$38,567.54).
(2) At the end of year 10, at a rate of 20%, Annuity D has a greater value (\$68,531.76 vs. \$64,896.71).

(c)

Annuity C (Ordinary)	Annuity D (Due)
$PVA_{i\%,n} = PMT \times (FVIFA_{i\%,n})$	$PVA_{due} = PMT \times [FVIFA_{i\%,n} \times (1 + i)]$
(1) $PVA_{10\%,10}$ = \$2,500 × 6.145 $PVA_{10\%,10}$ = \$15,362.50 Calculator solution: \$15,361.42	PVA_{due} = \$2,200 × (6.145 × 1.10) PVA_{due} = \$14,870.90 Calculator solution: \$14,869.85
(2) $PVA_{20\%,10}$ = \$2,500 × 4.192 $PVA_{20\%,10}$ = \$10,480 Calculator solution: \$10,481.18	PVA_{due} = \$2,200 × (4.192 × 1.20) PVA_{due} = \$11,066.88 Calculator solution: \$11,068.13

(d) (1) At the beginning of the 10 years, at a rate of 10%, Annuity C has a greater value ($15,362.50 vs. $14,870.90).

(2) At the beginning of the 10 years, at a rate of 20%, Annuity D has a greater value ($11,066.88 vs. $10,480.00).

(e) Annuity C, with an annual payment of $2,500 made at the end of the year, has a higher present value at 10% than Annuity D with an annual payment of $2,200 made at the beginning of the year. When the rate is increased to 20%, the shorter period of time to discount at the higher rate results in a larger value for Annuity D, despite the lower payment.

P4-21. LG 3: Retirement Planning

Challenge

(a) $FVA_{40} = \$2{,}000 \times (FVIFA_{10\%,40})$
$FVA_{40} = \$2{,}000 \times (442.593)$
$FVA_{40} = \$885{,}186$
Calculator solution: $885,185.11

(b) $FVA_{30} = \$2{,}000 \times (FVIFA_{10\%,30})$
$FVA_{30} = \$2{,}000 \times (164.494)$
$FVA_{30} = \$328{,}988$
Calculator solution: $328,988.05

(c) By delaying the deposits by 10 years the total opportunity cost is $556,198. This difference is due to both the lost deposits of $20,000 ($2,000 × 10yrs.) and the lost compounding of interest on all of the money for 10 years.

(d) **Annuity Due:**
$FVA_{40} = \$2{,}000 \times (FVIFA_{10\%,40}) \times (1 + 0.10)$
$FVA_{40} = \$2{,}000 \times (486.852)$
$FVA_{40} = \$973{,}704$
Calculator solution: $973,703.62

$FVA_{30} = \$2{,}000 \times (FVIFA_{10\%,30}) \times (1.10)$
$FVA_{30} = \$2{,}000 \times (180.943)$
$FVA_{30} = \$361{,}886$
Calculator solution: $361,886.85

Both deposits increased due to the extra year of compounding from the beginning-of-year deposits instead of the end-of-year deposits. However, the incremental change in the 40 year annuity is much larger than the incremental compounding on the 30 year deposit ($88,518 versus $32,898) due to the larger sum on which the last year of compounding occurs.

P4-22. LG 3: Value of a Retirement Annuity

Intermediate

$PVA = PMT \times (PVIFA_{9\%,25})$
$PVA = \$12{,}000 \times (9.823)$
$PVA = \$117{,}876.00$
Calculator solution: $117,870.96

P4-23. LG 3: Funding Your Retirement

Challenge

(a) $PVA = PMT \times (PVIFA_{11\%,30})$
$PVA = \$20{,}000 \times (8.694)$
$PVA = \$173{,}880.00$
Calculator solution: $173,875.85

(b) $PV = FV \times (PVIF_{9\%,20})$
$PV = \$173{,}880 \times (0.178)$
$PV = \$30{,}950.64$
Calculator solution: $31,024.82

(c) Both values would be lower. In other words, a smaller sum would be needed in 20 years for the annuity and a smaller amount would have to be put away today to accumulate the needed future sum.

P4-24. LG 2, 3: Value of an Annuity versus a Single Amount

Intermediate

(a) $PVA_n = PMT \times (PVIFA_{i\%,n})$
$PVA_{25} = \$40{,}000 \times (PVIFA_{5\%,25})$
$PVA_{25} = \$40{,}000 \times 14.094$
$PVA_{25} = \$563{,}760$
Calculator solution: $563,757.78

At 5%, taking the award as an annuity is better; the present value is $563,760, compared to receiving $500,000 as a lump sum.

(b) $PVA_n = \$40{,}000 \times (PVIFA_{7\%,25})$
$PVA_{25} = \$40{,}000 \times (11.654)$
$PVA_{25} = \$466{,}160$
Calculator solution: $466,143.33

At 7%, taking the award as a lump sum is better; the present value of the annuity is only $466,160, compared to the $500,000 lump sum payment.

