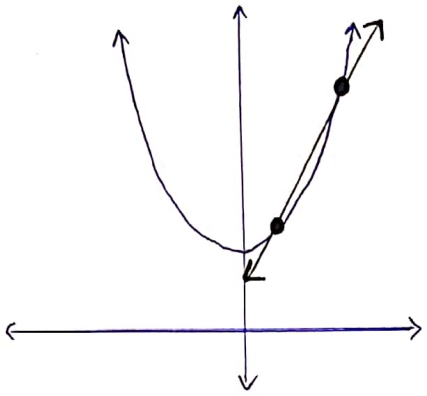
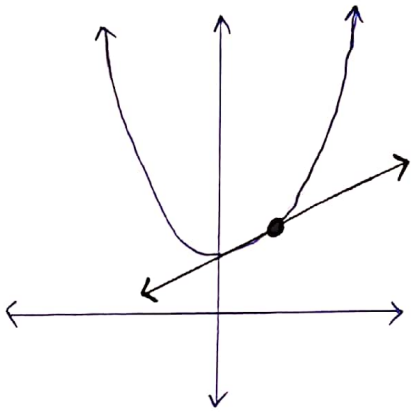
Secant Line



"Secant line is one that intersects two points in a line".

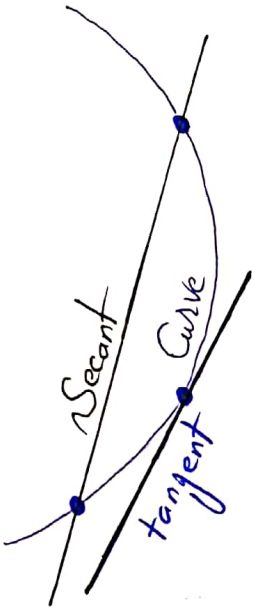
A secant line intersects two or more points on a curve.

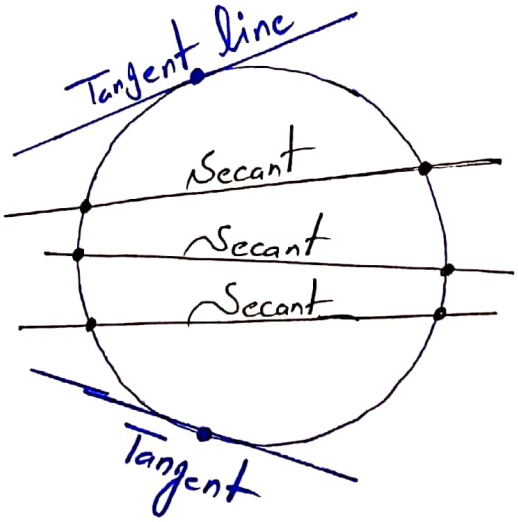
Tangent Line



"Tangent line is one that intersects exactly one point on the curve".

A tangent line just touches a curve at a point.





Secant Line

Slope of a Secant Line

$$m_{sec} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Eq. of Secant Line

$$y - f(x_0) = m_{sec}(x - x_0)$$

Example:

Find the equation of Secant line passing through the points $P(1,1)$ and $Q(3,9)$ on $f(x) = x^2$, and draw the graph.

Slope:

Here $x_0 = 1$, $x_1 = 3$

$f(x_0) = f(1) = (1)^2 = 1$

$f(x_1) = f(3) = (3)^2 = 9$

$$m_{sec} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$m_{sec} = \frac{9-1}{3-1} = \frac{8}{2} = 4$$

