EXERCISE

Discuss the continuity of the following functions at the indicated points sets **(Problems 1 – 7):**

1. $f(x) = |x - 3|$ at $x = 3$

2. $f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 0 & \text{if } x = 3 \end{cases}$

   at $x = 3$

3. $f(x) = \begin{cases} x - 4 & \text{if } -1 < x \leq 2 \\ x^2 - 6 & \text{if } 2 < x < 5 \end{cases}$

   at $x = 2$

4. $f(x) = \begin{cases} \dfrac{x^3 - 27}{x^2 - 9} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

   at $x = 3$

5. $f(x) = \begin{cases} \sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

   at $x = 0$

6. $f(x) = \sin x$ for all $x \in \mathbf{R}$.

7. $f(x) = \begin{cases} \dfrac{x^2}{a} - a & \text{if } 0 < x < a \\ 0 & \text{if } x = a \\ a - \dfrac{a^2}{x} & \text{if } x > a \end{cases}$

   at $x = a$

8. Determine the points of continuity of the function $f(x) = x - [x]$ for all $x \in \mathbf{R}$.

9. Discuss the continuity of $x - |x|$ at $x = 1$.

10. Show that the function $f : \mathbf{R} \longrightarrow \mathbf{R}$ defined by

$$f(x) = \begin{cases} x & \text{if } x \text{ is irrational} \\ 1 - x & \text{if } x \text{ is rational} \end{cases}$$

is continuous at $x = \dfrac{1}{2}$.

11. Show that the function $f :]0, 1] \longrightarrow \mathbf{R}$ defined by

$$f(x) = \frac{1}{x}$$

is continuous on $]0, 1]$. Is $f(x)$ bounded on this interval? Explain.

12. Let $f(x) = \begin{cases} \cos\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$

Is f continuous at $x = 0$?

13. Let $f(x) = \begin{cases} (x - a)\ \sin\left(\dfrac{1}{x - a}\right) & \text{if } x \neq 0 \quad x \neq a \\ 0 & \text{if } x = 0 \quad x = a \end{cases}$

Discuss the continuity of f at $x = a$

14. Let $f(x) = \begin{cases} x \cos\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Show that f is continuous at $x = 0$

15. Let $f(x) = \begin{cases} x^2\ \sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Discuss the continuity of f at $x = 0$.

16. Let $f(x) = \begin{cases} x\ \sin\left(\dfrac{|x|}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Discuss the continuity of f at $x = 0$

17. Find c such that the function

$$f(x) = \begin{cases} \dfrac{1 - \sqrt{x}}{x - 1} & \text{if } 0 \leq x < 1 \\ c & \text{if } x = 1 \end{cases}$$

is continuous for all $x \in [0, 1]$

In Problems 18 – 20, find the points of discontinuity of the given function

18. $f(x) = \begin{cases} x + 4 & \text{if } -6 \leq x < -2 \\ x & \text{if } -2 \leq x < 2 \\ x - 4 & \text{if } 2 \leq x \leq 6 \end{cases}$

19. $g(x) = \begin{cases} x^3 & \text{if } x < 1 \\ -4 + x^2 & \text{if } 1 \leq x \leq 10 \\ 6x^2 + 46 & \text{if } x > 10 \end{cases}$

20. $f(x) = \begin{cases} x + 2 & \text{if } 0 \leq x < 1 \\ x & \text{if } 1 \leq x < 2 \\ x + 5 & \text{if } 2 \leq x < 3 \end{cases}$

21. Find constants a and b such that the function f defined by

$$f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ ax + b & \text{if } -1 \leq x < 1 \\ x^2 + 2 & \text{if } x \geq 1 \end{cases}$$

is continuous for all x.

Find the interval on which the given function is continuous. Also find points where it is discontinuous. **(Problems 22–26) :**

22. $f(x) = \dfrac{x^2 - 5}{x - 1}$ **23.** $f(x) = \dfrac{x}{|x|}$

24. $f(x) = \dfrac{\sin x}{x}$ **25.** $f(x) = \tan x$

26. $f(x) = \begin{cases} \sin x & \text{if } x \le \pi/4 \\ \cos x & \text{if } x > \pi/4 \end{cases}$

In Problems 27 – 34, examine whether the given function is continuous at $x = 0$

27. $f(x) = \begin{cases} (1 + 3x)^{1/x} & \text{if } x \ne 0 \\ e^2 & \text{if } x = 0 \end{cases}$

28. $f(x) = \begin{cases} (1 + x)^{1/x} & \text{if } x \ne 0 \\ 1 & \text{if } x = 0 \end{cases}$