(c) Because the annuity is worth more than the lump sum at 5% and less at 7%, try 6%:
$PV_{25} = \$40{,}000 \times (PVIFA_{6\%,25})$
$PV_{25} = \$40{,}000 \times 12.783$
$PV_{25} = \$511{,}320$

The rate at which you would be indifferent is greater than 6%; about 6.25% Calculator solution: 6.24%

P4-25. LG 3: Perpetuities: $PV_n = PMT \times (PVIFA_{i\%,\infty})$

Basic

(a)

Case	PV Factor
A	1 ÷ 0.08 = 12.50
B	1 ÷ 0.10 = 10.00
C	1 ÷ 0.06 = 16.67
D	1 ÷ 0.05 = 20.00

(b)

PMT × (PVIFA$_{i\%,\infty}$)			**= PMT × (1 ÷ i)**
$20,000	×	12.50	= $250,000
$100,000	×	10.00	= $1,000,000
$3,000	×	16.67	= $50,000
$60,000	×	20.00	= $1,200,000

P4-26. LG 3: Creating an Endowment

Intermediate

(a) $PV = PMT \times (PVIFA_{i\%,\infty})$

$PV = (\$600 \times 3) \times (1 \div i)$

$PV = \$1,800 \times (1 \div 0.06)$

$PV = \$1,800 \times (16.67)$

$PV = \$30,006$

(b) $PV = PMT \times (PVIFA_{i\%,\infty})$

$PV = (\$600 \times 3) \times (1 \div i)$

$PV = \$1,800 \times (1 \div 0.09)$

$PV = \$1,800 \times (11.11)$

$PV = \$19,998$

P4-27. LG 4: Value of a Mixed Stream

Challenge

(a)

Cash Flow Stream	**Year**	**Number of Years to Compound**	**FV = CF × FVIF$_{12\%,n}$**		**Future Value**
A	1	3	\$900 × 1.405	=	\$1,264.50
	2	2	1,000 × 1.254	=	1,254.00
	3	1	1,200 × 1.120	=	1,344.00
					\$3,862.50
			Calculator Solution:		\$3,862.84
B	1	5	\$30,000 × 1.762	=	\$52,860.00
	2	4	25,000 × 1.574	=	39,350.00
	3	3	20,000 × 1.405	=	28,100.00
	4	2	10,000 × 1.254	=	12,540.00
	5	1	5,000 × 1.120	=	5,600.00
					\$138,450.00
			Calculator Solution:		\$138,450.79.
C	1	4	\$1,200 × 1.574	=	\$1,888.80
	2	3	1,200 × 1.405	=	1,686.00
	3	2	1,000 × 1.254	=	1,254.00
	4	1	1,900 × 1.120	=	2,128.00
					\$6,956.80
			Calculator Solution:		\$6,956.53

(b) If payments are made at the beginning of each period the present value of each of the end-of-period cash flow streams will be multiplied by (1 + i) to get the present value of the beginning-of-period cash flows.

A $3,862.50 (1 + 0.12) = $4,326.00

B $138,450.00 (1 + 0.12) = $155,064.00

C $6,956.80 (1 + 0.12) = $7,791.62

P4-28. LG 4: Value of a Single Amount Versus a Mixed Stream

Intermediate

Lump Sum Deposit

$FV_5 = PV \times (FVIF_{7\%,5})$

$FV_5 = \$24,000 \times (1.403)$

$FV_5 = \$33,672.00$

Calculator solution: $33,661.24

Mixed Stream of Payments

Beginning of Year	Number of Years to Compound	$FV = CF \times FVIF_{7\%,n}$		Future Value
1	5	$2,000 × 1.403	=	$2,806.00
2	4	$4,000 × 1.311	=	$5,244.00
3	3	$6,000 × 1.225	=	$7,350.00
4	2	$8,000 × 1.145	=	$9,160.00
5	1	$10,000 × 1.070	=	$10,700.00
				$35,260.00
		Calculator Solution:		$35,257.74

Gina should select the stream of payments over the front-end lump sum payment. Her future wealth will be higher by $1,588.

P4-29. LG 4: Value of Mixed Stream

Basic

Cash Flow Stream	Year	CF	×	$PVIF_{12\%,n}$	=	Present Value
A	1	–$2000	×	0.893	=	– $1,786
	2	3,000	×	0.797	=	2,391
	3	4,000	×	0.712	=	2,848
	4	6,000	×	0.636	=	3,816
	5	8,000	×	0.567	=	4,536
						$11,805
				Calculator solution		$11,805.51
B	1	$10,000	×	0.893	=	$8,930
	2–5	5,000	×	2.712*	=	13,560
	6	7,000	×	0.507	=	3,549
						$26,039
				Calculator solution:		$26,034.59
* Sum of PV factors for years 2–5						
C	1–5	–$10,000	×	3.605*		$36,050
	6–10	8,000	×	2.045**		16,360
						$52,410
				Calculator Solution		$52,411.34

* PVIFA for 12% 5 years

** (PVIFA for 12%,10 years) – (PVIFA for 12%,5 years)

P4-30. LG 4: Present Value-Mixed Stream

Intermediate

(a)

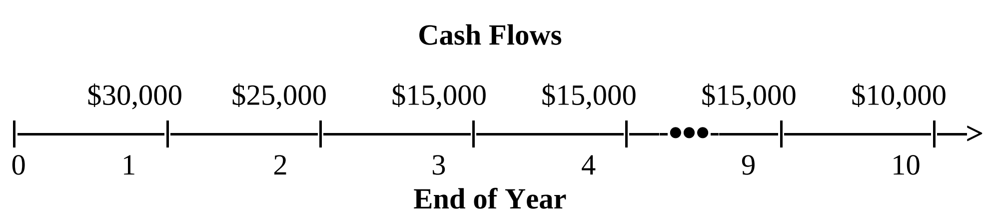
Cash Flow Stream	**Year**	**CF**	×	**PVIF$_{15\%,n}$**	=	**Present Value**
A	1	$50,000	×	0.870	=	$43,500
	2	40,000	×	0.756	=	30,240
	3	30,000	×	0.658	=	19,740
	4	20,000	×	0.572	=	11,440
	5	10,000	×	0.497	=	4,970
						$109,890
				Calculator solution		$109,856.33
B	1	$10,000	×	0.870	=	$8,700
	2	20,000	×	0.756	=	15,120
	3	30,000	×	0.658	=	19,740
	4	40,000	×	0.572	=	22,880
	5	50,000	×	0.497	=	24,850
						$91,290
				Calculator solution		$91,272.98

(b) Cash flow stream A, with a present value of $109,890, is higher than cash flow stream B's present value of $91,290 because the larger cash inflows occur in A in the early years when their present value is greater, while the smaller cash flows are received further in the future.