Eq. of Secant Line

The eq. of Secant line is

$$y - f(x_0) = m_{sec}(x - x_0)$$

$$y - 1 = 4(x - 1)$$

$$y - 1 = 4x - 4$$

$$y = 4x - 3$$

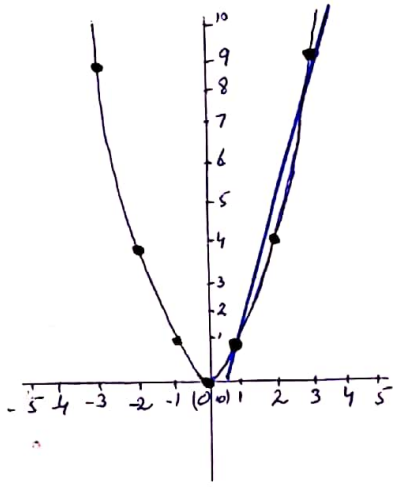
$f(x) = x^2$

-4	-3	-2	-1	0	1	2	3	4
16	9	4	+1	0	1	4	9	16

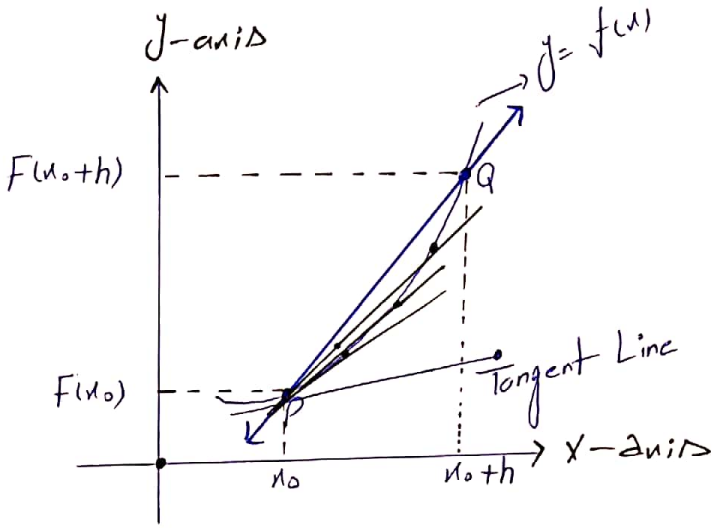
$y = 4x - 3$

x	1	3
y	1	9

Graph



Tangent Line



$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{f(x_0+h) - f(x_0)}{x_0+h-x_0}$$

$$m_{PQ} = \frac{f(x_0+h) - f(x_0)}{h}$$

$$m_{tan} = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} = \text{Slope of tangent line}$$

Example: Find the slope of tangent line to $f(x) = \sqrt{x}$ at any point.

$$f(x) = \sqrt{x}$$

$$f(x+h) = \sqrt{x+h}$$

$$\therefore m = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$m = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$\because\ a^2 - b^2 = (a-b)(a+b)$

$$\therefore\ m = \lim_{h\to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$m = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Example: Find the equation of tangent line of $F(x) = 3/x$ at Point $(3, 1)$.

$$m = \lim_{h\to 0} \frac{F(x_0+h) - F(x_0)}{h} \longrightarrow (1)$$

$$F(x_0) = \frac{3}{x_0}, \quad F(x_0+h) = \frac{3}{x_0+h}$$

Putting the above values in eq. (1), We get,

$$m = \lim_{h\to 0} \frac{\frac{3}{x_0+h} - \frac{3}{x_0}}{h} = \lim_{h\to 0} \frac{3x_0 - 3(x_0+h)}{x_0(x_0+h)} \times \frac{1}{h}$$

$$m = \lim_{h\to 0} \frac{3x_0 - 3x_0 - 3h}{hx_0(x_0+h)} = \lim_{h\to 0} \frac{-3h}{hx_0(x_0+h)}$$

$$m = \lim_{h\to 0} \frac{-3}{x_0(x_0+h)} = \frac{-3}{x_0^2}$$

$$m|_{P(3,1)} = \frac{-3}{(3)^2} = \frac{-3}{9} = -\frac{1}{3}$$

Eq. of tangent => Point slope formula = $y - y_1 = m(x - x_1)$

$m = -1/3, \quad x_1 = 3, \quad y_1 = 1$

$y - 1 = -\frac{1}{3}(x - 3) \Rightarrow y = -\frac{1}{3}x + 1 + 1$

Eq of tangent line = $y + \frac{1}{3}x = 2$