29. $f(x) = \begin{cases} (1 + 2x)^{1/x} & \text{if } x \ne 0 \\ e^2 & \text{if } x = 0 \end{cases}$

30. $f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \ne 0 \\ 1 & \text{if } x = 0 \end{cases}$

31. $f(x) = \begin{cases} \dfrac{e^{1/x}}{1 + e^{1/x}} & \text{if } x \ne 0 \\ 1 & \text{if } x = 0 \end{cases}$

32. $f(x) = \begin{cases} \dfrac{e^{1/x^2}}{e^{1/x^2} - 1} & \text{if } x \ne 0 \\ 1 & \text{if } x = 0 \end{cases}$

33. $f(x) = \begin{cases} \dfrac{\sin 2x}{x} & \text{if } x \ne 0 \\ 1 & \text{if } x = 0 \end{cases}$

34. $f(x) = \begin{cases} \dfrac{\sin 3x}{\sin 2x} & \text{if } x \ne 0 \\ \dfrac{2}{3} & \text{if } x = 0 \end{cases}$

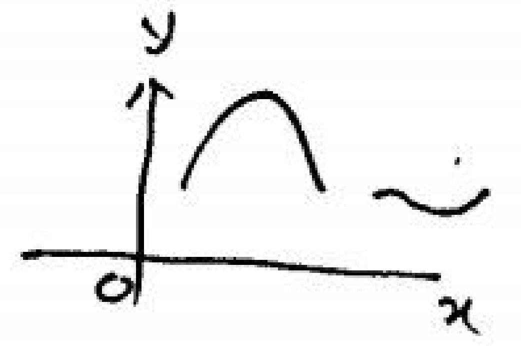
35. Let $f(x) = x^2$ and

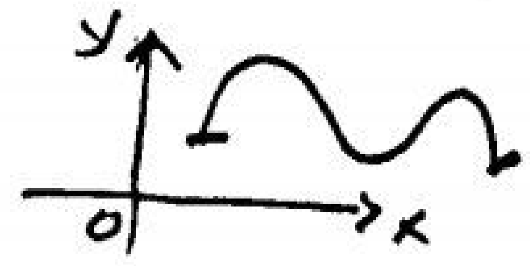
$$g(x) = \begin{cases} -4 & \text{if } x \le 0 \\ |x - 4| & \text{if } x > 0. \end{cases}$$

Determine whether fog and gof are continuous at $x = 0$.

Continuity

A function $y = f(x)$ is Said to be Continuous at a point $x = a \in D_f$





if (i) $f(x)$ is defined at $x = a$

(ii) $L.H.\lim\limits_{x \to a} f(x) = R.H \lim\limits_{x \to a} f(x)$

i.e Limit of $f(x)$ when $x \to a$ is Exist

(iii) $\lim\limits_{x \to a} f(x) = f(a)$

Q_1

$f(x) = |x-3| \to (i)$

at $x = 3$

<u>Value</u>

$f(x) = |x-3|$

Put $x = 3$

$f(3) = |3-3|$

$= |0|$

$= 0 \to (ii)$

R.H.L

$\lim\limits_{x \to 3+0} f(x) = \lim\limits_{x \to 3+0} |x-3|$

Put $x = 3+h$

$= \lim\limits_{h \to 0} |3+h-3|$

$= 0 \to (iii)$

$\lim\limits_{x \to 3-0} f(x) = \lim\limits_{x \to 3-0} |x-3|$

Put $x = 3-h$

$= \lim\limits_{h \to 0} |3-h-3|$

$= 0 \to (iv)$

(ii), (iii) & (iv)

Value = R.H.L = L.H.L

$f(x)$ is Cont: at $x = 3$

Q_2, $f(x) = \dfrac{x^2-9}{x-3}$ if $x \neq 3$

$= 0$ if $x = 3$

<u>Value</u> at $x = 3$ is given

$f(3) = 0 \to (i)$

$\lim\limits_{x \to 3} \dfrac{x^2-9}{x-3} = \lim\limits_{x \to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$= \lim\limits_{x \to 3} (x+3)$

$= 6$

(i) (ii)

$f(3) \neq \lim\limits_{x \to 3} f(x)$

$f(x)$ is Dis-Cont: at $x = 3$

(3) $f(x) = \begin{cases} x-4 & \text{if } -1 < x \leq 2 \\ x^2-6 & \text{if } 2 < x < 5 \end{cases}$ at $x = 2$