P4-31. LG 1, 4: Value of a Mixed Stream

Intermediate

(a)



(b)

Cash Flow Stream	Year	CF	×	$PVIF_{12\%,n}$	=	Present Value
A	1	$30,000	×	0.893	=	$26,790
	2	25,000	×	0.797	=	19,925
	3–9	15,000	×	3.639*	=	54,585
	10	10,000	×	0.322	=	3,220
						$104,520
				Calculator solution		$104,508.28

* The PVIF for this 7-year annuity is obtained by summing together the PVIFs of 12% for periods 3 through 9. This factor can also be calculated by taking the $PVIFA_{12\%,7}$ and multiplying by the $PVIF_{12\%,2}$.

(c) Harte should accept the series of payments offer. The present value of that mixed stream of payments is greater than the $100,000 immediate payment.

P4-32. LG 5: Funding Budget Shortfalls

Intermediate

(a)

Year	Budget Shortfall	×	$PVIF_{8\%,n}$	=	Present Value
1	$5,000	×	0.926	=	$4,630
2	4,000	×	0.857	=	3,428
3	6,000	×	0.794	=	4,764
4	10,000	×	0.735	=	7,350
5	3,000	×	0.681	=	2,043
					$22,215
			Calculator solution:		$22,214.03

A deposit of $22,215 would be needed to fund the shortfall for the pattern shown in the table.

(b) An increase in the earnings rate would reduce the amount calculated in part (a). The higher rate would lead to a larger interest being earned each year on the investment. The larger interest amounts will permit a decrease in the initial investment to obtain the same future value available for covering the shortfall.

P4-33. LG 4: Relationship between Future Value and Present Value-Mixed Stream
Intermediate

(a) **Present Value**

Year	CF	×	$PVIF_{5\%,n}$	=	Present Value
1	$800	×	0.952	=	$761.60
2	900	×	0.907	=	816.30
3	1,000	×	0.864	=	864.00
4	1,500	×	0.822	=	1,233.00
5	2,000	×	0.784	=	1,568.00
					$5,242.90
			Calculator Solution:		$5,243.17

(b) The maximum you should pay is $5,242.90.

(c) A higher 7% discount rate will cause the present value of the cash flow stream to be lower than $5,242.90.

P4-34. LG 5: Changing Compounding Frequency
Intermediate

(1) Compounding Frequency: $FV_n = PV \times FVIF_{i\%/m,n\times m}$

(a) **Annual**
12 %, 5 years
FV_5 = $5,000 × (1.762)
FV_5 = $8,810
Calculator solution: $8,811.71

Semiannual
12% ÷ 2 = 6%, 5 × 2 = 10 periods
FV_5 = $5,000 × (1.791)
FV_5 = $8,955
Calculator solution: $8,954.24

Quarterly
12% ÷ 4 = 3%, 5 × 4 = 20 periods
FV_5 = $5,000 (1.806)
FV_5 = $9,030
Calculator solution: $9,030.56

(b) **Annual**
16%, 6 years
FV_6 = $5,000 (2.436)
FV_6 = $12,180
Calculator solution: $12,181.98

Semiannual
16% ÷ 2 = 8%, 6 × 2 = 12 periods
FV_6 = $5,000 (2.518)
FV_6 = $12,590
Calculator solution: $12,590.85

Quarterly
16% ÷ 4 = 4%, 6 × 4 = 24 periods
FV_6 = $5,000 (2.563)
FV_6 = $12,815
Calculator solution: $12,816.52

