New Topic For Today.

⇒ The Gaussian Method of Inverting a Matrix.

The Gaussian method can also be used to invert a matrix, it means that this is another method to find inverse matrix. Simply set up an augmented matrix with the identity matrix on the right side the matrix. Then apply row operations until the coefficient matrix on the left is reduced to an identity matrix. Then the matrix on the right side is the inverse matrix.

Q: No: 01 :- To use the Gaussian elimination method to find the inverse for matrix. $A = \begin{bmatrix} 7 & 9 \\ 6 & 12 \end{bmatrix}$.

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Solution :— Set up The augmented matrix with the identity matrix on the right, as follows.

$$\left[\begin{array}{cc|cc} 7 & 9 & 1 & 0 \\ 6 & 12 & 0 & 1 \end{array}\right]$$

After This applying row operations until the original matrix is reduced to an identity matrix.

To Transform the Coefficient matrix to an identity matrix, 1st we obtain a 1 in the a_{11} position of the Coefficient matrix; Then use row operations to obtain 0s everywhere else in the 1st column.

Like That

$$\xrightarrow{\frac{1}{7}R_1}$$

$$\left[\begin{array}{cc|cc} 1 & \frac{9}{7} & 1/7 & 0 \\ 6 & 12 & 0 & 1 \end{array}\right]$$

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subtract 6 times R_1 from R_2 i.e. $R_2 - 6R_1$

$$\left[\begin{array}{cc|cc} 1 & 9/7 & \frac{1}{7} & 0 \\ 0 & 30/7 & -6/7 & 1 \end{array}\right]$$

$$\begin{array}{cccc} 6 & 12 & 0 & 1 \\ 6 & \frac{54}{7} & \frac{6}{7} & 0 \\ - & - & - & - \\ \hline 0 & 30/7 & -6/7 & 1 \end{array}$$

Next obtain a 1 in the a_{22} position, and use row operations to obtain 0s everywhere else in the 2nd column.

For this Xing row 2 by $\frac{7}{30}$ i.e $\frac{7}{30}R_2$

$$\left[\begin{array}{cc|cc} 1 & 9/7 & 1/7 & 0 \\ 0 & 1 & -1/5 & 7/30 \end{array}\right]$$

Subtract $\frac{9}{7}$ times R_2 from R_1. i.e $R_1 - \frac{9}{7}R_2$

$$\left[\begin{array}{cc|cc} 1 & 0 & 2/5 & -3/10 \\ 0 & 1 & -1/5 & 7/30 \end{array}\right]$$

$$\begin{array}{cccc} 1 & \frac{9}{7} & \frac{1}{7} & 0 \\ 0 & \frac{9}{7} & \frac{-9}{35} & \frac{3}{10} \\ - & - & + & - \\ \hline 1 & 0 & & \end{array}$$

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Hence

$$A^{-1} = \begin{bmatrix} 2/5 & -3/10 \\ -1/5 & 7/30 \end{bmatrix}$$

<u>Check</u> :- $A \cdot A^{-1} = I$

$$A \cdot A^{-1} = \begin{bmatrix} 7 & 9 \\ 6 & 12 \end{bmatrix} \cdot \begin{bmatrix} 2/5 & -3/10 \\ -\frac{1}{5} & 7/30 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{14}{5} - \frac{9}{5} & \frac{-21}{10} + \frac{63}{10} \\ \frac{12}{5} - \frac{12}{5} & \frac{-18}{10} + \frac{84}{30} \end{bmatrix} = \begin{bmatrix} \frac{5}{5} & \frac{-63+63}{10} \\ \frac{12-12}{5} & \frac{-54+84}{30} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I .$$

So $\underline{\underline{A A^{-1} = I}}$

For Practice

1. $A = \begin{bmatrix} 3 & 6 \\ 5 & 4 \end{bmatrix}$, 2. $B = \begin{bmatrix} 4 & 10 \\ 6 & 25 \end{bmatrix}$

3. $A = \begin{bmatrix} 3 & 7 \\ 2 & 9 \end{bmatrix}$

4. $A = \begin{bmatrix} 3 & 2 & 6 \\ 2 & 4 & 3 \\ 5 & 3 & 4 \end{bmatrix}$

5. $B = \begin{bmatrix} 6 & 2 & 5 \\ 7 & -3 & 1 \\ 4 & 8 & -9 \end{bmatrix}$

6. $A = \begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 6 & 7 \end{bmatrix}$

Gaussian Method of Solving Linear Equations

To use the G. E. M of solving linear equations, simply express the system of equations as an augmented matrix and apply repeated row operations to the augmented matrix until the coefficient matrix is reduced to an identity matrix

Q. No. 1:- Use the G. E. M to solve the following system of L. equations.

$$3x + 6y = 60$$
$$5x + 4y = 52.$$

Solution:- Its matrix form is

$$\begin{bmatrix} 3 & 6 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 60 \\ 52 \end{bmatrix}$$

In augmented Matrix form (9)

$$\left[\begin{array}{cc|c} 3 & 6 & 60 \\ 5 & 4 & 52 \end{array}\right]$$

Xing R_1 by $\frac{1}{3}$ i.e $\frac{1}{3}R_1$

$$\left[\begin{array}{cc|c} 1 & 2 & 20 \\ 5 & 4 & 52 \end{array}\right]$$

$R_2 - 5R_1$

$$\left[\begin{array}{cc|c} 1 & 2 & 20 \\ 0 & -6 & -48 \end{array}\right]$$

$-\frac{1}{6}R_2$

$$\left[\begin{array}{cc|c} 1 & 2 & 20 \\ 0 & 1 & 8 \end{array}\right]$$

(10)

$R_1 - 2R_2$

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 8 \end{array}\right]$$

Thus solution values are

$x = 4,$ & $y = 8$.

For Practice

(1) $2x + 8y = 34$
$4x + 12y = 56$

(2) $4x_1 + 10x_2 = 30$
$6x_1 + 25x_2 = 67$

(3) $3x_1 + 7x_2 = 67$
$2x_1 + 9x_2 = 75$

(4) $3x_1 + 2x_2 + 6x_3 = 24$
$2x_1 + 4x_2 + 3x_3 = 23$
$5x_1 + 3x_2 + 4x_3 = 33$

(5) $6x + 2y + 5z = 73$
$7x - 3y + z = -1$
$4x + 8y - 9z = -9$