<u>Value</u> $f(x) = x-4$ at $x = 2$

$f(2) = 2-4 = -2 \to (i)$

R.H.L $\lim\limits_{x \to 2+0} f(x) = \lim\limits_{x \to 2+0} (x^2-6)$ Put $x = 2+h$

$= \lim\limits_{h \to 0} [(2+h)^2-6] = (2+0)^2-6 = 4-6$

LHL
$\displaystyle\lim_{x\to 2-0} f(x) = \lim_{x\to 2-0} (x-4)$ put $x=2-h$

$= \lim_{h\to 0}(2-h-4)$

$= (2-0-4) = -2 \longrightarrow iii.$

i, ii. and iii,

$\Rightarrow f(x)$ is Continuous at $x=2$

(4) $f(x) = \begin{cases} \dfrac{x^3-27}{x^2-9} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

at $x=3$

Value at $x=3$ is given and is 6

$f(3) = 6 \longrightarrow i.$

Limit

$\displaystyle\lim_{x\to 3} f(x) = \lim_{x\to 3}\left(\frac{x^3-27}{x^2-9}\right)$

$\displaystyle= \lim_{x\to 3}\frac{x^3-3^3}{x^2-3^2}$

$\displaystyle= \lim_{x\to 3}\frac{\cancel{(x-3)}(x^2+9+3x)}{\cancel{(x-3)}(x+3)}$

$= \dfrac{3^2+9+3(3)}{3+3} = \dfrac{27}{6} = \dfrac{9}{2} \rightarrow ii.$

i, and ii.

$f(3) \neq \lim_{x\to 3} f(x)$

$\Rightarrow f(x)$ is discontinous at $x=3$

Q5

Hint $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$ at $x=0$

Ex 1.2 Q 11 Page (4) Limit

$\lim_{x\to 0} f(x)$ does not exist.

Hence The given function is not Continuous at $x=0$

Q6 $f(x) = \sin x \;\; \forall x \in R$

Sol

Let $a \in R$ we discuss the Continuity at $x=a$

$\because$ given that $x \in R$

Value $f(x) = \sin x$ Put $x=a$

$f(a) = \sin a \rightarrow i.$

RHL

$\displaystyle\lim_{x\to a+0} f(x) = \lim_{x\to a+0} \sin x$ Put $x=a+h$

$= \lim_{h\to 0} \sin(a+h)$

$= \sin(a+0) = \sin a \rightarrow ii.$

LHL

$\displaystyle\lim_{x\to a-0} f(x) = \lim_{x\to a-0} \sin x$ Put $x=a-h$

$= \lim_{h\to 0} \sin(a-h)$

$= \sin a \rightarrow iii$

i, ii, and iii,

$f(x)$ is Continuous at $x=a \in R$

But: as 'a, is arbitary Real number. So f is Continous at all $x \in R$.

(7) $f(x) = \begin{cases} \dfrac{x^2}{a} - a & \text{if } 0 < x < a \\ 0 & \text{if } x = a \\ a - \dfrac{a^2}{x} & \text{if } x > a \end{cases}$

at $x=a$

Sol: i. $f(a) = 0$ (given)

$f(x)$ is defined at $x=a$

ii. LH Limit $\displaystyle\lim_{x\to a} f(x) = \lim_{\substack{x\to a-h \\ h\to 0}} \left(\frac{x^2}{a} - a\right)$

$$\underset{h\to 0}{Lt}\left[\frac{(a-h)^2}{a}-a\right]$$

$$= \frac{a^2}{a} - a = 0$$

R.H. Limit $\underset{x\to a}{f(x)} = \underset{\substack{x\to a+h \\ h\to 0}}{Lt}\left(a-\frac{a^2}{x}\right)$

$$= \underset{h\to 0}{Lt}\left(a-\frac{a^2}{a+h}\right)$$

$$= a-\frac{a^2}{a} = 0$$

$\Rightarrow$ LH $\underset{x\to a}{\lim} f(x) =$ RH. $\underset{x\to a}{\lim} f(x) = 0$

$\Rightarrow \underset{x\to a}{Lt} f(x) = 0$

Limit of $f(x)$ exists at $x=0$

iii. $\underset{x\to c}{Lt} f(x) = f(a) = 0$

All Three Cond.s are Satisfied

$\therefore$ $f(x)$ is Cont: at $x=0$

Q(8)

Determine the points of Continuity of the function $f(x) = x-[x]$ for all $x \in R$.