(c) **Annual**

20%, 10 years

$FV_{10} = \$5{,}000 \times (6.192)$

$FV_{10} = \$30{,}960$

Calculator solution: \$30,958.68

Semiannual

20% ÷ 2 = 10%, 10 × 2 = 20 periods

$FV_{10} = \$5{,}000 \times (6.727)$

$FV_{10} = \$33{,}635$

Calculator solution: \$33,637.50

Quarterly

20% ÷ 4 = 5%, 10 × 4 = 40 periods

$FV_{10} = \$5{,}000 \times (7.040)$

$FV_{10} = \$35{,}200$

Calculator solution: \$35,199.94

(2) Effective Interest Rate: $i_{eff} = (1 + i/m)^m - 1$

(a) **Annual**

$i_{eff} = (1 + 0.12/1)^1 - 1$

$i_{eff} = (1.12)^1 - 1$

$i_{eff} = (1.12) - 1$

$i_{eff} = 0.12 = 12\%$

Semiannual

$i_{eff} = (1 + 12/2)^2 - 1$

$i_{eff} = (1.06)^2 - 1$

$i_{eff} = (1.124) - 1$

$i_{eff} = 0.124 = 12.4\%$

Quarterly

$i_{eff} = (1 + 12/4)^4 - 1$

$i_{eff} = (1.03)^4 - 1$

$i_{eff} = (1.126) - 1$

$i_{eff} = 0.126 = 12.6\%$

(b) **Annual**

$i_{eff} = (1 + 0.16/1)^1 - 1$

$i_{eff} = (1.16)^1 - 1$

$i_{eff} = (1.16) - 1$

$i_{eff} = 0.16 = 16\%$

Semiannual

$i_{eff} = (1 + 0.16/2)^2 - 1$

$i_{eff} = (1.08)^2 - 1$

$i_{eff} = (1.166) - 1$

$i_{eff} = 0.166 = 16.6\%$

Quarterly

$i_{eff} = (1 + 0.16/4)^4 - 1$

$i_{eff} = (1.04)^4 - 1$

$i_{eff} = (1.170) - 1$

$i_{eff} = 0.170 = 17\%$

(c) **Annual**

$i_{eff} = (1 + 0.20/1)^1 - 1$

$i_{eff} = (1.20)^1 - 1$

$i_{eff} = (1.20) - 1$

$i_{eff} = 0.20 = 20\%$

Semiannual

$i_{eff} = (1 + 0.20/2)^2 - 1$

$i_{eff} = (1.10)^2 - 1$

$i_{eff} = (1.210) - 1$

$i_{eff} = 0.210 = 21\%$

Quarterly

$i_{eff} = (1 + 0.20/4)^4 - 1$

$i_{eff} = (1.05)^4 - 1$

$i_{eff} = (1.216) - 1$

$i_{eff} = 0.216 = 21.6\%$

P4-35. LG 5: Compounding Frequency, Time Value, and Effective Annual Rates
Intermediate

(a) Compounding Frequency: $FV_n = PV \times FVIF_{i\%,n}$

A
$FV_5 = \$2{,}500 \times (FVIF_{3\%,10})$
$FV_5 = \$2{,}500 \times (1.344)$
$FV_5 = \$3{,}360$
Calculator solution: $3,359.79

B
$FV_3 = \$50{,}000 \times (FVIF_{2\%,18})$
$FV_3 = \$50{,}000 \times (1.428)$
$FV_3 = \$71{,}400$
Calculator solution: $71,412.31

C
$FV_{10} = \$1{,}000 \times (FVIF_{5\%,10})$
$FV_{10} = \$1{,}000 \times (1.629)$
$FV_{10} = \$16{,}290$
Calculator solution: $1,628.89

D
$FV_6 = \$20{,}000 \times (FVIF_{4\%,24})$
$FV_6 = \$20{,}000 \times (2.563)$
$FV_6 = \$51{,}260$
Calculator solution: $51,266.08

(b) Effective Interest Rate: $i_{eff\,=}\,(1 + i\%/m)^m - 1$

A
$i_{eff} = (1 + 0.06/2)^2 - 1$
$i_{eff} = (1 + 0.03)^2 - 1$
$i_{eff} = (1.061) - 1$
$i_{eff} = 0.061 = 06.1\%$

B
$i_{eff} = (1 + 0.12/6)^6 - 1$
$i_{eff} = (1 + 0.02)^6 - 1$
$i_{eff} = (1.126) - 1$
$i_{eff} = 0.126 = 12.6\%$

C
$i_{eff} = (1 + 0.05/1)^1 - 1$
$i_{eff} = (1 + 0.05)^1 - 1$
$i_{eff} = (1.05) - 1$
$i_{eff} = 0.05 = 5\%$

D
$i_{eff} = (1 + 0.16/4)^4 - 1$
$i_{eff} = (1 + 0.04)^4 - 1$
$i_{eff} = (1.170) - 1$
$i_{eff} = 0.17 = 17\%$

(c) The effective rates of interest rise relative to the stated nominal rate with increasing compounding frequency.

P4-36. LG 5: Continuous Compounding: $FVcont. = PV \times e^x$ ($e = 2.7183$)
Intermediate

A $FV_{cont.} = \$1{,}000 \times e^{0.18} = \$1{,}197.22$
B $FV_{cont.} = \$\ 600 \times e^1 = \$1{,}630.97$
C $FV_{cont.} = \$4{,}000 \times e^{0.56} = \$7{,}002.69$
D $FV_{cont.} = \$2{,}500 \times e^{0.48} = \$4{,}040.19$

Note: If calculator doesn't have e^x key, use y^x key, substituting 2.7183 for y.

P4-37. LG 5: Compounding Frequency and Time Value
Challenge

(a) (1)
$FV_{10} = \$2{,}000 \times (FVIF_{8\%,10})$
$FV_{10} = \$2{,}000 \times (2.159)$
$FV_{10} = \$4{,}318$
Calculator solution: $4,317.85

(2)
$FV_{10} = \$2{,}000 \times (FVIF_{4\%,20})$
$FV_{10} = \$2{,}000 \times (2.191)$
$FV_{10} = \$4{,}382$
Calculator solution: $4,382.25

(3)
$FV_{10} = \$2{,}000 \times (FVIF_{0.022\%,3650})$
$FV_{10} = \$2{,}000 \times (2.232)$
$FV_{10} = \$4{,}464$
Calculator solution: $4,450.69

(4)
$FV_{10} = \$2{,}000 \times (e^{0.8})$
$FV_{10} = \$2{,}000 \times (2.226)$
$FV_{10} = \$4{,}452$
Calculator solution: $4,451.08

(b) (1) $i_{eff} = (1 + 0.08/1)^1 - 1$
$i_{eff} = (1 + 0.08)^1 - 1$
$i_{eff} = (1.08) - 1$
$i_{eff} = 0.08 = 8\%$

(2) $i_{eff} = (1 + 0.08/2)^2 - 1$
$i_{eff} = (1 + 0.04)^2 - 1$
$i_{eff} = (1.082) - 1$
$i_{eff} = 0.082 = 8.2\%$

(3) $i_{eff} = (1 + 0.08/365)^{365} - 1$
$i_{eff} = (1 + 0.00022)^{365} - 1$
$i_{eff} = (1.0833) - 1$
$i_{eff} = 0.0833 = 8.33\%$

(4) $i_{eff} = (e^k - 1)$
$i_{eff} = (e^{0.08} - 1)$
$i_{eff} = (1.0833 - 1)$
$i_{eff} = 0.0833 = 8.33\%$

(c) Compounding continuously will result in \$134 more dollars at the end of the 10 year period than compounding annually.

