EXERCISE

Evaluate the indicated limits **(Problems 1 – 30):**

1. $\lim\limits_{x \to 2} \dfrac{x-2}{\sqrt{2+x}}$

2. $\lim\limits_{x \to 1} \dfrac{x^3-1}{x-1}$

3. $\lim\limits_{x \to 1} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3} \right)$

4. If $P_n(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n$, prove that

$$\lim_{x \to a} P_n(x) = P_n(a)$$

5. $\lim\limits_{x \to 0} \dfrac{\csc x - \cot x}{x}$

6. $\lim\limits_{x \to 0} \dfrac{\sin ax}{\sin bx}$

7. $\lim\limits_{x \to 0} \dfrac{1-\cos x}{x^2}$

8. $\lim\limits_{y \to x} \dfrac{y^{2/3} - x^{2/3}}{y-x}$

9. $\lim\limits_{x \to \pi} \dfrac{\tan(\sin x)}{\sin x}$

10. $\lim\limits_{x \to 0} x \sin\left(\dfrac{1}{x}\right)$

11. $\lim\limits_{x \to 0} \sin\left(\dfrac{1}{x}\right)$

12. $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2+1}}{x+1}$

13. $\lim\limits_{x \to \infty} \dfrac{4x^3 - 2x^2 + 1}{3x^3 - 5}$

14. $\lim\limits_{x \to \infty} \left(1 + \dfrac{2}{x}\right)^x$

15. $\lim\limits_{x \to \infty} \left(1 - \dfrac{1}{x}\right)^x$

16. $\lim\limits_{x \to \infty} \left(\dfrac{x}{1+x}\right)^x$

17. $\lim\limits_{x \to \infty} \dfrac{a^x - 1}{x}, \quad (a > 1)$

18. $\lim\limits_{x \to \infty} \dfrac{x^4 - 2x^2 + 6}{x^2 + 7}$

19. $\lim\limits_{x \to \pm\infty} \left[\dfrac{x^2}{x+1} - \dfrac{x^2}{x+3} \right]$

20. $\lim\limits_{x \to \infty} \left(x - \sqrt{x^2 - a^2} \right)$

21. $\lim\limits_{x \to \infty} \dfrac{x^2+1}{x^{3/2}}$

22. $\lim\limits_{x \to \pm\infty} \dfrac{5x^3 + 3x^2 - 1}{x - 4x^4}$

23. $\lim\limits_{x \to \infty} \dfrac{3 - 2x^4}{1+x}$

24. $\lim\limits_{x \to \pm\infty} (x^4 - x^3 + x)$

25. $\lim\limits_{x \to \pm\infty} \left[\dfrac{x^2}{x+3} - \dfrac{x^2}{x+5} \right]$

26. $\lim\limits_{x \to \pm\infty} \left[4x^3 - 3x^2 + x - 1 \right]$

27. $\lim\limits_{x \to 1-0} \dfrac{\sqrt{1-x^2}}{1-x}$

28. $\lim\limits_{x \to 1+0} \dfrac{x-1}{\sqrt{x^2-1}}$

29. $\lim\limits_{x \to 2-0} \dfrac{\sqrt{4-x^2}}{\sqrt{6-5x+x^2}}$

30. $\lim\limits_{x \to \infty} \dfrac{x + \sin x}{x}$

31. Let $f(x) = \begin{cases} x^2+3 & \text{if } x \le 1 \\ x+1 & \text{if } x > 1 \end{cases}$

Find $f(1+0)$ and $f(1-0)$.

