Elementry Operations: (Elementry row opetaions)

i) Add a constant times 1 row(equation) to another

ii) Interchange any two row's(equations)

iii) Multiply a row(equation) by a non-zero constant.

Elimination method:

In the elimination method you either add or subtract the equation's to get an equation in one variable.

When the coefficients of one variable are opposites you add the eqs to eliminate a variable and when the coefficient's of one variable are equal you subtract the eqs to eliminate a variable.

Example:

Solve the given system of equation by Using "Elimination Method".

$$x + 2y = 8$$
$$3x - 4y = 4$$

Sol:

$$x + 2y = 8 \quad \text{---} (i)$$
$$3x - 4y = 4 \quad \text{---} (ii)$$

$\times$ eq (i) by 2

$$2x + 4y = 16 \longrightarrow (iii)$$

Adding eq. (ii) and eq. (iii)

$$2x + \cancel{4y} = 16$$
$$3x - \cancel{4y} = 4$$
$$5x = 20$$

$\div$ both sides by 5, we get

$$x = 4$$

Put $x = 4$ in equation (i)

$$x + 2y = 8$$
$$4 + 2y = 8$$
$$2y = 8 - 4$$
$$2y = 4$$
$$y = 2$$

$$S.S = \{(x, y)\}$$
$$S.S = \{(4, 2)\}$$

(Unique solution)

Question:

Solve the given system of given eq. by Using Elimination method.

$$2x - 3y + 4z = -12 \quad \text{---} (i)$$

$$x - 2y + z = -5 \quad \text{----} (ii)$$

$$3x + y + 2z = 1 \quad \text{----} (iii)$$

Multiplying eq (ii) by (4)

$$4x - 8y + 4z = -20 \quad \text{----} (iv)$$

Multiplying eq (iii) by 2

$$2x - 4y + 2z = -10 \quad \text{----} (v)$$

Subtract eq (i) from (iv)

$$4x - 8y + \cancel{4z} = -20$$

$$\underline{\overset{-}{2x} \overset{+}{-} 3y \pm \cancel{4z} = \overset{+}{-}12}$$

$$2x - 5y = -8 \quad \text{----} (vi)$$

$$2x - 4y \cancel{+2z} = -10$$

$$\underline{\overset{-}{3x} \pm y \pm \cancel{2z} = \overset{-}{1}}$$

$$-x - 5y = -11 \longrightarrow (vii)$$

Subtract eq (vii) from eq. (vi)

$$2x - \cancel{5y} = -8$$

$$\underline{\overset{-}{+}x \overset{-}{+} \cancel{5y} = \overset{-}{+}11}$$

$$3x = 3$$

$$\boxed{x = 1}$$

Using eq (vi)

$$2(1) - 5y = -8$$

$$-5y = -8 - 2 \Rightarrow y = \frac{-10}{-5} = 2$$

$$\boxed{y = 2}$$

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Using eq.(i)

$2x - 3y + 4z = -12$

$2(1) - 3(2) + 4z = -12$

$2 - 6 + 4z = -12$

$-4 + 4z = -12$

$4z = -12 + 4 \Rightarrow 4z = -8$

$\boxed{z = -2}$

Question

Using Elimination method (No Solution)

$x + y = 5 \longrightarrow (i)$

$3x + 3y = 10 \longrightarrow (ii)$

$\times$ eq (i) by 3

$3x + 3y = 15 \longrightarrow (iii)$

Subtract eq (ii) from eq (iii)

$\cancel{3x} + \cancel{3y} = 15$

$\underline{-\cancel{3x} \pm \cancel{3y} = -10}$

$0 = 5$

$\boxed{0 \neq 5}$ No solution

Question: Find the solution by E.M.