(d) The more frequent the compounding the larger the future value. This result is shown in part a by the fact that the future value becomes larger as the compounding period moves from annually to continuously. Since the future value is larger for a given fixed amount invested, the effective return also increases directly with the frequency of compounding. In part b we see this fact as the effective rate moved from 8% to 8.33% as compounding frequency moved from annually to continuously.

P4-38. LG 5: Comparing Compounding Periods

Challenge

(a) $FV_n = PV \times FVIF_{i\%,n}$

(1) **Annually:** $FV = PV \times FVIF_{12\%,2} = \$15,000 \times (1.254) = \$18,810$
Calculator solution: \$18,816

(2) **Quarterly:** $FV = PV \times FVIF_{3\%,8} = \$15,000 \times (1.267) = \$19,005$
Calculator solution: \$19,001.55

(3) **Monthly:** $FV = PV \times FVIF_{1\%,24} = \$15,000 \times (1.270) = \$19,050$
Calculator solution: \$19,046.02

(4) **Continuously:** $FV_{cont.} = PV \times e^{xt}$
$FV = PV \times 2.7183^{0.24} = \$15,000 \times 1.27125 = \$19,068.77$
Calculator solution: \$19,068.74

(b) The future value of the deposit increases from \$18,810 with annual compounding to \$19,068.77 with continuous compounding, demonstrating that future value increases as compounding frequency increases.

(c) The maximum future value for this deposit is \$19,068.77, resulting from continuous compounding, which assumes compounding at every possible interval.

P4-39. LG 3, 5: Annuities and Compounding: $FVA_n = PMT \times (FVIFA_{i\%,n})$

Intermediate

(a)

(1) **Annual**
$FVA_{10} = \$300 \times (FVIFA_{8\%,10})$
$FVA_{10} = \$300 \times (14.487)$
$FVA_{10} = \$4,346.10$
Calculator solution: = \$4,345.97

(2) **Semiannual**
$FVA_{10} = \$150 \times (FVIFA_{4\%,20})$
$FVA_{10} = \$150 \times (29.778)$
$FVA_{10} = \$4,466.70$
Calculator Solution: \$4,466.71

(3) **Quarterly**

$FVA_{10} = \$75 \times .(FVIFA_{2\%,40})$

$FVA_{10} = \$75 \times (60.402)$

$FVA_{10} = \$4,530.15$

Calculator solution: \$4,530.15

(b) The sooner a deposit is made the sooner the funds will be available to earn interest and contribute to compounding. Thus, the sooner the deposit and the more frequent the compounding, the larger the future sum will be.

P4-40. LG 6: Deposits to Accumulate Growing Future Sum: $PMT = \dfrac{FVA_n}{FVIFA_{i\%,n}}$

Basic

Case	Terms	Calculation	Payment
A	12%, 3 yrs.	PMT = \$5,000 ÷ 3.374 =	\$1,481.92
		Calculator solution:	\$1,481.74
B	7%, 20 yrs.	PMT = \$100,000 ÷ 40.995 =	\$2,439.32
		Calculator solution:	\$2,439.29
C	10%, 8 yrs.	PMT = \$30,000 ÷ 11.436 =	\$2,623.29
		Calculator solution:	\$2,623.32
D	8%, 12 yrs.	PMT = \$15,000 ÷ 18.977 =	\$790.43
		Calculator solution:	\$790.43

P4-41. LG 6: Creating a Retirement Fund

Intermediate

(a) $PMT = FVA_{42} \div (FVIFA_{8\%,42})$

$PMT = \$220,000 \div (304.244)$

$PMT = \$723.10$

(b) $FVA_{42} = PMT \times (FVIFA_{8\%,42})$

$FVA_{42} = \$600 \times (304.244)$

$FVA_{42} = \$182,546.40$

P4-42. LG 6: Accumulating a Growing Future Sum

Intermediate

$FV_n = PV \times (FVIF_{i\%,n})$

$FV_{20} = \$185,000 \times (FVIF_{6\%,20})$

$FV_{20} = \$185,000 \times (3.207)$

$FV_{20} = \$593,295$ = Future value of retirement home in 20 years.

Calculator solution: \$593,320.06

$PMT = FV \div (FVIFA_{i\%,n})$

$PMT = \$272,595 \div (FVIFA_{10\%,20})$

$PMT = \$272,595 \div (57.274)$

$PMT = \$4,759.49$

Calculator solution: \$4,759.61 = annual payment required.