(24) $\lim\limits_{x \to -1} \dfrac{x^{1/3}+1}{x+1}$

(25) $\lim\limits_{x \to 3^-} \left(\dfrac{1}{x-3} - \dfrac{1}{|x-}\right.$

32. $f(x) = \begin{cases} 3 & \text{if } x \le -2 \\ -\dfrac{1}{2}x^2 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \ge 2 \end{cases}$

Find $f(-2-0)$, $f(-2+0)$, $f(2-0)$ and $f(2+0)$

33. Let $f(x) = \begin{cases} x^2-1 & \text{if } x \le 2 \\ \sqrt{x+7} & \text{if } x > 2 \end{cases}$

(26) $\lim\limits_{x \to 2} \dfrac{x^2+2x-8}{x^2-4}$

Find $\lim f(x)$ as $x \to 2$.

34. Let $f(x) = \begin{cases} \cos x & \text{if } x \le 0 \\ 1-x & \text{if } x > 0 \end{cases}$

Find $\lim\limits_{x \to 0} f(x)$.

35. Let $f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ x^3 & \text{if } x > 1 \end{cases}$

Show that $\lim\limits_{x \to 1} f(x) = 1$

36. Let $f(x) = \begin{cases} x+2 & \text{if } x \le -1 \\ ax^2 & \text{if } x > -1 \end{cases}$

Find a so that $\lim\limits_{x \to -1} f(x)$ exists.

37. Evaluate $\lim\limits_{x \longrightarrow 3+0} \dfrac{3-x}{|x-3|}$

38. Evaluate $\lim\limits_{x \longrightarrow 0-0} \dfrac{x}{x-|x|}$.

39. Find $\lim\limits_{h \longrightarrow 0-0} \dfrac{|-1+h|-1}{h}$

EXERCISE

Q.No (1) $\underset{x\to 2}{Lt}\ \dfrac{x-2}{\sqrt{2+x}} = \dfrac{2-2}{\sqrt{2+2}} = \dfrac{0}{2} = 0$

Q.No (2).

$\underset{x\to 1}{Lim}\ \dfrac{x^3-1}{x-1}$ ... $(\frac{0}{0})$ form

$\underset{x\to 1}{Lt}\ \dfrac{(x-1)(x^2+x+1)}{(x-1)}$

$= \underset{x\to 1}{Lt}\ (x^2+x+1)$

$= 1^2+1+1$

$= 3$

(3) $\underset{x\to 1}{Lt}\ \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3}\right)$

So taking LCM, we get

$\underset{x\to 1}{Lt}\ \left(\dfrac{x^2+x+1-3}{(1-x)(x^2+x+1)}\right)$

$\underset{x\to 1}{Lt}\ \left(\dfrac{x^2+x-2}{(1-x)(x^2+x+1)}\right)$

$= \underset{x\to 1}{Lt}\ \dfrac{x^2+2x-x-2}{(1-x)(x^2+x+1)}$

$= \underset{x\to 1}{Lt}\ \dfrac{(x+2)\cancel{(x-1)}}{-\cancel{(x-1)}(x^2+x+1)}$

$= \underset{x\to 1}{Lt}\ \dfrac{x+2}{-(x^2+x+1)}$

$= \dfrac{1+2}{-(1+1+1)} = \dfrac{3}{-3} = -1$ Ans

(4) $P_n(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n \rightarrow (i)$

taking the limit of both sides

$\underset{x\to a}{Lt}\ P_n(x) = \underset{x\to a}{Lt}\ (a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n)$

$= a_0a^n + a_1a^{n-1} + \dots + a_{n-1}a + a_n$

$= P_n(a)$ by (i)

(5) $\underset{x\to 0}{Lt}\ \dfrac{\operatorname{cosec} x - \cot x}{x}$

$= \underset{x\to 0}{Lt}\ \dfrac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x}$

$= \underset{x\to 0}{Lt}\ \dfrac{1}{x}\left[\dfrac{1-\cos x}{\sin x}\right]$

$= \underset{x\to 0}{Lt}\ \dfrac{1}{x}\left[\dfrac{1-\cos x}{\sin x} \times \dfrac{1+\cos x}{1+\cos x}\right]$

$= \underset{x\to 0}{Lt}\ \dfrac{1}{x}\left[\dfrac{1-\cos^2 x}{\sin x\,(1+\cos x)}\right]$