$x + y - 2z = 5 \longrightarrow (i)$

$2x + 3y + 4z = 2 \longrightarrow (ii)$

'x' eq (i) by 2

$2x + 2y - 4z = 10 \longrightarrow (iii)$

Subtract eq (ii) from eq (iii)

$\cancel{2x} + 2y - 4z = 10$

$\underline{-\cancel{2x} \pm 3y \pm 4z = -2}$

$-y - 8z = 8$

$-y = 8 + 8z$

$y = -8 - 8z$

let $z = r$ (where 'r' is any real no)

$y = -8 - 8r$

Using ev (i)

$x + y - 2z = 5$

$x + (-8 - 8r) - 2r = 5$

$x - 8 - 8r - 2r = 5$

$x - 8 - 10r = 5$

$x = 13 + 10r$

Infinite solution.

Question (Unique Solution)

Solve by Using E.M.

$2x + 3y = 13 \longrightarrow (i)$

$x - 2y = 3 \longrightarrow (ii)$

$5x + 2y = 27 \longrightarrow (iii)$

X ev (ii) by 2

$2x - 4y = 6 \longrightarrow (iv)$

subtract ev (i) from (iv)

$$\begin{array}{r} \cancel{2x} - 4y = 6 \\ -\cancel{2x} \pm 3y = -13 \\ \hline -7y = -7 \end{array}$$

$\boxed{y = 1}$

Using ev. (ii)

$x - 2y = 3$

$x - 2(1) = 3$

$$\boxed{x = 5}$$

Using eq (iii)

$$5x + 2y = 27$$

$$5(5) + 2(1) = 27$$

$$27 = 27$$

Question: (No solution)

Find the solution by E.M.

$$x - 5y = 6 \longrightarrow (i)$$

$$3x + 2y = 1 \longrightarrow (ii)$$

$$5x + 2y = 2 \longrightarrow (iii)$$

$\times$ eq (i) by 3

$$3x - 15y = 18 \longrightarrow (iv)$$

subtract eq (iv) from (ii)

$$\begin{array}{r} 3x + 2y = 1 \\ \underline{\cancel{3x} \underset{+}{-} 15y = \underset{-}{1}8} \\ 17y = -17 \end{array}$$

$$\boxed{y = -1}$$

Using eq (i)

$$x - 5y = 6$$

$$x - 5(-1) = 6$$

$$x + 5 = 6$$

$$\boxed{x = 1}$$

Eq (iii) $\Rightarrow$

$$5x + 2y = 2$$

$$5(1) + 2(-1) = 2$$

$$3 \neq 2 \quad \text{(No solution)}$$

Substitution Method:

The substitution method is the algebraic method to solve simultaneous linear equations. As in this method, the value of one variable from one eq is substituted in the other eq. In this way, a pair of the linear eqs gets transformed into the linear eq with only one variable, which can then easily be solved.

Difference b/w Substitution and Elimination method:

The major difference b/w the S.M and E.M is that the substitution method is the process of replacing the variable with a value, where as, the elimination method is the process of removing the variable from the system of a linear eqs.

Substitution Method:

Procedure for S.M.

i) Solve one of the equations for one of the variable.

ii) Substitute the expression found in step 1 into the other equation.

iii) Now, solve for the remaining variable.

iv) Substitute the value from step 2 into the eq written in step 1. and solve for the remaining variable.

Question

Solve the following system of eqs by substitution.

$$y = x + 3 \quad \longrightarrow (i)$$

$$x + y = -5 \quad \longrightarrow (ii)$$

Substitute $x+3$ into eq (ii) and solve,

$$x + (x+3) = -5$$

$$2x + 3 = -5$$

$$x = -4$$

Substitute -4 into eq (i) and solve

$$y = x + 3$$

$$y = -4 + 3$$

$$y = -1$$

The answer: $(-4, -1)$

Question:

Solve the system by S.M.

$$x + y = 5 \longrightarrow (i)$$
$$y = 3 + x \longrightarrow (ii)$$

Put $y = 3+x$ in (i)

$$x + (3+x) = 5$$
$$2x + 3 = 5$$
$$2x = 2$$
$$x = 1$$

Substitute $x=1$ in eq (ii)

$$y = 3+1$$
$$y = 4$$

Check your solution.

$$(1, 4)$$

i) => $1+4 = 5$ ✓

$4 = 3+1$ ✓

Question

Solve the system by S.M.

