- System of linear Equation

a) ELIMINATION METHOD

b) SUBSTITUTION METHOD

b) CRAMER'S RULE

ELIMINATION METHOD

1

$3x_1 - x_2 = 1 \rightarrow 1$

$x_1 + x_2 = 3 \rightarrow 2$

$\underbrace{x_1, x_2}_{\text{unknown}} = ?$

eq(1) + eq(2)

$$3x_1 - \cancel{x_2} = 1$$
$$+\; x_1 + \cancel{x_2} = 3$$
$$4x_1 = 4$$
$$x_1 = \frac{4}{4}$$
$$\boxed{x_1 = 1}$$

eq(2)

$$x_1 + x_2 = 3$$
$$1 + x_2 = 3$$
$$x_2 = 3 - 1$$
$$\boxed{x_2 = 2}$$

This system of eq has solution $\{x_1, x_2\} = \{1, 2\}$ Ans

2

$x_1 + 2x_2 = 4 \rightarrow 1$

$2x_1 + 4x_2 = 12 \rightarrow 2$

2 x eq(1) - eq(2)

$$\cancel{2x_1} + \cancel{4x_2} = 8$$
$$\pm\; \cancel{2x_1} \pm \cancel{4x_2} = \pm 12$$
$$0 = -4$$

has not solution

Q

Exp:- $2x_1 - x_2 = 1 \rightarrow (1)$
$x_1 + x_2 = 3 \rightarrow (2)$

eq (2)
$x_1 + x_2 = 3$
$$\boxed{x_1 = 3 - x_2}$$
or
$$\boxed{x_2 = 3 - x_1}$$

eq (1) $2x_1 - x_2 = 1$
put $x_1 = 3 - x_2$
$2(3 - x_2) - x_2 = 1$
$6 - 2x_2 - x_2 = 1$
$6 - 3x_2 = 1$
$-3x_2 = 1 - 6$
$-3x_2 = -5$
$$\boxed{x_2 = 5/3}$$

$\Rightarrow 2x_1 - 5/3 = 1$
$2x_1 = 1 + 5/3$
$2x_1 = \frac{3+5}{3} = 8/3$
$x_1 = \frac{\cancel{8}4}{3 \times \cancel{2}} = 4/3$
$x_1 = 4/3$
$$\boxed{x_1 = 4/3, \; x_2 = 5/3}$$

(3) Cramer's Rule :-

$Ax = B$

$x = \frac{D_x}{D}$
$y = \frac{D_y}{D}$

$x_1 = \frac{D_{x_1}}{D}$
$x_2 = \frac{D_{x_2}}{D}$

$D = |A|$

$D_2 =$

solve the system

$x = ?$

$y = ?$

$2x + 3y = 8 \rightarrow (1)$

$x - 2y = -3 \rightarrow (2)$

Eq :- $A x = B$

$$\begin{array}{cc} x & y \\ \end{array}$$

$$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$$

Check :-

$2x + 3y = 8$

$2(1) + 3(2) = 8$

$2 + 6 = 8$

$8 = 8$

L.H.S = R.H.S

$$D = |A| = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = 2(-2) - 3(1)$$

$$= -4 - 3$$

$$= -7 \neq 0$$

$$\boxed{D = -7}$$

$$\cancel{\frac{Dx}{D} \rightarrow \infty}$$

$$B = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$$

$$Dx = \begin{vmatrix} 8 & 3 \\ -3 & -2 \end{vmatrix} = 8(-2) - 3(-3)$$

$$= -16 + 9$$

$$= -7$$

$$\boxed{Dx = -7}$$

$$Dy = \begin{vmatrix} 2 & 8 \\ 1 & -3 \end{vmatrix} = 2(-3) - 8(1)$$

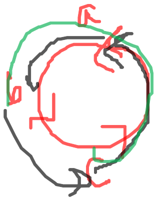
$$= -6 - 8$$

$$= -14$$

$$\boxed{x = \frac{Dx}{D}} = \frac{\cancel{-7}}{\cancel{-7}} = 1$$

$$\boxed{y = \frac{Dy}{D}} = \frac{\cancel{-14}\;2}{\cancel{-7}} = 2$$

$$\boxed{x = 1}, \boxed{y = 2}$$



$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

DETERMINANTS OF THIRD ORDER

consisting of nine numbers arranged in three rows and three columns is called a determinant of third order. By definition, the value of this determinant is given by

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix}$$

$$a_1 b_2 c_3 + b_1 c_2 a_3 + \boxed{c_1 a_2 b_3} - c_1 b_2 a_3 - a_1 c_2 b_3 - b_1 a_2 c_3$$

is called the ~~expansion of the determinant~~

Exp:-

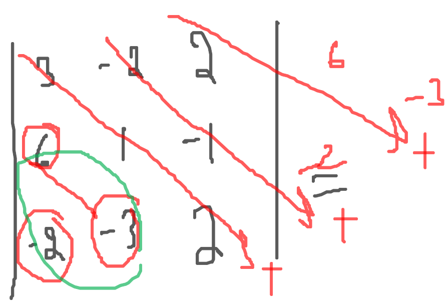
$$\begin{array}{ccc} a & b & c \\ 1 & 1 & 1 \end{array}$$

$$\begin{vmatrix} 3 & -2 & 2 \\ 6 & 1 & -1 \\ -2 & -3 & 2 \end{vmatrix}$$

$$|A| = a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3 - c_1 b_2 a_3 - a_1 c_2 b_3 - b_1 a_2 c_3$$

$$= (3)(1)(2) + (-2)(-1)(-2) + (2)(6)(-3) - 2(1)(-2) - (3)(-1)(-3) - (-2)(6)(2)$$

$$= -15$$



$$(3)(1)(2) + (-2)(-1)(-2) + (2)(6)(-3)$$

$$-(2)(1)(-2) - (3)(-1)(-3) - (2)(6)(-2)$$