$= \underset{x\to 0}{Lt}\ \dfrac{\sin^2 x}{x \sin x\,(1+\cos x)}$

$= \underset{x\to 0}{Lt}\left(\dfrac{\sin x}{x}\right) \times \dfrac{1}{\underset{x\to 0}{Lt}\ 1+\cos x}$

$= (1) \times \dfrac{1}{1+1} = \dfrac{1}{2}$ Ans

(6) $\underset{x\to 0}{Lt}\ \dfrac{\sin ax}{\sin bx}$

$= \underset{x\to 0}{Lt}\ \dfrac{\sin ax}{\sin bx} \times \dfrac{ax}{ax} \times \dfrac{bx}{bx}$

$= \underset{x\to 0}{Lt}\ \dfrac{\sin ax}{ax} \times \dfrac{bx}{\sin bx} \times \dfrac{a\cancel{x}}{b\cancel{x}}$

$= \left(\underset{x\to 0}{Lt}\ \dfrac{\sin ax}{ax}\right)\left(\underset{x\to 0}{Lt}\ \dfrac{\sin bx}{bx}\right)^{-1} \times \dfrac{a}{b}$

$= (1)\,(1)^{-1}\left(\dfrac{a}{b}\right) = \dfrac{a}{b}$

(7) $\underset{x\to 0}{Lt}\ \dfrac{1-\cos x}{x^2} = \underset{x\to 0}{Lt}\ \dfrac{1-\cos x}{x^2} \times \dfrac{1+\cos x}{1+\cos x}$ —— A.

$= \underset{x\to 0}{Lt}\ \dfrac{1-\cos^2 x}{x^2(1+\cos x)} = \underset{x\to 0}{Lt}\ \dfrac{\sin^2 x}{x^2} \times \dfrac{1}{1+\cos x}$

$= 1 \times \dfrac{1}{1+1} = \dfrac{1}{2}$ Ans

(8) $\lim_{y\to x} \dfrac{y^{2/3}-x^{2/3}}{y-x}$

$= \lim_{y\to x} \dfrac{(y^{1/3})^2-(x^{1/3})^2}{(y^{1/3})^3-(x^{1/3})^3}$

$= \lim_{y\to x} \dfrac{\cancel{(y^{1/3}-x^{1/3})}(y^{1/3}+x^{1/3})}{\cancel{(y^{1/3}-x^{1/3})}((y^{1/3})^2+y^{1/3}x^{1/3}+(x^{1/3})^2)}$

$= \lim_{y\to x} \dfrac{y^{1/3}+x^{1/3}}{y^{2/3}+y^{1/3}x^{1/3}+x^{2/3}}$

$= \dfrac{x^{1/3}+x^{1/3}}{x^{2/3}+x^{1/3}\cdot x^{1/3}+x^{2/3}}$

$= \dfrac{2x^{1/3}}{3x^{2/3}}$

$= \dfrac{2}{3(x)^{2/3-1/3}} = \dfrac{2}{3(x)^{1/3}}$ Ans

2nd Method Put $y = x+h$ using B. Th

(9) $\lim_{x\to\pi} \dfrac{\tan(\sin x)}{\sin x}$

Let $\sin x = \theta$

When $x\to\pi$, $\theta\to 0$

$= \lim_{\theta\to 0} \dfrac{\tan\theta}{\theta}$

$= \lim_{\theta\to 0} \dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\theta}$

$= \lim_{\theta\to 0} \dfrac{\sin\theta}{\theta}\cdot\dfrac{1}{\cos\theta}$

$= (1) \times \dfrac{1}{\cos 0} = 1\times\dfrac{1}{1}$

$= 1$ Ans

(10) $\lim_{x\to 0} x\sin\frac{1}{x}$

$= \left(\lim_{x\to 0} x\right)\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$