$$3y + x = 7 \longrightarrow (i)$$
$$4x - 2y = 0 \longrightarrow (ii)$$

i) $x = 7-3y$

ii) => $4(-3y+7) - 2y = 0$

$$-12y + 28 - 2y = 0$$
$$-14y + 28 = 0$$
$$\boxed{y = 2}$$

Substitute $y = 2$ in eq (ii), we get

$$4x - 2(2) = 0$$
$$4x - 4 = 0$$
$$4x = 4 \Rightarrow \boxed{x = 1}$$

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Check your answer (Sol)

$(1, 2)$

$3(2) + 1 = 7$ ✓

$4(1) - 2(2) = 0$ ✓

Question:

Solve the system by S.M.

$3x + 2y = 11 \longrightarrow (i)$

$-x + y = 3 \longrightarrow (ii)$

$y = x + 3 \longrightarrow iii)$

Substitute $y = x + 3$ in (i)

$3x + 2(x+3) = 11$

$3x + 2x + 6 = 11$

$5x = 5$

$x = 1$

Subst. $x = 1$ in eq iii)

$y = 1 + 3$

$y = 4$

$S.S = \{(1, 4)\}$

Guass Elimination method

Guass elimination method is also known as row reduction, is an algorithm in linear algebra for solving a system of L.E.
It is usually understood as a sequence of operations performed on the corresponding matrix of coefficients.

Guass Jordan method

Guass Jordan elimination is an algorithm that can be used to solve system of l.E and to find the inverse of any invertible matrix.
It relies upon three elementary row operations one can use on a matrix: Swap the operations of two of the rows.
Multiply one of the rows by non-zero scalar.

Gauss Elimination Method

(Echelon Form)

$$x - y + 4z = 4$$
$$2x + 2y - z = 2$$
$$3x - 2y + 3z = -3$$

$$\begin{bmatrix} 1 & -1 & 4 \\ 2 & 2 & -1 \\ 3 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$$

$$A \qquad\qquad X \qquad\qquad B$$

$$[A \,\vdots\, B] = \left[\begin{array}{ccc|c} 1 & -1 & 4 & 4 \\ 2 & 2 & -1 & 2 \\ 3 & -2 & 3 & -3 \end{array}\right]$$

(Augmented matrix)

Using Row operation

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 4 \\ 2 & 2 & -1 & 2 \\ 3 & -2 & 3 & -3 \end{array}\right]$$

$$\begin{array}{cccc} 2 & 2 & -1 & 2 \\ -2 & \mp 2 & \pm 8 & -8 \\ \hline 0 & 4 & -9 & -6 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 4 \\ 2-2 & 2+2 & -1-8 & 2-8 \\ 3 & -2 & 3 & -3 \end{array}\right] R_3 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 4 \\ 0 & 4 & -9 & -6 \\ 3 & -2 & 3 & -3 \end{array}\right] R_3 - 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 4 \\ 0 & 4 & -9 & -6 \\ 0 & 1 & -9 & -15 \end{array}\right] \sim 4R_3$$

$$\begin{array}{cccc} 3 & -2 & 3 & -3 \\ -3 & \mp 3 & \mp 12 & \pm 12 \\ \hline 0 & 1 & -9 & -15 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 4 \\ 0 & 4 & -9 & -6 \\ 0 & 4 & -36 & -60 \end{array}\right] \sim R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 4 & 4 \\ 0 & 4 & -9 & -6 \\ 0 & 0 & -27 & -54 \end{array}\right]$$

$$\begin{array}{cccc} 0 & 4 & -36 & -60 \\ -0 & -4 & \mp 9 & \mp 6 \\ \hline 0 & 0 & -27 & -54 \end{array}$$

Backward Substitution

$0x + 0y - 27z = -54$

$-27z = -54$

$\boxed{z = 2}$

$0x + 4y - 9z = -6$

$4y - 9(2) = -6$

$4y - 18 = -6$

$4y = 12$

$\boxed{y = 3}$

$1x - 1.y + 4z = 4$

$x - 3 + 4(2) = 4$

$x - 3 + 8 = 4$

$\boxed{x = -1}$

Question

$$x + y + 2z + 3w = 13 \longrightarrow (i)$$
$$x - 2y + z + w = 8 \longrightarrow (ii)$$
$$3x + y + z - w = 1 \longrightarrow (iii)$$