Note $y=[x]$ is called Bracket fn (Greatest integral value of x But not greater than x (If x is Decimal)

Case-I Let $x = 2.5$ (Take Fractional Value $\in R$)

Then $f(2.5) = 2.5 - [2.5]$

$= 2.5 - 2 = .5 \rightarrow$ (1)

and $\underset{x\to 2.5}{Lt} f(x) = \underset{x\to 2.5}{Lt} x-[x]$

$= 2.5 - [2.5]$

$= 2.5 - 2 = .5 \rightarrow$ (2)

From (1) and (2), we get

$f(2.5) = \underset{x\to 2.5}{Lt} f(x) = .5$

fn: is Cont: at any fractional value of $x \in R$

Case-II

When 'c' is Integer either +ve or -ve

Suppose that $x=c=5$

Then $f(x) = x-[x]$ will

Then $f(5) = 5-[5] = 5-5=0$

and $\underset{x\to 5-0}{Lt} f(x) = \underset{x\to 5-0}{Lt}(x-[x])$

$= 5-[5-0]$

$= 5-4 = 1$

$\underset{x\to 5+0}{Lt} f(x) = \underset{x\to 5+0}{Lt}(x-[x])$

$= 5-[5+0]$

$= 5-5 = 0$

$\underset{x\to 5-0}{Lt} f(x) \neq \underset{x\to 5+0}{Lt} f(x)$

Limit does not Exist at $x = c = 5 \in R$.

i.e for any +ve or -ve Integral value of $x \in R$ fn: does not exist. Implies that fn: is discontinuous for all Integral values of x

However it is Continuous at any other Real value of x

$\therefore$ fn: is Cont: for all decimal values

(9) $f(x) = x - |x|$ at $x=1$

$f(1) = 1 - |1| = 1-1 = 0$

$\lim_{x \to 1-0} f(x) = \lim_{x \to 1-0} x - |x|$

$= 1-1 = 0$

$\lim_{x \to 1+0} f(x) = \lim_{x \to 1+0} x - |x|$

$= 1-1$

$= 0$

$\because \lim_{x \to 1-0} f(x) = \lim_{x \to 1+0} f(x) = f(1)$

fn: is Cont: at $x=1$

(10) Show that the fn. $f: R \to R$ defined by

$$f(x) = \begin{cases} x & \text{if } x \text{ is irrational} \\ 1-x & \text{if } x \text{ is Rational} \end{cases}$$

is Cont: at $x = \frac{1}{2}$

$\because$ Note The numbers those can be written to the form of p/q, p and q are integer when $q \neq 0$ is called Rational no:

i. $f(\frac{1}{2}) = 1 - x$

$= 1 - \frac{1}{2} = \frac{1}{2}$

$f(x)$ is defined at $x = \frac{1}{2}$

ii. L.H.L $\lim_{x \to \frac{1}{2}} f(x) = \lim_{\substack{x \to \frac{1}{2}-h \\ h \to 0}} f(x)$

$= \lim_{\substack{x \to \frac{1}{2}-h \\ h \to 0}} (x)$

$= \lim_{h \to 0} (\frac{1}{2} - h) = \frac{1}{2}$

(if x is irrational)

& $\lim_{\substack{x \to \frac{1}{2}-h \\ h \to 0}} (1-x)$ if x is rational

$= 1 - (\frac{1}{2} - h)$

$= \frac{1}{2} + h = \frac{1}{2}$

R.H.L $\lim_{x \to \frac{1}{2}} f(x) = \lim_{\substack{x \to \frac{1}{2}+h \\ h \to 0}} f(x)$

$= \lim_{\substack{x \to \frac{1}{2}+h \\ h \to 0}} (x)$ x is irrational

$= \lim_{h \to 0} (\frac{1}{2} + h) = \frac{1}{2}$

& $\lim_{\substack{x \to \frac{1}{2}-h \\ h \to 0}} (1-x)$ x is Rational

$= \lim_{h \to 0} (1 - (\frac{1}{2} - h))$

$= \lim_{h \to 0} (\frac{1}{2} - h) = \frac{1}{2}$

$\because$ Value = Limit. (Obvious)

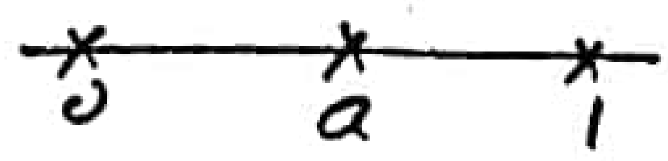
$\lim_{x \to \frac{1}{2}} f(x) = f(\frac{1}{2}) = \frac{1}{2}$

All the Three Conds are Satisfied

$\therefore f(x)$ is Cont: at $x = \frac{1}{2}$

1) Show that the fn: $f:]0,1] \to R$ defined by $f(x) = \frac{1}{x}$ is Cont: on $]0,1]$. Is $f(x)$ bounded on this interval? Explain