$= 0 \times$ Something

$= 0$

$\because$ Sin is a bounded function

value of $\sin x$ lies b/w 1 & -1

(11) $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$

Now $\lim_{x\to 0+0} \sin\frac{1}{x}$ Put $x = 0+h$

$= \lim_{h\to 0} \sin\left(\frac{1}{0+h}\right)$

$= \lim_{h\to 0} \sin\left(\frac{1}{h}\right) = l$ (Say) $\to$ i

$\because l \in [-1, 1]$

Also $\lim_{x\to 0-0} \sin\left(\frac{1}{x}\right)$ Put $x = 0-h$

$= \lim_{h\to 0} \sin\left(\frac{1}{0-h}\right)$

$= \lim_{h\to 0} \sin\left(-\frac{1}{h}\right)$

$= -\lim_{h\to 0} \sin\left(\frac{1}{h}\right) = -l$ (Say) $\to$ ii

$\because l \in [-1, 1]$

From i, & ii

$\lim_{x\to 0+0} \sin\frac{1}{x} \neq \lim_{x\to 0-0} \sin\frac{1}{x}$

Hence $\lim_{x\to 0} \sin\frac{1}{x}$ does not Exist

(12) $\lim_{x\to\infty} \dfrac{\sqrt{x^2+1}}{x+1} = \lim_{x\to\infty} \dfrac{x\sqrt{1+\frac{1}{x^2}}}{x\left(1+\frac{1}{x}\right)}$

$= \dfrac{\sqrt{1+0}}{(1+0)} = \dfrac{1}{1} = 1$

Ans

(13) $\lim_{x\to\infty} \dfrac{4x^3-2x^2+1}{3x^3-5}$

$= \lim_{x\to\infty} \dfrac{x^3\left(4-\frac{2}{x}+\frac{1}{x^3}\right)}{x^3\left(3-\frac{5}{x^3}\right)}$

$= \lim_{x\to\infty} \dfrac{4-\frac{2}{x}+\frac{1}{x^3}}{3-\frac{5}{x^3}}$

$= \dfrac{4-0+0}{3-0} = \dfrac{4}{3}$ Ans

Alt. Put $x=\frac{1}{y}$ $\lim_{x\to\infty}, y\to 0$

(14) $\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^x$

$\lim_{x\to\infty} \left[\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^2$

$= e^2$ $\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

(15) $\lim_{x\to\infty} \left(1-\frac{1}{x}\right)^x$ — (1)

(1) Can be written as

$\lim_{x\to\infty} \left(1+\left(-\frac{1}{x}\right)\right)^x$

$= \left[\lim_{x\to\infty}\left(1+\left(-\frac{1}{x}\right)\right)^{-x}\right]^{-1}$

$= e^{-1} = 1/e$

(16) $\lim_{x\to\infty} \left[\dfrac{x}{1+x}\right]^x$

$\lim_{x\to\infty} \left(\dfrac{1+x}{x}\right)^{-x}$

$\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right]^{-1}$

$= e^{-1} = 1/e$

(17) $\lim_{x\to\infty} \dfrac{a^x-1}{x}$ $\because a>1$

Let $y = a^x - 1$

When $\lim x\to\infty$, $y\to\infty$

$a^x = y+1$

Taking ln of both sides/ $\ln$ get

$x\ln a = \ln(y+1)$

$x = \dfrac{\ln(1+y)}{\ln a}$

$\lim_{x\to\infty} \dfrac{a^x-1}{x} = \lim_{y\to\infty} \dfrac{y}{\frac{\ln(1+y)}{\ln a}}$

(*) $= \lim_{y\to\infty} \dfrac{\ln a}{\frac{1}{y}\ln(1+y)} = \dfrac{\ln a}{\ln \lim_{y\to\infty}(1+y)^{1/y}}$

$\because \lim_{y\to\infty}(1+y)^{\frac{1}{y}} = (\infty)^0 = 1$ (assume)

$\Rightarrow 5^0 = 1$

$= \dfrac{\ln a}{\ln 1} = \dfrac{\ln a}{0} = \infty$

2nd Solution (*)

