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Class: B.S.IT. 3rd (S)
Math-203 Linear Algebra

Curve in $\mathbb{R}^n$:-

A curve in $\mathbb{R}^n$ is a continuous Function

$$\alpha : [a, b] \longrightarrow \mathbb{R}^n$$

such that For each $t \in [a, b]$, there is a point in $\mathbb{R}^n$ that is

$$\alpha(t) = (\alpha_1(t), \alpha_2(t), ---, \alpha_n(t))$$

Plane Curve:

$$\alpha : [a, b] \longrightarrow \mathbb{R}^2$$

$$\alpha(t) = (\alpha_1(t), \alpha_2(t))$$

Example:

$$\alpha(t) = (a\cos t, a\sin t)$$

Space Curve:

$$\alpha : [a, b] \longrightarrow \mathbb{R}^3$$

$$\alpha(t) = (\alpha_1(t), \alpha_2(t), \alpha_3(t))$$

Example:

$$\alpha(t) = (a\cos t, a\sin t, bt)$$

Differentiable Curve:

A curve $\alpha(t) \in \mathbb{R}^n$ is differentiable if $\alpha_1(t), \alpha_2(t), \alpha_3(t), \_\_, \alpha_n(t)$ are differentiable function of t.

Example:

$$\alpha(t) = (6t, 3t^2, 3t^3)$$

$$\bar{\alpha}(t) = 6t\hat{i} + 3t^2\hat{j} + 3t^3\hat{k})$$

$$\bar{\alpha}'(t) = 6\hat{i} + 6t\hat{j} + 9t^2\hat{k}$$

Tangent to Curve:

If curve $\alpha(t) \in \mathbb{R}^n$ is differentiable, then $\alpha'(t) = \bar{T} = (\alpha_1'(t), \alpha_2'(t), \ldots, \alpha_n'(t))$ is called tangent to that differentiable curve $\alpha(t)$.

Example:

$$\bar{\alpha}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$$

$$\bar{\alpha}'(t) = \hat{i} + 2t\hat{j} + 3t^2\hat{k}$$

Unit tangent to Curve:

- If $\alpha(t) \in \mathbb{R}^n$ is differentiable curve then unit tangent curve is defined as.

$$\hat{T} = \frac{\alpha'(t)}{\|\alpha'(t)\|} = \frac{\bar{T}}{\|T\|}$$

Example:

$$\alpha(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}, \quad t=2$$

$$\bar{T} = \alpha'(t) = \hat{i} + 2t\hat{j} + 3t^2\hat{k}$$

Put $t=2$, we get

$$\bar{T} = i + 4\hat{j} + 12\hat{k}$$

$$\|\bar{T}\| = \sqrt{(1)^2 + (4)^2 + (12)^2} = \sqrt{1+16+144}$$

$$= \sqrt{161}$$

So,

$$\hat{T} = \frac{T}{\|T\|} = \frac{\hat{i} + 4\hat{j} + 12\hat{k}}{\sqrt{161}}$$

Ans

Line:

A set of points that extends infinitely in two directions with one dimension is called line

Parametric equation of line in $\mathbb{R}^3$:

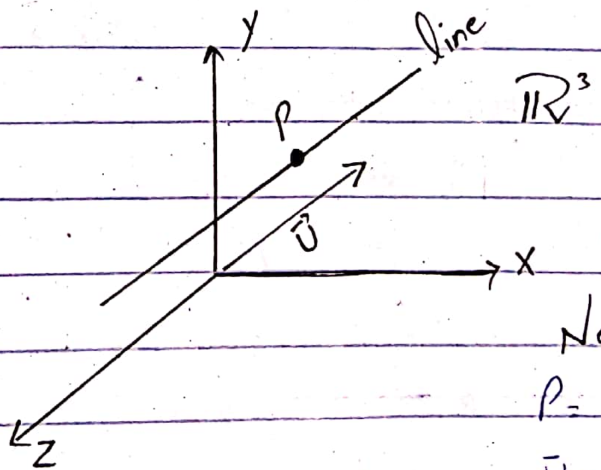
Parametric equation of line passing from point $P(a_1, a_2, a_3)$ and in direction of vector $\bar{u} = (b_1, b_2, b_3)$ is

$$X = P + t\bar{u}$$

$$x_1 = a_1 + t b_1$$

$$x_2 = a_2 + t b_2$$

$$x_3 = a_3 + t b_3$$



Note that:

$P = (a_1, a_2, ---- a_n)$

$\bar{U} = (b_1, b_2, \_\_, b_n)$

$X = P + t\bar{U}$

$\mathbb{R}^n$

Example:

Equation of line = ?

$P(1, 2, -3)$, $\bar{U} = (4, 5, -7)$

Sol:

$P(a_1, a_2, a_3)$

$\bar{U}(b_1, b_2, b_3)$

$$X = P + t\bar{U}$$

$$x_1 = a_1 + t b_1$$

$$x_2 = a_2 + t b_2$$

$$x_3 = a_3 + t b_3$$

$$\Rightarrow x_1 = 1 + 4t$$

$$\Rightarrow x_2 = 2 + 5t$$

$$\Rightarrow x_3 = -3 - 7t$$

<u>or</u>

$$X = P + t\bar{v}$$

$$X - P = t\bar{v}$$

$$(x_1, x_2, x_3) - (1, 2, -3) = t(4, 5, -7)$$

$$(x_1 - 1, x_2 - 2, x_3 + 3) = (4t, 5t, -7t)$$

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Matrix:

Matrix is a rectangular array of numbers enclosed by a pair of square bracket

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \rightarrow \text{Row}$$

↓ column

General Representation:

$A = [a_{ij}]$ i = rows ; j = columns

for m-rows and n-columns

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

order of A = m by n

rows by column

Example: 2 by 3 = 2×3

Types Of Matrices

1) Row matrix:

$A = [1 \;\; 2 \;\; 3]$

$A = [2]$

2) Column Matrix:-

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} , \quad A = [2]$$

3) Equal Matrix:

Two matrices A and B are said

to be equal if

→ A and B has same order

→ Their corresponding entries are same.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

so

$$A = B$$

$$A \neq C \neq B$$

4) Null / Zero matrix:

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $A = [0]$

5) Square matrix:

Rows = Columns

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $R = 2$, $C = 2$

6) Rectangular matrix:

Rows ≠ Columns

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $R = 2$, $C = 3$

7) Transpose of matrix:

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$, $A^t = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

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8) Symmetric matrix:

$A^t = A$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \quad A^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 6 \end{bmatrix}$$

so $= A^t = A$

9) Skew-Symmetric matrix:

$A^t = -A$

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & -3 \\ 2 & 3 & 0 \end{bmatrix} \Rightarrow A^t = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix} \Rightarrow A^t = -\begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix}$$

$= -A$

So, $A^t = -A$

10) Hermitian matrix:

A square matrix with complex entries is called Hermitian matrix if $A = (\bar{A})^t$. It is denoted by $A^H = A$

$$A = \begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix} \Rightarrow \bar{A} = \begin{bmatrix} 1 & 1+i \\ 1-i & 2 \end{bmatrix} \Rightarrow (\bar{A})^t = \begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix} = A$$

11) Skew-Hermitian matrix:

$A = -(\bar{A})^t = -A^H$ or $A^H = -A$

$$A = \begin{bmatrix} 0 & 2-3i \\ -2-3i & 0 \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} 0 & 2+3i \\ -2+3i & 0 \end{bmatrix}$$

$$(\bar{A})^t = \begin{bmatrix} 0 & -2+3i \\ 2+3i & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2-3i \\ -2-3i & 0 \end{bmatrix}$$

$A^H = -A$

Remember:

If square matrix A consist of real entries then $A = \bar{A}$ which implies $A^H = A^t \Rightarrow \bar{z} = z$