P4-43. LG 3, 6: Deposits to Create a Perpetuity
Intermediate

(a) Present value of a perpetuity = PMT × (1 ÷ i)
= \$6,000 × (1 ÷ 0.10)
= \$6,000 × 10
= \$60,000

(b) PMT = FVA ÷ ($FVIFA_{10\%,10}$)
PMT = \$60,000 ÷ (15.937)
PMT = \$3,764.82
Calculator solution: \$3,764.72

P4-44. LG 2, 3, 6: Inflation, Time Value, and Annual Deposits
Challenge

(a) FV_n = PV × ($FVIF_{i\%,n}$)
FV_{20} = \$200,000 × ($FVIF_{5\%,25}$)
FV_{20} = \$200,000 × (3.386)
FV_{20} = \$677,200 = Future value of retirement home in 25 years.
Calculator solution: \$677,270.99

(b) PMT = FV ÷ ($FVIFA_{i\%,n}$)
PMT = \$677,200 ÷ ($FVIFA_{9\%,25}$)
PMT = \$677,200 ÷ (84.699)
PMT = \$7,995.37
Calculator solution: \$7,995.19 = annual payment required.

(c) Since John will have an additional year on which to earn interest at the end of the 25 years his annuity deposit will be smaller each year. To determine the annuity amount John will first discount back the \$677,200 one period.

$$PV_{24} = \$677,200 \times 0.9174 = \$621,263.28$$

John can solve for his annuity amount using the same calculation as in part (b).

PMT = FV ÷ ($FVIFA_{i\%,n}$)
PMT = \$621,263.28 ÷ ($FVIFA_{9\%,25}$)
PMT = \$621,263.28 ÷ (84.699)
PMT = \$7,334.95
Calculator solution: \$7,334.78 = annual payment required.

P4-45. LG 6: Loan Payment: $PMT = \dfrac{PVA}{PVIFA_{i\%,\,n}}$

Basic

Loan

A PMT = \$12,000 ÷ ($PVIFA_{8\%,3}$)
PMT = \$12,000 ÷ 2.577
PMT = \$4,656.58
Calculator solution: \$4,656.40

B PMT = \$60,000 ÷ ($PVIFA_{12\%,10}$)
PMT = \$60,000 ÷ 5.650
PMT = \$10,619.47
Calculator solution: \$10,619.05

C PMT = \$75,000 ÷ ($PVIFA_{10\%,30}$)
PMT = \$75,000 ÷ 9.427
PMT = \$7,955.87
Calculator Solution: \$7,955.94

D PMT = \$4,000 ÷ ($PVIFA_{15\%,5}$)
PMT = \$4,000 ÷ 3.352
PMT = \$1,193.32
Calculator solution: \$1,193.26

P4-46. LG 6: Loan Amortization Schedule

Intermediate

(a) PMT = \$15,000 ÷ ($PVIFA_{14\%,3}$)
PMT = \$15,000 ÷ 2.322
PMT = \$6,459.95
Calculator solution: \$6,460.97

(b)

End of Year	Loan Payment	Beginning of Year Principal	Payments		End of Year Principal
			Interest	Principal	
1	\$6,459.95	\$15,000.00	\$2,100.00	\$4,359.95	\$10,640.05
2	\$6,459.95	10,640.05	1,489.61	4,970.34	5,669.71
3	\$6,459.95	5,669.71	793.76	5,666.19	0

(The difference in the last year's beginning and ending principal is due to rounding.)

(c) Through annual end-of-the-year payments, the principal balance of the loan is declining, causing less interest to be accrued on the balance.

P4-47. LG 6: Loan Interest Deductions
Challenge

(a) PMT = $10,000 ÷ ($PVIFA_{13\%,3}$)
PMT = $10,000 ÷ (2.361)
PMT = $4,235.49
Calculator solution: $4,235.22

(b)

End of Year	Loan Payment	Beginning of Year Principal	Payments		End of Year Principal
			Interest	Principal	
1	$4,235.49	$10,000.00	$1,300.00	$2,935.49	$7,064.51
2	4,235.49	7,064.51	918.39	3,317.10	3,747.41
3	4,235.49	3,747.41	487.16	3,748.33	0

(The difference in the last year's beginning and ending principal is due to rounding.)

P4-48. LG 6: Monthly Loan Payments
Challenge

(a) PMT = $4,000 ÷ ($PVIFA_{1\%,24}$)
PMT = $4,000 ÷ (21.243)
PMT = $188.28
Calculator solution: $188.29

(b) PMT = $4,000 ÷ ($PVIFA_{0.75\%,24}$)
PMT = $4,000 ÷ (21.889)
PMT = $182.74
Calculator solution: $182.74

P4-49. LG 6: Growth Rates
Basic

(a) $PV = FV_n \times PVIF_{i\%,n}$

Case

A $PV = FV_4 \times PVIF_{k\%,4yrs.}$
$\$500 = \$800 \times PVIF_{k\%,4yrs}$
$0.625 = PVIF_{k\%,4yrs}$
12% < k < 13%
Calculator Solution: 12.47%

B $PV = FV_9 \times PVIF_{i\%,9yrs.}$
$\$1,500 = \$2,280 \times PVIF_{k\%,9yrs.}$
$0.658 = PVIF_{k\%,9yrs.}$
4%<k<5%
Calculator solution: 4.76%

C $PV = FV_6 \times PVIF_{i\%,6}$
$\$2,500 = \$2,900 \times PVIF_{k\%,6\ yrs.}$
$0.862 = PVIF_{k\%,6yrs.}$
2% < k < 3%
Calculator solution: 2.50%

(b)

Case

A Same as in (a)

B Same as in (a)

C Same as in (a)

(c) The growth rate and the interest rate should be equal, since they represent the same thing.