Sol: Let a is an arbitrary real no belonging to $]0,1]$



i. Value

$f(a) = \frac{1}{a} \in R$

Limit

$\lim\limits_{x \to a^-} f(x) = \lim\limits_{h \to 0} \left(\frac{1}{x}\right)$ Put $x = a-h$

$= \lim\limits_{h \to 0} \left(\frac{1}{a-h}\right)$

$= \frac{1}{a}$

$\lim\limits_{x \to a^+} f(x) = \lim\limits_{h \to 0} \left(\frac{1}{x}\right)$ Put $x = a+h$

$= \lim\limits_{h \to 0} \frac{1}{a+h} = \frac{1}{a}$

Limit exist

iii $f(a) = \lim\limits_{x \to a} f(x) = \frac{1}{a}$

a is arbitrary real no $\in (0,1]$

$f(x)$ is Cont: on $(0,1]$

Explain $x \to$ any value $\in]0,1]$

we see that $f(x) = 1$ when $x = 1$

$x = 1$ is its Lower bound.

But value of x become decreases from 1. The value of $f(x) \to \infty$ (as $x \to 0$)

So fn: has not upper bound.

Thus fn: $f(x)$ is not bounded above.

Hence $f(x)$ is Unbounded.

(12)

Let $f(x) = \begin{cases} \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & x = 0 \end{cases}$

f is Cont: at $x=0$

Value $f(0) = 0$ (given)

Limit

$\lim\limits_{x \to 0+0} f(x) \neq \lim\limits_{x \to 0-0} f(x)$

$\because \cos\frac{1}{x}$ (may be any value b/w $[-1,1]$

Limit does not unique.

$\Rightarrow$ Limit does not Exist

$f(x)$ is dis Cont: at $x = 0$

---

(13)

$f(x) = \begin{cases} (x-a)\sin\left(\frac{1}{x-a}\right) & x \neq a \\ 0 & x = a \end{cases}$

Cont: at $x = 0$

Sol:

$f(a) = 0$ (given)

LH Limit $\lim\limits_{x \to a} f(x) = \lim\limits_{\substack{x \to a-h \\ h \to 0}} (x-a)\sin\left(\frac{1}{x-a}\right)$

$= \lim\limits_{h \to 0} (a-h-a)\sin\left(\frac{1}{a-h-a}\right)$

$= \lim\limits_{h \to 0} -h \sin\left(-\frac{1}{h}\right) = \lim\limits_{h \to 0} h \sin\frac{1}{h}$

$= 0 \times$ any values $[-1,1] = 0$

RH Limit $\lim\limits_{x \to a} f(x) = \lim\limits_{\substack{x \to a+h \\ h \to 0}} (x-a)\sin\left(\frac{1}{x-a}\right)$

$= (a+h-a)\sin\left(\frac{1}{a+h-a}\right)$

Limit exist

ii $\lim_{x \to a} f(x) = f(a) = 0$

$f(x)$ is Cont: at $x = a$

Same Q 14, 15 as 13

Q (16) $f(x) = \begin{cases} x \sin \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & x = 0 \end{cases}$

$\because f(x) = x \sin \frac{x}{x} = x \sin 1$ if $x \geq 0$

$= x \sin \frac{-x}{x} = x \sin(-1)$ $x < 0$

(1) Value $f(0) = 0$ given

(2) Limit $\lim_{x \to 0^+} f(x) = \lim_{x \to 0} x \sin 1$

$= 0 \times \sin 1 = 0$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} x \sin(-1)$

$= 0 \times \sin(-1) = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0} f(x) = 0$

$\lim_{x \to 0} f(x) = 0$

(3) $\lim_{x \to 0} f(x) = f(0)$

fn: is Cont: at $x = 0$

(17) Find C s.t the fn

$f(x) = \begin{cases} \frac{1 - \sqrt{x}}{x - 1} & \text{if } 0 \leq x < 1 \\ C & \text{if } x = 1 \end{cases}$

$f(x)$ is cont: $x \in [0, 1]$

$f(1) = C \quad \to (i)$

$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{1 - \sqrt{x}}{x - 1}$

$= \lim_{x \to 1} \frac{-\cancel{(\sqrt{x} - 1)}}{\cancel{(\sqrt{x} - 1)}(\sqrt{x} + 1)}$

$= \lim_{x \to 1} \frac{-1}{\sqrt{x} + 1} = -\frac{1}{2} \quad \to ii$

Given that $f(x)$ is Continous at $x = 1$

Therefor $f(1) = \lim_{x \to 1} f(x)$

using i. & ii.