Matrix form

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & -2 & 1 & 1 \\ 3 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 13 \\ 8 \\ 1 \end{bmatrix}$$

Augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 1 & -2 & 1 & 1 & 8 \\ 3 & 1 & 1 & -1 & 1 \end{array}\right] \sim R_2 - R_1$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -3 & -1 & -2 & -5 \\ 3 & 1 & 1 & -1 & 1 \end{array}\right] \sim R_3 - 3R_1$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -3 & -1 & -2 & -5 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] \begin{array}{l} \sim -2R_2 \\ \sim 3R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 6 & 2 & 4 & 10 \\ 0 & -6 & -15 & -30 & 114 \end{array}\right] \sim R_3 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 6 & 2 & 4 & 10 \\ 0 & 0 & -13 & -26 & 124 \end{array}\right]$$

Using backward substitution

$$0x + 0y - 13z - 26w = -104$$

$$-13z = -104 + 26w$$

$$z = 8 - 2w$$

Let $w = s$

$$z = 8 - 2s$$

$$0x + 6y + 2z + 4w = 10$$

$$6y + 2(8 - 2s) + 4s = 10$$

$$6y + 16 - \cancel{4s} + \cancel{4s} = 10$$

$$y = -1$$

Similarly

$$1.x + 1.y + 2z + 3w = 13$$

$$x - 1 + 2(8 - 2s) + 3s = 13$$

$$x - 1 + 16 - 4s + 3s = 13$$

$$x - s + 15 = 13$$

$$x = 13 - 15 + s$$

$$x = s - 2$$

infinite solution

Guass Jorden Method

(Reduced Echelon form)

$$x + y + 2z = -1$$
$$x - 2y + z = -5$$
$$3x + y + z = 3$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right] \begin{array}{l} \\ \sim R_2 - R_1 \\ \sim R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -3 & -1 & 4 \\ 0 & -2 & -5 & 6 \end{array}\right] \begin{array}{l} \\ \sim -2R_2 \\ \sim 3R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 6 & 2 & 8 \\ 0 & 6 & -15 & 18 \end{array}\right] \sim R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 6 & +2 & 8 \\ 0 & 0 & -13 & 26 \end{array}\right] \sim 6R_1$$

$$\left[\begin{array}{ccc|c} 6 & 6 & 12 & -6 \\ 0 & 6 & 2 & 8 \\ 0 & 0 & -13 & 26 \end{array}\right] \sim R_1 - R_2$$

$$\left[\begin{array}{ccc|c} 6 & 0 & 10 & -14 \\ 0 & 6 & 2 & 8 \\ 0 & 0 & -13 & 26 \end{array}\right] \sim R_3/-13$$

$$\left[\begin{array}{ccc|c} 6 & 0 & 10 & -14 \\ 0 & 6 & 2 & 8 \\ 0 & 0 & 1 & -2 \end{array}\right] \sim R_2 - 2R_3$$

$$\left[\begin{array}{ccc|c} 6 & 0 & 10 & -14 \\ 0 & 6 & 0 & -4 \\ 0 & 0 & 1 & -2 \end{array}\right] \sim R_1 - 10R_3$$

$$\left[\begin{array}{ccc|c} 6 & 0 & 0 & 6 \\ 0 & 6 & 0 & -4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$\Rightarrow \quad 6x = 6$

$\quad\quad x = 1$

$\Rightarrow \quad 6y = -4$

$\quad\quad y = -2/3$

$\Rightarrow \quad z = -2$

Inverse of a matrix by Guass Jordan Elimination method:

Example:

Find the inverse of given matrix by Guass Jordan Elimination.

$$A = \begin{bmatrix} 1 & 1 & +3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