$= \ln a \lim_{y\to\infty} \dfrac{y}{\ln(1+y)}$

Apply L-Hospital Rule

$= \ln a \cdot \dfrac{\infty}{\infty}$

$= \ln a \cdot \lim_{y\to\infty} \dfrac{1}{\frac{1}{1+y}}$

$= \ln a \lim_{y\to\infty}(1+y) = \ln a \times \infty = \infty$

OR 3rd way $\lim_{x\to\infty} \dfrac{a^x-1}{x} = \dfrac{a^x\ln a - 0}{1}$ LH Rule.

$= \lim_{x\to\infty}(a^x\ln a) = \infty \times \ln a = \infty$

4th way $\lim_{x\to\infty} \dfrac{a^x-1}{x}$ Put $a = 1+h$

$\lim_{x\to\infty} \dfrac{(1+h)^x-1}{x} = \lim_{x\to\infty} \dfrac{1+xh+\frac{x(x-1)}{2!}h^2+\cdots-1}{x} = \lim_{x\to\infty} \dfrac{x\left[h+\frac{x-1}{2!}h^2+\cdots\right]}{x}$

$= h+\infty = \infty$

(18) $\lim_{x\to\infty} \dfrac{x^4-2x^2+6}{x^2+7}$

Dividing N & D by x^4

$\lim_{x\to\infty} \dfrac{1-\frac{2}{x^2}+\frac{6}{x^4}}{\frac{1}{x^2}+\frac{7}{x^4}}$

$= \dfrac{1-0+0}{0+0} = \dfrac{1}{0} = \infty$

(19) $\lim_{x\to\pm\infty} \left[\dfrac{x^2}{x+1} - \dfrac{x^2}{x+3}\right]$

$= \lim_{x\to\pm\infty} \left[\dfrac{x^2(x+3)-x^2(x+1)}{(x+1)(x+3)}\right]$

$= \lim_{x\to\pm\infty} \dfrac{\cancel{x^3}+3x^2-\cancel{x^3}-x^2}{x^2+4x+3}$

$= \lim_{x\to\pm\infty} \dfrac{2x^2}{x^2+4x+3}$

$= \lim_{x\to\pm\infty} \dfrac{2\cancel{x^2}}{\cancel{x^2}(1+\frac{4}{x}+\frac{3}{x^2})}$

$= \dfrac{2}{1+0+0} = 2$ Ans

(20) M-1

$\lim_{x\to\infty} \left[x-\sqrt{x^2-a^2}\right]$

$= \lim_{x\to\infty} \left[x - x\left(1-\frac{a^2}{x^2}\right)^{\frac{1}{2}}\right]$

$= \lim_{x\to\infty} \left[x - x\left\{1+\frac{1}{2}\left(-\frac{a^2}{x^2}\right)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}\left(-\frac{a^2}{x^2}\right)^2+\dots\right\}\right]$

$= \lim_{x\to\infty}\left[\cancel{x}-\cancel{x}-\frac{a^2}{2x}-\frac{1}{8}\cdot\frac{a^4}{x^3}+\dots\right]$

$= 0-0+0 \dots = 0$

M-2 $\lim_{x\to\infty}\left[x-\sqrt{x^2-a^2}\right] \times \dfrac{\left[x+\sqrt{x^2-a^2}\right]}{\left[x+\sqrt{x^2-a^2}\right]}$

$= \lim_{x\to\infty} \dfrac{\cancel{x^2}-\cancel{x^2}+a^2}{x+\sqrt{x^2-a^2}}$

$= \dfrac{a^2}{\infty} = 0$ Ans

(21) $\lim_{x\to\infty} \dfrac{x^2+1}{x^{3/2}}$

$\div$ by x^2 N & D

$\lim_{x\to\infty} \dfrac{\left[1+\frac{1}{x^2}\right]}{x^{-1/2}}$

$\frac{3}{2}-2$

$\frac{3-4}{2} = -\frac{1}{2}$

$\lim_{x\to\infty} x^{\frac{1}{2}}\left[1+\frac{1}{x^2}\right]$

$= \infty(1+0) = \infty$ Ans.