$C = -\frac{1}{2}$ Ans

---

(18) Find the points of discontinuity of the given fn.

$f(x) = \begin{cases} x + 4 & \text{if } -6 \leq x < -2 \\ x & \text{if } -2 \leq x < 2 \\ x - 4 & \text{if } 2 \leq x \leq 6 \end{cases}$

$\because$ Continuity of the fn: at the changing pt $x = -2, 2$.

At $\Rightarrow x = -2$

i. $f(-2) = (-2) = -2$

ii $\lim_{x \to -2^+} f(x) = \lim_{x \to -2} (x)$

$= (-2) = -2$

$\lim_{x \to -2^-} f(x) = \lim_{x \to -2} (x + 4)$

$= -2 + 4 = 2$

$\lim_{x \to -2^+} f(x) \neq \lim_{x \to -2^-} f(x)$

$f(x)$ is discont: at $x = -2$

At $\Rightarrow x = 2$

$f(2) = 2 - 4 = -2$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (x - 4) = 2 - 4 = -2$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (x) = 2$

$\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$

$f(x)$ is discont: at $x = 2$

Thus the pt: of discont: at $x = 2, -2$

(19)

$$g(x) = \begin{cases} x^3 & \text{if } x<1 \\ -4-x^2 & \text{if } 1 \le x \le 10 \\ 6x^2+46 & \text{if } x>10 \end{cases}$$

At $x=1$: $g(1) = -4-(1)^2 = -4-1 = -5$

$\lim_{x\to 1^-} g(x) = \lim_{x\to 1} (x^3) = (1)^3 = 1$

$\lim_{x\to 1^+} g(x) = \lim_{x\to 1} -4-x^3 = -4-(1)^3 = -5$

$\lim_{x\to 1^-} g(x) \neq \lim_{x\to 1^+} g(x)$

$g(x)$ is dis Cont: at $x=1$

At $x=10$: $g(10) = -4-(10)^2 = -4-100 = -104$

$\lim_{x\to 10^-} g(x) = \lim_{x\to 10} -4-x^2$

$= -4-(10)^2$

$= -104$

$\lim_{x\to 10^+} g(x) = \lim_{x\to 10^+} 6(10)^2+46$

$= 6(100)+46$

$= 600+46$

$= 646$

$\lim_{x\to 10^+} g(x) \neq \lim_{x\to 10^-} g(x)$

$g(x)$ is also disCont: at $x=10$

Same Q: 20

Check dis: at $x = 1, 2$.

(21)

$$f(x) = \begin{cases} x^3 & \text{if } x<-1 \\ ax+b & \text{if } -1 \le x < 1 \\ x^2+2 & \text{if } x \ge 1 \end{cases}$$

is Cont: for all x.

at $x=1$

$f(1) = x^2+2 = (1)^2+2 = 3 \rightarrow$ (i)

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) \rightarrow$ ii

$\lim_{x\to 1} x^2+2 = \lim_{x\to 1} (ax+b)$

$(1)^2+2 = a(1)+b$

$3 = a+b \rightarrow$ (iii)

Since fn: is Cont: (given)

from i, ii & iii

$a+b = 3 \longrightarrow$ (1)

At $x=-1$

$f(-1) = a(-1)+b = -a+b \rightarrow$ i.

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1} (x)^3 = (-1)^3 = -1 \rightarrow$ ii

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1} ax+b$

$= (a)(-1)+b = -a+b \rightarrow$ iii

from i. ii & iii

$-a+b = -1 \longrightarrow$ (2)

Adding (1) & (2)

$\cancel{-a}+b=-1$

$\cancel{a}+b=3$

$2b = 2 \Rightarrow \boxed{b=1}$

Using (1) $a+1 = 3$

$\boxed{a = 2}$

Find the interval on which the given function is Continous. Also find points where it is Discontinous (22 – 26)

(22) $f(x) = \frac{x^2-5}{x-1}$

Clearly at $x=1$, The Value of $f(x)$ doesnot Exist

$f(x)$ is not Continous at $x=1$

and Continous for all $x \in \mathbb{R} - \{1\}$

(23) $f(x) = \frac{x}{|x|}$

As function is not defined at $x=0$

So is discontinuous at $x=0$

The function is Cont: at Every Value of x when $x \in \mathbb{R} - \{0\}$

(24) $f(x) = \frac{\sin x}{x}$

$f(x)$ is not defined at $x=0$ (DisCont: Pt)