12) **Diagonal matrix.**

$A = [a_{ij}]$ be square matrix.
If $a_{ij} = 0$ for $i \neq j$ and $a_{ij} \neq 0$ for $i = j$ then A is called diagonal matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{ or } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

13) **Scaler matrix:**

$A = [a_{ij}]$ be square matrix. If $a_{ij} = 0$ For $i \neq j$ and $a_{ij} = k$ For $i = j$, then A is called scaler matrix, where 'k' is any scaler.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

14) **Unit / Identity matrix:**

$A = [a_{ij}]$ be square matrix, $a_{ij} = 0$ For $i \neq j$ and $a_{ij} = 1$ for $i = j$, then A is called unit matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

15) Trace of matrix:

Let $A = [a_{ij}]$ square matrix. Sum of diagonal element of $A = [a_{ij}]$.

$$tr(A) = a_{11} + a_{22} + a_{33} + \dots + a_{nn}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad tr(A) = 1 + 5 + 9 = 15$$

16) Upper triangular matrix:

A square matrix $A = [a_{ij}]$ is upper triangular matrix or simply triangular if $a_{ij} = 0$ for $i > j$

that is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 6 \end{bmatrix}$$

17) Lower triangular matrix:

A square matrix $A = [a_{ij}]$ is lower triangular if $a_{ij} = 0$ for $i < j$

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Remember:

Upper Unit triangular

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

Lower Unit triangular

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$$

Addition of matrix:

Suppose A and B are two matrices then addition of A and B is possible if both A and B have same order.

Example:

$$A = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Solution:

$$A+B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 3+6 \\ 4+2 & 6+0 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 6 & 6 \end{bmatrix}$$

A+C / B+C is not possible

$$A+C = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 3+2 & ?+3 \\ 4+4 & 6+5 & ?+6 \end{bmatrix} \neq \text{not possible}$$

Remember:

1) $A+B = B+A$ that is addition of matrices A and B is commutative
2) Addition of two matrices with different order is not defined

Multiplication of two matrices:

Two matrices A and B are said to be suitable for product AB if

No. of columns of A = No. of rows of B

Example: Check whether multP. is Possible (AB)

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

No of columns A = 2 , No of rows B = 2 So AB is Pos

$A = \begin{bmatrix} 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

No of columns A = 2 , No of rows B = 2 So AB is Possible

$A = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \end{bmatrix}$

No of columns of A = 1 , No of rows B = 1 So AB is Possible

$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

No of columns A = 1 , No of rows B = 2 AB is not Possible

How to multiply two matrices:

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Example:

$A = \begin{bmatrix} 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+2 \end{bmatrix} = \begin{bmatrix} 2 \end{bmatrix}$

$A = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}$

Order of Product of A & B matrices

Let order of A = $m \times n$

order of B = $n \times p$

$\Rightarrow$ order of AB = $m \times p$

$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $A = [A_{ij}]_{2\times1}$

$B = \begin{bmatrix} 4 & 6 \end{bmatrix}$ $B = [b_{ij}]_{1\times2}$

$AB = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 4 & 6 \end{bmatrix}$

$$AB = \begin{bmatrix} 4 & 6 \\ 8 & 12 \end{bmatrix}_{2\times2}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$$

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 1 \end{bmatrix}_{3\times2}$$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1+6+0 & 2+8+3 \\ 4+15+0 & 8+20+6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 & 13 \\ 19 & 34 \end{bmatrix}_{2\times2}$$

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18) ## Invertible matrix:

A square matrix A is said to be invertible if there exist a matrix B such that

$$AB = BA = I$$

B is called inverse of A and denoted as $B = A^{-1}$

Example:

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

How to find Inverse of 2x2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Let

$$A^{-1} = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}$$ s.that

$$AA^{-1} = I$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} ax_1 + by_1 & ax_2 + by_2 \\ cx_1 + dy_1 & cx_2 + dy_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $ax_1 + by_1 = 1$ $\qquad$ $ax_2 + by_2 = 0$

