21\.

OPTIMIZATION OF COST AND REVENUE Functions

Optimization of cost and revenue functions is a process of finding maximum or minimum value of the functions.

* **Optimization of Revenue : (TR or R)**

Producer wants to maximize his revenue. For this he tried to produce such a quantity at which revenue can be maximized.

⇒ **How to find Maxima or Minima.**

When 1st Derivative is set equal to zero.

* Maxima	* Minima
if 2nd Derivative is less than zero(−)	if 2nd derivative is greater than zero(+)

NOTATIONS : AR (average revenue)

TR (Total revenue)

Related Question :

Q # 01 Demand function is given $P(Q) = 500 - 0.1Q$

TR" (total revenue) $= (P)(Q)$

$TR = (500 - 0.1Q)Q \Rightarrow 500Q - 0.1Q^2$

Necessary condition: FOC Take 1st derivative and set it $= 0$

$$\frac{d(TR)}{dQ} = \frac{d}{dQ}(500Q - 0.1Q^2) \Rightarrow 500 - 0.2Q$$

set it as $500 - 0.2Q = 0$ for $Q = ?$

$500 = 0.2Q$

$Q = 2500$

Sufficient Condition: SOC Taking $\frac{d^2(TR)}{dQ^2} = \frac{d}{dQ}(500 - 0.2Q)$

$$\frac{d^2TR}{dQ^2} = -0.2 < 0 \quad \text{(So maxima)}$$

22. Then $Q = 2500$ is Maximum Quantity where

$TR = 500(2500) - 0.1(2500)^2 = 1250000 - 625000$

$\boxed{TR = 625000}$

⇒ Question : Demand function $P = 10 - Q/2$

Find level of production at which R is maximum

As we know $R = P \times Q$

So, $R = (10 - Q/2)Q \Rightarrow 10Q - Q^2/2$

Set 1st Derivative '0' to find Q so

Necessary Condition ⇒ FOC $\frac{dR}{dQ} = 0$, So $\frac{d}{dQ}(10Q - Q^2/2) = 0$

$= 10 - 2Q/2 = 0 \Rightarrow 10 - Q = 0$

$\boxed{Q = 10}$

Sufficient Condition ⇒ SOC $\frac{d^2R}{dQ^2} \Rightarrow \frac{d}{dQ}(10 - Q)$

$\frac{d^2R}{dQ^2} = -1 < 0$ (So max)

Maximum revenue attained at $Q = 10$ where

$TR = 10(10) - (10)^2/2 \Rightarrow 100 - 50 = 50$

⇒ QUESTION : 03

Revenue of product $R = 100 - \frac{400}{x+5} - x$

Find x for maximum revenue.

Total Revenue $R = 100 - \frac{400}{x+5} - x$

Necessary Condition ⇒ FOC $\frac{dR}{dQ} = \frac{d}{dQ}\left(100 - \frac{400}{x+5} - x\right)$

Set 1st Deri $= 0$

$\frac{400}{(x+5)^2} - 1 = 0 \Rightarrow \frac{400}{(x+5)^2} = 1$

$400 = (x+5)^2 \Rightarrow (x+5)^2 = (20)^2$

26. QUESTION

⇒ Relation of price & Quantity is given as $P = 20 - 3Q$. This relation is basically known as Demand function. Find revenue function and Marginal revenue and level of production.

Sol : Given $P = 20 - 3Q$

i) Revenue is obtained by multiplying 'P' and Q. So, $R = PQ$

$R = (20 - 3Q)Q = 20Q - 3Q^2$

i) Revenue function

$$\boxed{R = 20Q - 3Q^2}$$

ii) Next, to find **Marginal revenue** Differentiate the revenue function.

$$\frac{dR}{dQ} = \frac{d}{dQ}(20Q - 3Q^2)$$

$$\boxed{\frac{dR}{dQ} = 20 - 6Q}$$

iii) Level of Production at which R is maximum:-

Set $\frac{dR}{dQ} = 0$ to find Q

So, $20 - 6Q = 0$ $Q = 20/6$

So $Q = 3.34$

Taking $d^2R/dQ^2 = \frac{d}{dQ}(20 - 6Q) = -6 < 0$

So maximum revenue at $Q = 3.34$

where $TR = R = 20(3.34) - 3(3.34)^2 \Rightarrow 66.8 - 33.4$

$$\boxed{TR = 33.4}$$

$x = 20 - 5 = 15$

Sufficient Condition SOC $\frac{d^2R}{dx^2} = \frac{-2}{(x+5)^3} < 0$

So at $x=15$ TR is $100 - \frac{400}{15+5} - 15$

$TR = 100 - 20 - 15 = 65$

OPTIMIZATION OF COST FUNCTION:

Cost optimization is a business-focused, continuous discipline to drive spending and cost reduction while maximizing business value. It includes:

Obtaining best price and terms for all business purchases.

$TC = TFC + TVC$

Maximum Condition set $\frac{dy}{dx} = 0$ then 2nd derivative $d^2y/dx^2 < 0$

QUESTION: A company estimate $C = 800 + 0.04Q + 0.002Q^2$ Find production level that minimize AC per unit.

$C = 800 + 0.04Q + 0.002Q^2$

Average cost AC as we know $AC = \frac{TC}{Q}$

$AC = \frac{800 + 0.04Q + 0.002Q^2}{Q}$

$AC = 800/Q + 0.04 + 0.002Q$

Now Take 1st Derivative & set it '0'

$\frac{dAC}{dQ} = \frac{d}{dQ}\left(\frac{800}{Q} + 0.04 + 0.02Q\right)$

$= -\frac{800}{Q^2} + 0.02 = 0$

24. $\frac{800}{Q^2} = 0.02 \Rightarrow Q^2 = 800/0.002$

$Q = \sqrt{400000} = 632.455$

Now 2nd derivative $\frac{d^2AC}{dQ^2} = \left(\frac{d}{dQ}\left(-\frac{800}{Q^2} + 0.002\right)\right)$

$= \frac{1600}{Q^3} > 0$

So production level is $Q = 632$ units.

⇒ **QUESTION: 02**

Minimum Marginal Cost

$AC = Q^2 - 5Q + 14$

As we know $MC = \frac{dTC}{dQ}$

So $TC = (AC)Q = Q^3 - 5Q^2 + 14Q$

Now $MC = \frac{d}{dQ}(Q^3 - 5Q^2 + 14Q)$

$MC = 3Q^2 - 10Q + 14$

set $\frac{dMC}{dQ} = 0$ $\quad \frac{d}{dQ}(3Q^2 - 10Q + 14) = 0$

$6Q - 10 = 0$

$\boxed{Q = 1.67}$

Now taking 2nd Derivative

$\frac{d^2MC}{dQ^2} = 6 > 0$ so Min marginal cost.

at $Q = 1.67$

$MC = 3(1.67)^2 - 10(1.67) = \boxed{5.63 = MC}$

⇒ **QUESTION: 03**

Find Minimum Total Cost:

$TC = Q^3 - 25Q^2 + 10Q + 7$

FOC $\frac{d(TC)}{dQ} = \frac{d}{dQ}(Q^3 - 25Q^2 + 10Q + 7)$

$= 3Q^2 - 50Q + 10$

put $\frac{dTC}{dQ} = 0$