Every value of $\sin x$ and x is Continuous when $x \neq 0$

i.e $x \in R - \{0\}$

(25) $f(x) = \tan x$

Since

$\tan (2n+1)\frac{\pi}{2} = \infty$

Value of $\tan x$ at $x = (2n+1)\frac{\pi}{2}$ does not Exist, Therefore $f(x)$ is discontinous at $x = (2n+1)\frac{\pi}{2}$

$f(x)$ is Cont: for all $x \in R - \{(2n+1)\frac{\pi}{2}\}$

(26) $f(x) = \sin x \quad x \leq \frac{\pi}{4}$

$= \cos x \quad x > \frac{\pi}{4}$

(i) $f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

(ii) $\lim_{x \to \frac{\pi}{4}^+} f(x) = \lim_{x \to \frac{\pi}{4}} \cos x = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{x \to \frac{\pi}{4}} \sin x = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\lim_{x \to \frac{\pi}{4}^+} f(x) = \lim_{x \to \frac{\pi}{4}^-} f(x)$

(iii) $f(\frac{\pi}{4}) = \lim_{x \to \frac{\pi}{4}} f(x)$

$f(x)$ is Cont: at $x = \frac{\pi}{4}$

both $\sin x$ & $\cos x$ are continous at Every Value of $x \in R$. Hence $f(x)$ is Continuous at every value of $x \in R$.

Examine whether the given Function is Continuous at $x=0$

(27) $f(x)=\begin{cases}(1+3x)^{\frac{1}{x}} & \text{if } x\neq 0\\ e^{2} & \text{if } x=0\end{cases}$

Value $f(0)=e^{2} \rightarrow i$

Limit

$\lim_{x\to 0} f(x)=\lim_{x\to 0}(1+3x)^{\frac{1}{x}}$

$=\left[\lim_{x\to 0}(1+3x)^{\frac{1}{3x}}\right]^{3}$

$=e^{3} \rightarrow ii$

i, and ii.

$f(x)$ is not Continuous at $x=0$

(28) $f(x)=\begin{cases}(1+x)^{1/x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$

Value $f(0)=1 \rightarrow i$

Limit $\lim_{x\to 0} f(x)=\lim_{x\to 0}(1+x)^{1/x}$

$=e \rightarrow ii$

from i. & ii.

Function $f(x)$ is discontinuous at $x=0$.

(29) $f(x)=(1+2x)^{1/x} \quad x\neq 0$

$=e^{2} \quad \text{if } x=0$

Value $f(0)=e^{2} \rightarrow i$

Limit $\lim_{x\to 0} f(x)=\lim_{x\to 0}(1+2x)^{\frac{1}{x}}$

$=\left[\lim_{x\to 0}(1+2x)^{\frac{1}{2x}}\right]^{2}=e^{2}$

$\rightarrow ii$

from i & ii

$f(x)$ is Continuous at $x=0$

(30) $f(x)=e^{-1/x^{2}} \quad x\neq 0$

$=1 \quad \text{if } x=0$

Value at $x=0$ is given and is 1

$f(0)=1 \longrightarrow i.$

Limit

$\lim_{x\to 0} f(x)=\lim_{x\to 0} e^{-1/x^{2}}$

$=\lim_{x\to 0}\frac{1}{e^{1/x^{2}}}=\frac{1}{e^{\infty}}$

$=\frac{1}{\infty}=0 \rightarrow ii$

OR

L.H.L $\lim_{x\to 0-0}=\lim_{x\to 0-0} e^{-1/x^{2}}$ Put $x=0-h$

$=\lim_{h\to 0} e^{-\frac{1}{(0-h)^{2}}}=\lim_{h\to 0}\frac{1}{e^{\frac{1}{(0-h)^{2}}}}$

$=\lim_{h\to 0}\frac{1}{e^{\left(\frac{1}{0-0}\right)^{2}}}=\frac{1}{e^{\infty}}=\frac{1}{\infty}$

$=0$

$f(x)$ is disContinuous at $x=0$.