OR Put $x = \frac{1}{y}$ $\lim x\to\infty$, $y\to 0$

(22) $\lim_{x\to\pm\infty} \dfrac{5x^3+2x^2-1}{x-4x^4}$

$\div$ by x^4 N & D

$\dfrac{\lim_{x\to\pm\infty}\left[\frac{5}{x}+\frac{2}{x^2}-\frac{1}{x^4}\right]}{\lim_{x\to\pm\infty}\left[\frac{1}{x^3}-4\right]}$

$= \dfrac{0+0-0}{0-4} = \dfrac{0}{-4} = 0$ Ans

2nd way Note Since the limit of quotient of Polynomial as $x\to\pm\infty$ is the same as the limit of the quotient of the highest Power terms.

$\lim_{x\to\pm\infty} \dfrac{5x^3}{-4x^4}$

$= \lim_{x\to\pm\infty} \dfrac{5}{-4x} = \dfrac{5}{\infty} = 0$

Note

$\lim_{x\to\pm\infty}(x^4-x^3-x)$

$= (\pm\infty)^4-(\pm\infty)^3-(\pm\infty)$

$= \infty$

OR $\lim_{x\to\pm\infty} x^4 = \infty$

Best Take higher Power Term

Note if

$\lim_{x\to\pm\infty}(4x^3-3x^2-1)$

$=\lim_{x\to\pm\infty} 4x^3$

$=\pm\infty$

(23) $\lim_{x\to\infty}\frac{3-2x^4}{1+x}$

$=\lim_{x\to\infty}\frac{-2x^4}{x}$

$=\lim_{x\to\infty}-2x^3$

$=-\infty$

(24) $\lim_{x\to-1}\frac{x^{1/3}+1}{x+1}$ ... (0/0) form

Put $x=-1+h$

$x\to-1,\ h\to0$

$\lim_{h\to0}\frac{(-1+h)^{1/3}+1}{\cancel{-1}+h+\cancel{1}}$

$\lim_{h\to0}\frac{\cancel{-1}+\frac{1}{3}h-\frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}h^2+\dots\cancel{+1}}{h}$

$\lim_{h\to0}\frac{\cancel{h}\left[\frac{1}{3}-\frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}h+\dots\right]}{\cancel{h}}$

$=\frac{1}{3}-0+0\dots=\frac{1}{3}$ Ans

(25) $\lim_{x\to3^-}\left(\frac{1}{x-3}-\frac{1}{|x-3|}\right)$

Put $x=3-h$

L.H.L.

$|x-3|$

$=-(x-3)$ $\quad x-3<0$

$\lim_{x\to3^-}\left(\frac{1}{x-3}-\frac{1}{-(x-3)}\right)$

$\lim_{x\to3^-}\left(\frac{2}{x-3}\right)$

$\lim_{h\to0}\left(\frac{2}{\cancel{3}-h-\cancel{3}}\right)=\lim_{h\to0}\left(\frac{2}{-h}\right)$

$=-\infty$ Ans

(26) $\lim_{x\to-2^-}\frac{x^2+2x-8}{x^2-4}$

$=\lim_{x\to-2^-}\frac{x^2+4x-2x-8}{(x-2)(x+2)}$

$=\lim_{x\to-2^-}\frac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$=\lim_{x\to-2^-}\frac{x+4}{x+2}$ $\quad$ Put $x=-2-h$, $h\to0,\ x\to-2$

$=\lim_{h\to0}\frac{-2-h+4}{-\cancel{2}-h+\cancel{2}}$

$=\lim_{h\to0}\frac{2-h}{-h}=\lim_{h\to0}\frac{2}{-h}+1=-\infty$

if $=\lim_{h\to0}\frac{h-2}{h}=\infty$ Ans (Correct)