Solution:

$$[A \;\vdots\; I]$$

A: make it I by guass Jordan elimination

I: it will become inverse

$$A = A \cdot I$$
$$I = A\, A^{-1}$$

$$[A \,\vdots\, I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \sim R_2 - R_1 \\ \sim R_3 + 2R_1 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1/2 \end{array}\right] \begin{array}{l} \\ \sim R_2/2 \\ \sim R_3/2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3/2 & -1/2 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & -2 & 1/2 & 1/2 & 1/2 \end{array}\right] \begin{array}{l} \sim R_1 - R_2 \\ \sim R_3 + R_2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & -2 & 1/2 & 1/2 & 1/2 \end{array}\right] \begin{array}{l} \sim R_1 + 3R_3 \\ \sim R_2 - \frac{3}{2} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right] \sim R_3/-2$$

Hence

$$A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

Now check whether the answer is right or wrong.

We apply this relation

$$A \cdot A^{-1} = I$$

Take AA^{-1}

$$AA^{-1} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ 2 & -4 & -4 \end{bmatrix} \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

$$\begin{bmatrix} 3-5/4-3/4 & 1-1/4-3/4 & 3/2-3/4-3/4 \\ 3-15/4+3/4 & 1-3/4+3/4 & 3/2-9/4+3/4 \\ -6+20/4+1 & -2+1+1 & -6/2+12/4+1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = R.H.S$$

LU Decomposition Method
(Factorization method)

Method:-

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$AX = B$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = LU$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$A = LU$$
$$LUX = B$$

Let $UX = Y$

$$LY = B$$

Incomputing element's of L and U.

First row of U i-e u_{11}, u_{12}, u_{13}

First Column of L i-e l_{21}, l_{31}

Second row of U i-e u_{22}, u_{23}

Second Column of L i-e l_{32}

Third row of U i-e u_{33}

Factorization method / LU Decomposition method.

Example:-

Solve the following system of equations by LU method.

$$x + 5y + z = 14$$
$$2x + y + 3z = 13$$
$$3x + y + 4z = 17$$

Solution:

$$AX = B$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 7 \end{bmatrix} = LU$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

$$u_{11} = 1, \quad u_{12} = 5, \quad u_{13} = 1$$

$l_{21}u_{11} = 2, \quad l_{31}u_{11} = 3, \quad l_{21}u_{12} + u_{22} = 1$

$l_{21}(1) = 2, \quad l_{31}(1) = 3, \quad (2)(5) + u_{22} = 1$

$l_{21} = 2, \quad l_{31} = 3, \quad u_{22} = -9$

$l_{21}u_{13} + u_{23} = 3$

$(2)(1) + u_{23} = 3$

$u_{23} = 1$

$l_{31}u_{12} + l_{32}u_{22} = 1$

$(3)(5) + l_{32}(-9) = 1$

$l_{32}(-9) = -14$

$l_3 = 14/9$

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 4$

$(3)(1) + (14/9)(1) + u_{33} = 4$

$u_{33} = 4 - 3 - 14/9$

$u_{33} = 1 - 14/9$

$u_{33} = -5/9$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix}$$

$AX = B$

$LUX = B$

Let $UX = Y$

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

$y_1 = 14$

$2y_1 + y_2 = 13$,

$2(14) + y_2 = 13$,

$28 + y_2 = 13$,

$y_2 = -15$,

$3y_1 + \frac{14}{9} y_2 + y_3 = 17$

$3(14) + \frac{14}{9}(-15) + y_3 = 17$

$y_3 = -5/3$

$$Ux = Y$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -15 \\ -5/3 \end{bmatrix}$$

$$x + 5y + z = 14$$
$$-9y + z = -15$$
$$-\frac{5}{9} z = -5/3$$

$$z = 3$$
$$-9y + z = -15$$
$$-9y + 3 = -15$$
$$-9y = -18$$
$$y = 2$$

$$x + 5y + z = 14$$
$$x + 10 + 3 = 14$$
$$x = 1$$

Hence

$$x = 1$$
$$y = 2$$
$$z = 3$$

Check ans:

$$x + 5y + z = 14$$
$$1 + 10 + 3 = 14$$
$$14 = 14 \quad \text{Satisfied}$$