P4-50. LG 6: Rate of Return: $PV_n = FV_n \times (PVIF_{i\%,n})$

Intermediate

(a) $PV = \$2,000 \times (PVIF_{i\%,3yrs.})$

$\$1,500 = \$2,000 \times (PVIF_{i\%,3\ yrs.})$

$0.75 = PVIF_{i\%,3\ yrs.}$

$10\% < i < 11\%$

Calculator solution: 10.06%

(b) Mr. Singh should accept the investment that will return $2,000 because it has a higher return for the same amount of risk.

P4-51. LG 6: Rate of Return and Investment Choice

Intermediate

(a) **A**

$PV = \$8,400 \times (PVIF_{i\%,6yrs.})$

$\$5,000 = \$8,400 \times (PVIF_{i\%,6\ yrs.})$

$0.595 = PVIF_{i\%,6\ yrs.}$

$9\% < i < 10\%$

Calculator solution: 9.03%

B

$PV = \$15,900 \times (PVIF_{i\%,15yrs.})$

$\$5,000 = \$15,900 \times (PVIF_{i\%,15yrs.})$

$0.314 = PVIF_{i\%,15yrs.}$

$8\% < i < 9\%$

Calculator solution: 8.02%

C

$PV = \$7,600 \times (PVIF_{i\%,4yrs.})$

$\$5,000 = \$7,600 \times (PVIF_{i\%,4\ yrs.})$

$0.658 = PVIF_{i\%,4\ yrs.}$

$11\% < i < 12\%$

Calculator solution: 11.04%

D

$PV = \$13,000 \times (PVIF_{i\%,10\ yrs.})$

$\$5,000 = \$13,000 \times (PVIF_{i\%,10\ yrs.})$

$0.385 = PVIF_{i\%,10\ yrs..}$

$10\% < i < 11\%$

Calculator solution: 10.03%

(b) Investment C provides the highest return of the 4 alternatives. Assuming equal risk for the alternatives, Clare should choose C.

P4-52. LG 6: Rate of Return-Annuity: $PVA_n = PMT \times (PVIFA_{i\%,n})$

Basic

$\$10,606 = \$2,000 \times (PVIFA_{i\%,10\ yrs.})$

$5.303 = PVIFA_{i\%,10\ yrs.}$

$13\% < i < 14\%$

Calculator solution: 13.58%

P4-53. LG 6: Choosing the Best Annuity: $PVA_n = PMT \times (PVIFA_{i\%,n})$

Intermediate

(a) **Annuity A**

$\$30,000 = \$3,100 \times (PVIFA_{i\%,20\ yrs.})$

$9.677 = PVIFA_{i\%,20\ yrs.}$

$8\% < i < 9\%$

Calculator solution: 8.19%

Annuity B

$\$25,000 = \$3,900 \times (PVIFA_{i\%,10\ yrs.})$

$6.410 = PVIFA_{i\%,10\ yrs.}$

$9\% < i < 10\%$

Calculator solution: 9.03%

Annuity C

$\$40,000 = \$4,200 \times (PVIFA_{i\%,15\ yrs.})$

$9.524 = PVFA_{i\%,15\ yrs.}$

$6\% < i < 7\%$

Calculator solution: 6.3%

Annuity D

$\$35,000 = \$4,000 \times (PVIFA_{i\%,12\ yrs.})$

$8.75 = PVIFA_{i\%,12\ yrs.}$

$5\% < i < 6\%$

Calculator solution: 5.23%

(b) Loan B gives the highest rate of return at 9% and would be the one selected based upon Raina's criteria.

P4-54. LG 6: Interest Rate for an Annuity

Challenge

(a) **Defendants interest rate assumption**

$\$2,000,000 = \$156,000 \times (PVIFA_{i\%,25\ yrs.})$

$12.821 = PVFA_{i\%,25\ yrs.}$

$5\% < i < 6\%$

Calculator solution: 5.97%

(b) **Prosecution interest rate assumption**

$\$2,000,000 = \$255,000 \times (PVIFA_{i\%,25\ yrs.})$

$7.843 = PVFA_{i\%,25\ yrs.}$

$i = 12\%$

Calculator solution: 12.0%

(c) $\$2,000,000 = PMT \times (PVIFA_{9\%,25yrs.})$

$\$2,000,000 = PMT\ (9.823)$

$PMT = \$203,603.79$

P4-55. LG 6: Loan Rates of Interest: $PVA_n = PMT \times (PVIFA_{i\%,n})$

Intermediate

(a) **Loan A**

$\$5{,}000 = \$1{,}352.81 \times (PVIFA_{i\%,5\text{ yrs.}})$

$3.696 = PVIFA_{i\%,5\text{ yrs.}}$

$i = 11\%$

Loan B

$\$5{,}000 = \$1{,}543.21 \times (PVIFA_{i\%,4\text{ yrs.}})$

$3.24 = PVIFA_{i\%,\ 4\text{ yrs.}}$

$i = 9\%$

Loan C

$\$5{,}000 = \$2{,}010.45 \times (PVIFA_{i\%,3\text{ yrs.}})$

$2.487 = PVIFA_{k\%,3\text{ yrs.}}$

$i = 10\%$

(b) Mr. Fleming should choose Loan B, which has the lowest interest rate.