(27) $\lim_{x\to1^-}\frac{\sqrt{1-x^2}}{1-x}$

$\lim_{x\to1^-}\frac{\sqrt{1-x}\sqrt{1+x}}{1-x}$

$\lim_{x\to1^-}\frac{\sqrt{1+x}}{\sqrt{1-x}}$

Put $x=1-h$ $\quad$ L.H. Limit

$\lim_{h\to0}\frac{\sqrt{1+1-h}}{\sqrt{\cancel{1}-\cancel{1}+h}}=\lim_{h\to0}\frac{\sqrt{2-h}}{\sqrt{h}}$

$=\frac{\sqrt{2-0}}{\sqrt{0}}=\infty$ Ans

2nd M Alt Put $x=1-h$

$\lim_{h\to0}\frac{\sqrt{1-(1-h)^2}}{1-(1-h)}$

$=\infty$ Ans

(28) $\lim_{x\to 1^+} \frac{x-1}{\sqrt{x^2-1}}$

$= \lim_{x\to 1^+} \frac{x-1}{\sqrt{(x-1)(x+1)}}$

$= \lim_{x\to 1^+} \frac{\sqrt{x-1}}{\sqrt{x+1}}$

Put $x = 1+h$ RHL

$= \lim_{h\to 0} \frac{\sqrt{\cancel{1}+h-\cancel{1}}}{\sqrt{1+h+1}}$

$= \lim_{h\to 0} \frac{\sqrt{h}}{\sqrt{2+h}} = \frac{0}{\sqrt{2+0}} = 0$ Ans

(29) $\lim_{x\to 2^-} \frac{\sqrt{4-x^2}}{\sqrt{6-5x+x^2}}$

$\lim_{x\to 2^-} \frac{\sqrt{(2-x)(2+x)}}{\sqrt{(2-x)(3-x)}}$

$\because 6-5x+x^2 = x^2-5x+6$
$= x^2-3x-2x+6$
$= x(x-3)-2(x-3)$
$= (x-3)(x-2)$
$= (2-x)(3-x)$

$= \lim_{x\to 2^-} \frac{\sqrt{2+x}}{\sqrt{3-x}}$

Put $x = 2-h$

$= \lim_{h\to 0} \frac{\sqrt{2+(2-h)}}{\sqrt{3-2+h}}$

$= \lim_{h\to 0} \frac{\sqrt{4-h}}{\sqrt{1+h}}$

$= \frac{\sqrt{4}}{\sqrt{1}} = 2$ Ans

OR Put $x = 2-h$

Hint $= \lim_{h\to 0} \frac{\sqrt{4-(2-h)^2}}{\sqrt{6-5(2-h)+(2-h)^2}} = 2$

(30) $\lim_{x\to\infty} \frac{x+\sin x}{x}$

$= \lim_{x\to\infty} \left[\frac{x}{x} + \frac{\sin x}{x}\right]$

$= 1 + \lim_{x\to\infty} \frac{\sin x}{x}$

$= 1 + \frac{l}{\infty}$ $\quad l \in [-1, 1]$

$= 1 + 0 = 1$ Ans

(31) $f(x) = \begin{cases} x^2+3 & \text{if } x \le 1 \\ x+1 & \text{if } x > 1 \end{cases}$

find $\lim_{x\to 1^+} f(x)$ & $\lim_{x\to 1^-} f(x)$

$f(1+0) = \lim_{x\to 1+0} f(x) = \lim_{x\to 1} (x+1)$
$= 1+1 = 2$

$f(1-0) = \lim_{x\to 1-0} f(x) = \lim_{x\to 1} (x^2+3)$
$= 1^2+3$
$= 4$

(32) $f(x) = \begin{cases} 3 & \text{if } x \le -2 \\ -\frac{1}{2}x^2 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \ge 2 \end{cases}$

find $\lim_{x\to \pm 2^+} f(x)$ and $\lim_{x\to \pm 2^-} f(x)$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} (3) = 3$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2} (-\frac{1}{2}x^2) = -\frac{1}{2}(2)^2 = -2$