$cx_1 + dy_1 = 0$ $\qquad$ $cx_2 + dy_2 = 1$

$x_1 = -\frac{d y_1}{c}$

$-\frac{ad}{c} y_1 + b y_1 = 1$

$\left(-\frac{ad}{c} + b\right) y_1 = 1$

$\left(\frac{-ad + bc}{c}\right) y_1 = 1$

$y_1 = \frac{c}{-ad + bc}$

$y_1 = \frac{-c}{-bc + ad} = \frac{-c}{|A|}$

$x_1 = -\frac{d}{c}\left[\frac{-c}{|A|}\right] = \frac{d}{|A|}$

Similarly

$x_2 = \frac{-b}{|A|}, \quad y_2 = \frac{a}{|A|}$

So

$$A^{-1} = \begin{bmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example:

$A = \begin{bmatrix} 2 & 3 \\ 6 & 10 \end{bmatrix}$, Find A^{-1}

As $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $\quad a = 2, b = 3$, $c = 6, d = 10$

$$= \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{20 - 18}\begin{bmatrix} 10 & -3 \\ -6 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 10 & -3 \\ -6 & 2 \end{bmatrix} = \begin{bmatrix} 5 & -\frac{3}{2} \\ -3 & 1 \end{bmatrix}$$

Minor of an element:

Let $A = [a_{ij}]_{3\times3}$ then minor of an element a_{ij} is the determinant of $(3-1)\times(3-1)$ matrix formed by deleting ith rows and jth column.

Minor of element is denoted by M_{ij}

Example:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix} \quad M_{12} = ? \quad M_{22} = ? \quad M_{32} = ?$$

$$M_{12} = \begin{vmatrix} -2 & 1 \\ 4 & 2 \end{vmatrix} = (-2)(2) - 4(1) = -8$$

$$M_{22} = \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} = (2)(1) - (3)(4) = -10$$

$$M_{32} = \begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = (1)(1) - (3)(-2) = 7$$

Cofactor of an element

cofactor of an element a_{ij} is defined as "singed minor of element a_{ij}". It is denoted by A_{ij}

$$A_{ij} \quad (-1)^{i+j} M_{ij}$$

Example: $A_{12} = ?$ $A_{32} = ?$

$$A_{12} = (-1)^{1+2} M_{12} = (-1)^{3}(-8) = 8$$

$$A_{32} = (-1)^{3+2} M_{32} \quad (-1)^{5}(7) = -7$$

Inverse of 3x3 matrix

Let $A = [a_{ij}]_{3\times3}$, then inverse of A is denoted by A^{-1} and defined as

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A \; ; \text{ where } A \text{ is non-singular}$$

Determinant of 3x3 matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Then det / expension of A using first row is denoted by $|A|$ and defined as

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

Adjoint of 3x3 matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

then adj of A is the "transpose of matrix of cofactor of A"

$$\text{adj of } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

Example: Find adj A and A^{-1}

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

Sol

$A_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix}$ $= (-1)^2 (2+1)$ $= 3$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix}$ $= (-1)^3 (0+2)$ $= -2$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix}$ $= (-1)^4 (0-4)$ $= -4$

Similarly

$A_{12} = 1$ $\quad A_{22} = -1$ $\quad A_{32} = -1$

$A_{13} = -2$ $\quad A_{23} = 1$ $\quad A_{33} = 2$

i $|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= (1)(3) + 0(1) + 2(-2)$

$= -1 \neq 0$

ii

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} 3 & 1 & -2 \\ -2 & -1 & 1 \\ -4 & -1 & 2 \end{bmatrix}^t$$

$$= \begin{bmatrix} 3 & -2 & -4 \\ 1 & -1 & -1 \\ -2 & 1 & 2 \end{bmatrix}$$

iii $A^{-1} = \frac{1}{|A|} \text{adj} A$

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 3 & 2 & -4 \\ 1 & -1 & -1 \\ -2 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 4 \\ -1 & 1 & 1 \\ 2 & -1 & -2 \end{bmatrix}$$