P4-56. LG 6: Number of Years to Equal Future Amount

Intermediate

A

$FV = PV \times (FVIF_{7\%,n\text{ yrs.}})$

$\$1{,}000 = \$300 \times (FVIF_{7\%,n\text{ yrs.}})$

$3.333 = FVIF_{7\%,n\text{ yrs.}}$

$17 < n < 18$

Calculator solution: 17.79

B

$FV = \$12{,}000 \times (FVIF_{5\%,n\text{ yrs.}})$

$\$15{,}000 = \$12{,}000 \times (FVIF_{5\%,n\text{ yrs.}})$

$1.250 = FVIF_{5\%,n\text{ yrs.}}$

$4 < n < 5$

Calculator solution: 4.573

C

$FV = PV \times (FVIF_{10\%,n\text{ yrs.}})$

$\$20{,}000 = \$12{,}000 \times (FVIF_{10\%,n\text{ yrs.}})$

$1.667 = FVIF_{10\%,n\text{ yrs.}}$

$5 < n < 6$

Calculator solution: 5.36

D

$FV = \$100 \times (FVIF_{9\%,n\text{ yrs.}})$

$\$500 = \$100 \times (FVIF_{9\%,n\text{ yrs.}})$

$5.00 = FVIF_{9\%,n\text{ yrs.}}$

$18 < n < 19$

Calculator solution: 18.68

E

$FV = PV \times (FVIF_{15\%,}n\text{ yrs.})$

$\$30{,}000 = \$7{,}500 \times (FVIF_{15\%,n\text{ yrs.}})$

$4.000 = FVIF_{15\%,n\text{ yrs.}}$

$9 < n < 10$

Calculator solution: 9.92

P4-57. LG 6: Time to Accumulate a Given Sum

Intermediate

(a) $20{,}000 = \$10{,}000 \times (FVIF_{10\%,n\text{ yrs.}})$

$2.000 = FVIF_{10\%,n\text{ yrs.}}$

$7 < n < 8$

Calculator solution: 7.27

(b) $20{,}000 = \$10{,}000 \times (FVIF_{7\%,n\text{ yrs.}})$

$2.000 = FVIF_{7\%,n\text{ yrs.}}$

$10 < n < 11$

Calculator solution: 10.24

(c) $20{,}000 = \$10{,}000 \times (FVIF_{12\%,n\text{ yrs.}})$

$2.000 = FVIF_{12\%,n\text{ yrs.}}$

$6 < n < 7$

Calculator solution: 6.12

(d) The higher the rate of interest the less time is required to accumulate a given future sum.

P4-58. LG 6: Number of Years to Provide a Given Return

Intermediate

A

$PVA = PMT \times (PVIFA_{11\%,n\ yrs.})$

$\$1,000 = \$250 \times (PVIFA_{11\%,n\ yrs.})$

$4.000 = PVIFA_{11\%,n\ yrs.}$

$5 < n < 6$

Calculator solution: 5.56

B

$PVA = PMT \times (PVIFA_{15\%,n\ yrs.})$

$\$150,000 = \$30,000 \times (PVIFA_{15\%,n\ yrs.})$

$5.000 = PVIFA_{15\%,n\ yrs.}$

$9 < n < 10$

Calculator solution: 9.92

C

$PVA = PMT \times (PVIFA_{10\%,n\ yrs.})$

$\$80,000 = \$30,000 \times (PVIFA_{10\%,n\ yrs.})$

$2.667 = PVIFA_{10\%,n\ yrs.}$

$3 < n < 4$

Calculator solution: 3.25

D

$PVA = PMT \times (PVIFA_{9\%,n\ yrs.})$

$\$600 = \$275 \times (PVIFA_{9\%,n\ yrs.})$

$2.182 = PVIFA_{9\%,n\ yrs.}$

$2 < n < 3$

Calculator solution: 2.54

E

$PVA = PMT \times (PVIFA_{6\%,n\ yrs.})$

$\$17,000 = \$3,500 \times (PVIFA_{6\%,n\ yrs.})$

$4.857 = PVIFA_{6\%,n\ yrs.}$

$5 < n < 6$

Calculator solution: 5.91

P4-59. LG 6: Time to Repay Installment Loan

Intermediate

(a) $\$14,000 = \$2,450 \times (PVIFA_{12\%,n\ yrs.})$

$5.714 = PVIFA_{12\%,n\ yrs.}$

$10 < n < 11$

Calculator solution: 10.21

(b) $\$14,000 = \$2,450 \times (PVIFA_{9\%,n\ yrs.})$

$5.714 = PVIFA_{9\%,n\ yrs.}$

$8 < n < 9$

Calculator solution: 8.37

(c) $\$14,000 = \$2,450 \times (PVIFA_{15\%,n\ yrs.})$

$5.714 = PVIFA_{15\%,n\ yrs.}$

$13 < n < 14$

Calculator solution: 13.92

(d) The higher the interest rate the greater the number of time periods needed to repay the loan fully.

P4-60. Ethics Problem

Intermediate

This is a tough issue. Even back in the Middle Ages, scholars debated the idea of a “just price.” The ethical debate hinges on (1) the basis for usury laws, (2) whether full disclosure is made of the true cost of the advance, and (3) whether customers understand the disclosures. Usury laws are premised on the notion that there is such a thing as an interest rate (price of credit) that is “too high.” A centuries-old fairness notion guides us into not taking advantage of someone in duress or facing an emergency situation. One must ask, too, why there are not market-supplied credit sources for borrowers, which would charge lower interest rates and receive an acceptable risk-adjusted return. On issues #2 and #3, there is no assurance that borrowers comprehend or are given adequate disclosures. See the box for the key ethics issues on which to refocus attention (some would view the objection cited as a smokescreen to take our attention off the true ethical issues in this credit offer).