(31) $f(x)=\frac{e^{-1/x}}{1+e^{1/x}} \quad \text{if } x\neq 0$

$=1 \quad \text{if } x=0$

Since $f(0)=1$ (given)

$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{e^{-1/x}}{1+e^{1/x}}$

$$= \lim_{x \to 0} \frac{1/e^{1/x}}{1+e^{1/x}}$$

$$= \frac{1}{\lim_{x\to 0} e^{1/x}(1+e^{1/x})}$$

$$= \frac{1}{e^{1/0}(1+e^{1/0})} = \frac{1}{\infty} = 0$$

Since

$f(0) \neq \lim_{x\to 0} f(x)$

function is not Continuous at $x = 0$

Note

$$\lim_{x \to 0^-} f(x) = \lim_{h\to 0} \frac{e^{\frac{1}{h}}}{1+e^{-1/h}}$$

Put $x = 0-h$ $\quad = \dfrac{e^{\frac{1}{h}}}{\frac{e^{1/h}+1}{1/e^{\frac{1}{h}}}} = \dfrac{e^{\frac{2}{h}}}{1+e^{1/h}}$

$\lim \Rightarrow \infty$ does not Exist

$$\lim_{x\to 0+0} = \lim_{h\to 0} \frac{e^{-1/h}}{1+e^{1/h}} = \frac{1}{e^{2/h}(1+e^{-1/h})}$$

$x = 0+h$ $\quad = 0$

---

(32)

$$f(x) = \frac{e^{1/x^2}}{e^{\frac{1}{x^2}}-1} \quad \text{if } x \neq 0$$

$$= 1 \quad \text{if } x = 0$$

Value $f(0) = 1$

Limit $\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{e^{\frac{1}{x^2}}}{e^{1/x^2}-1}$

$$= \lim_{x\to 0} \frac{\cancel{e^{\frac{1}{x^2}}}}{\cancel{e^{1/x^2}}\left[1-\frac{1}{e^{1/x^2}}\right]}$$

$$= \frac{1}{\lim_{x\to 0} 1-\frac{1}{e^{1/x^2}}} = \frac{1}{1-0}$$

$$= 1$$

$f(0) = \lim_{x\to 0} f(x)$

fn: is Continuous at $x=0$

(33)

$$f(x) = \frac{\sin 2x}{x} \quad \text{if } x \neq 0$$

$$= 1 \quad \text{if } x = 0$$

Value $f(0) = 1$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin 2x}{x}$$

$$= 2\left[\lim_{x\to 0}\frac{\sin 2x}{2x}\right]$$

$$= 2(1) = 2$$

$\lim_{x\to 0} f(x) \neq f(0)$

$f(x)$ is not Cont: at $x = 0$.

(34)

$$f(x) = \begin{cases} \dfrac{\sin 3x}{\sin 2x} & \text{if } x \neq 0 \\ \dfrac{2}{3} & \text{if } x = 0 \end{cases}$$

Value $f(0) = 2/3$

Limit $\lim_{x\to 0} \dfrac{\sin 3x}{\sin 2x}$

$$= \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \frac{3x}{1} \cdot \frac{1}{\frac{\sin 2x}{2x}} \cdot \frac{1}{2x}$$

$$= \lim_{x\to 0} \frac{3}{2}(1)\cdot\frac{1}{(1)}$$

$$= 3/2$$

$f(0) \neq \lim_{x \to 0} f(x)$

fn. is disCont. at $x = 0$

---

(35) Let $f(x) = x^2$

and

$$g(x) = \begin{cases} -4 & \text{if } x \le 0 \\ |x-4| & \text{if } x > 0 \end{cases}$$

Determine whether fog and gof are Continuous at $x=0$

$fog(x) = f(g(x))$

$$= \begin{cases} f(-4) = (-4)^2 = 16 & \text{if } x \le 0 \\ f|x-4| = (\pm(x-4))^2 = (x-4)^2 & \text{if } x > 0 \end{cases}$$

Value $fog(0) = f(g(0))$

$= f(-4)$

$= (-4)^2 = 16$

OR $fog(0) = 16$ given.

$\lim_{x \to 0^+} fog(x) = \lim_{x \to 0} (x-4)^2$

$= (0-4)^2 = 16$

$\lim_{x \to 0^-} fog(x) = \lim_{x \to 0} 16 = 16$

$\lim_{x \to 0} fog(x) = fog(0)$

$fog(x)$ is Continuous at $x = 0$

$$gof(x) = g(f(x)) = \begin{cases} g(f(x)) = g(x^2) = -4 & \text{if } x^2 \le 0 \\ g[f(x)] = g(x^2) = |x^2-4| & \text{if } x^2 > 0 \end{cases}$$

Value

$gof(0) = -4$

Limit

$\lim_{x \to 0^+} gof(x) = \lim_{x \to 0} |x^2-4|$

$= |0-4| = 4$

$\lim_{x \to 0^-} gof(x) = \lim_{x \to 0} (-4)$

$= -4$

$\lim_{x \to 0^+} gof(x) \neq \lim_{x \to 0^-} gof(x)$

$\lim_{x \to 0} gof(x) \neq gof(0)$

$gof(x)$ is disCont. at $x = 0$