$\lim_{x\to -2^-} f(x) = \lim_{x\to -2} 3 = 3$

$\lim_{x\to -2^+} f(x) = \lim_{x\to -2} (-\frac{1}{2}x^2)$
$= -\frac{1}{2}(-2)^2 = -2$

33 $f(x) = \begin{cases} x^2-1 & \text{if } x \le 2 \\ \sqrt{x+7} & \text{if } x > 2 \end{cases}$

find $f(x)$ as $x \to 2$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} \sqrt{x+7}$

$= \sqrt{2+7}$

$= \sqrt{9} = 3$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (x^2-1)$

$= 4-1$

$= 3$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = 3$

Then $\lim_{x \to 2} f(x) = 3$ Limit Exist

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(34)

$f(x) = \begin{cases} \cos x & \text{if } x \le 0 \\ 1-x & \text{if } x > 0 \end{cases}$

find $\lim_{x \to 0} f(x)$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0} (1-x)$

$= 1 - 0$

$= 1$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} \cos x$

$= \cos(0)$

$= 1$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = 1$

$\lim_{x \to 0} f(x) = 1$

Limit of fn. Exists.

(35)

$f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ x^3 & \text{if } x > 1 \end{cases}$

Show That

$\lim_{x \to 1} f(x) = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} x^3 = (1)^3 = 1$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} x^2 = (1)^2 = 1$

LHL = R.H.L = 1

$\lim_{x \to 1} f(x) = 1$

(36) v.Imp

$f(x) = \begin{cases} x+2 & \text{if } x \le -1 \\ ax^2 & \text{if } x > -1 \end{cases}$

find 'a', So that

$\lim_{x \to -1} f(x)$ Exists,

Because given that

$\lim_{x \to -1} f(x)$ exist

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$

$\lim_{x \to -1} (ax^2) = \lim_{x \to -1} (x+2)$

$a(-1)^2 = (-1+2)$

$a = 1$

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(37) $\lim_{x \to 3^-} \frac{3-x}{|x-3|}$

LHL. $x \to 3^-$

$|x-3| = -(x-3)$

$\lim_{x \to 3^-} \frac{3-x}{-(x-3)}$

$\lim_{x \to 3^-} \frac{-(x-3)}{-(x-3)} = \lim_{x \to 3^-} (1)$

$= 1$ Ans

(38) Evaluate

$$\lim_{x \to 0^-} \frac{x}{x-|x|}$$

for L.H. Limit

$x<0 \quad |x| = -x$

$$\lim_{x \to 0^-} \frac{x}{x-(-x)}$$

$$\lim_{x \to 0^-} \frac{x}{2x}$$

$$\lim_{x \to 0^-} \left(\frac{1}{2}\right) = \frac{1}{2} \text{ Ans}$$

(39)

$$\lim_{h \to 0} \frac{|-1+h|-1}{h}$$

$$\because |-(1-h)| \overset{\text{Mod.}}{=} (1-h)$$

$$\lim_{h \to 0} \frac{(1-h)-1}{h}$$

$$= -1 \text{ Ans}$$

Note

$$\lim_{h \to 0^-} \frac{|-1+h|-1}{h}$$

$$|-1+h| = -(-1+h)$$

$$= 1-h$$

$$= \lim_{h \to 0^-} \frac{1-h-1}{h}$$

$$= -1$$

if $\lim_{h \to 0^+} \frac{|-1+h|-1}{h}$

$$|-1+h| = +(-1+h)$$

$$\lim_{h \to 0^+} \frac{-1+h-1}{h}$$

$$\lim_{h \to 0^+} \frac{-2+h}{h}$$

$$= \frac{-2+0}{0} = -\infty$$