CONCEPT OF PARTIAL DERIVATIVES

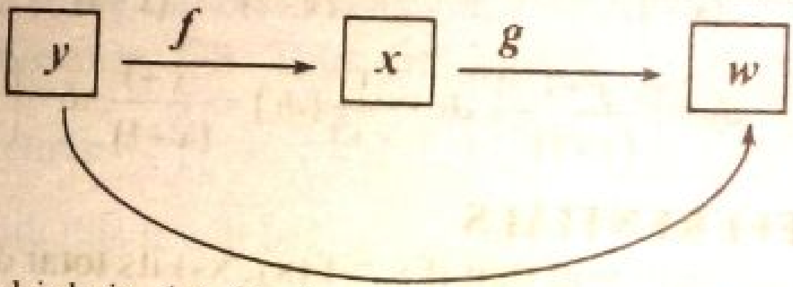
In connection with derivatives we told if the function is $y = f(x)$, then its derivative will be $\frac{dy}{dx}$. This denotes that the dependent variable y depends upon one independent variable x. Accordingly, $y = f(x)$ is a function having just one independent variable x. But in addition to such functions there are also functions which are furnished with two independent variables. In other words, in such functions the dependent variable (z) is a function of two independent variables (x , y). Thus it is written as: $z= f(x , y)$. Such like functions yield two partial derivatives. They are written as $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. It is reminded that the symbol (∂) also represents the change. For the sake of simple (ordinary) derivatives the symbol (d) is used to represent change. The symbol (∂) is known as **Partial** or **Curly**. Thus $\frac{\partial z}{\partial x}$ is read as "*Partial z over partial x*" or " *Curly z over curly x* ". Similarly, $\frac{\partial z}{\partial y}$ is read as "Partial z over partial y" or "Curly zover curly y ".

DIFFERENTIALS:

In respect of derivatives we told that if the function is y = f (x) its derivative will be $\frac{dy}{dx}$. The derivative $\frac{dy}{dx}$ was used as a single term. But in respect of differential $\frac{dy}{dx}$ is being used as a ratio of dy and dx. We rewrite the function y = f (x), its 'Difference Quotient' is $\frac{\Delta y}{\Delta x}$ which shows the rate of change in y w.r.t. x. But if we want to show just the change in y (Δy) it will be written as: $\Delta y = \frac{\Delta y}{\Delta x}(\Delta x)$. This equation shows that if we know $\frac{\Delta y}{\Delta x}$ and Δx we can find Δy. If Δx is very small the Δy will also be very small. As result, the different quotient will assume the form of derivative $(\frac{dy}{dx})$. Thus using $(\frac{dy}{dx})$ and dx in place of $\frac{\Delta y}{\Delta x}$ and Δx: $dy = \frac{dy}{dx}(dx)$ or dy = 'f(x) dx. Here the symbols dy and dx are called the differential of y and x. If the above equations are divided by dx we get derivative which we call quotient of two separate differentials dy and dx. Thus it means that the derivative $(\frac{dy}{dx})$ is like a converter which changes the smallest change dx into its corresponding change dy.

TOTAL DERIVATIVES

If the function is $y = f(x)$ where $x = g(w)$. This means that here three variables y, x and w are attached with each other. It is explained as :



Here w which is the final source of change affects y through two ways, as:

(1) It affects indirectly through function g and then through function f.

(2) It affects directly through f (shown by bent arrow).

In order to attain total derivative we get total differential. Then divide both sides of differential by dw. As $y = f(x, w)$ while $x = g(w)$

$$\Rightarrow dy = fx \, . \, dx + fw \, . \, dw$$

$$\frac{dy}{dw} = fx \, . \, \frac{dx}{dw} + fw \, . \, \frac{dw}{dw} \quad \text{or} \quad \frac{dy}{dw} = \frac{Zy}{Zx} \cdot \frac{dx}{dw} + \frac{Zy}{Zw}$$

It is clear that total derivative measures the direct effect of w on y and indirect affect of w on y.

DERIVATIVES OF IMPLICIT FUNCTIONS

In case $y = f(x)$ the y has explicitly been expressed in terms of y. On this line if we take the equation $y = 3x^4$, here by assuming values of x we can find values of y. Such function is called explicit function. If we write the above equation: $y - 3x^4 = 0$

It becomes implicit function. Such like functions are written generally, as :$F(x, y) = 0$

Here, neither y is clearly expressed in x nor it has any standard form. Here F is used instead of f. The differential of such function is as:

$$Fx \,.\, dx + Fy \,.\, dy = 0 \qquad \text{or} \qquad Fy \,.\, dy = -fx \,.\, dx$$

As derivative is the ratio of differentials. $\frac{dy}{dx} = -\frac{Fx}{Fy}$

By solving it we get the derivative of implicit function. If the implicit function is as:

$$F(y_1 x_1, x_2, x_3 \text{---------}, x_m)$$

We can get so many partial derivatives, as : $\frac{\partial y}{\partial z_1} = -\frac{Fi}{Fy}$

where $i = 1, 2, \text{---------}, m$

Thus simply, if the equation is $F(x, y) = 0$ the rule of implicit function derivative will be as:

$$\frac{dy}{dx} = -\frac{Fx}{Fy}$$