OPERATIONS WITH MATRICES

① Addition

If Matrix A an B have same size or order $m = n$

$A + B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}] = [c_{ij}]$

$m \times n \longrightarrow$ Column
Row

Example 01: find the sum $\begin{bmatrix} 2 & 3 & 4 \\ 6 & 0 & -1 \end{bmatrix}$ & $\begin{bmatrix} 0 & 3 & -2 \\ -1 & 1 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 0 & -1 \end{bmatrix}_{2\times3}$; $B = \begin{bmatrix} 0 & 3 & -2 \\ -1 & 1 & 2 \end{bmatrix}_{2\times3}$

$A + B = \begin{bmatrix} 2+0 & 3+3 & 4-2 \\ 6-1 & 0+1 & -1+2 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 2 \\ 5 & 1 & 1 \end{bmatrix} = C$

② Subtraction

If Matrix A an B have same size or order

$A - B = [a_{ij}] - [b_{ij}] = [a_{ij} - b_{ij}] = [c_{ij}]$

Example 02:

$A = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 3 & -2 \\ -1 & 1 & 2 \end{bmatrix}$

$A - B = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 3 & -2 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2-0 & 3-3 & 4-(-2) \\ 6-(-1) & 0-1 & -1-2 \end{bmatrix}$

$A - B = \begin{bmatrix} 2 & 0 & 6 \\ 7 & -1 & -3 \end{bmatrix}$

$- , + \quad =$

$- , - \quad = +$

$+ , + \quad = +$
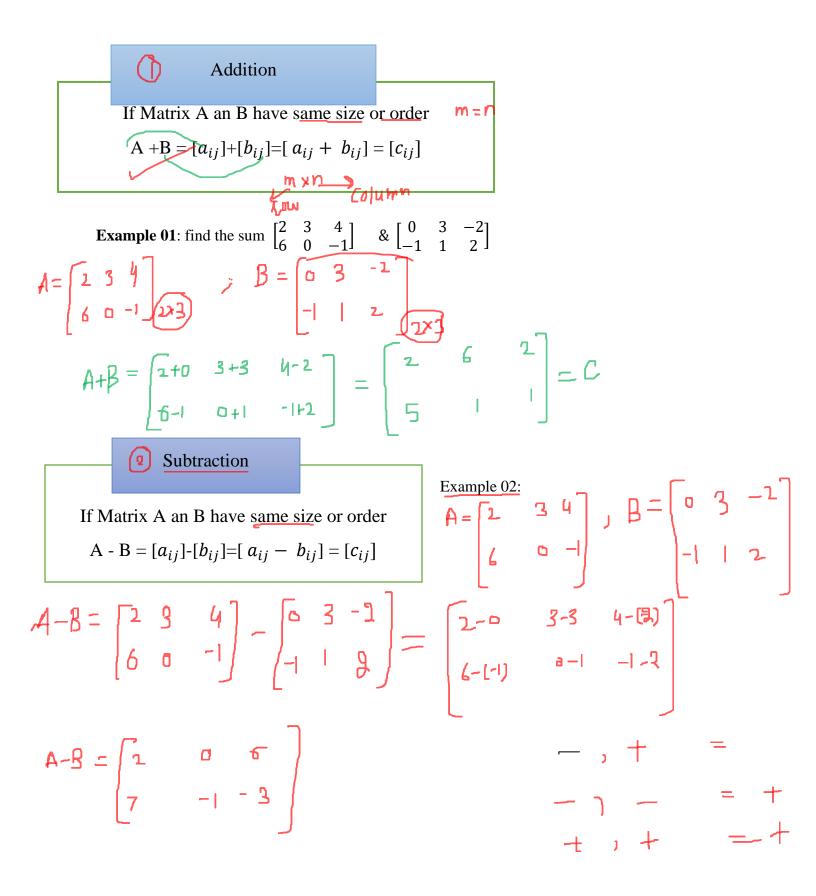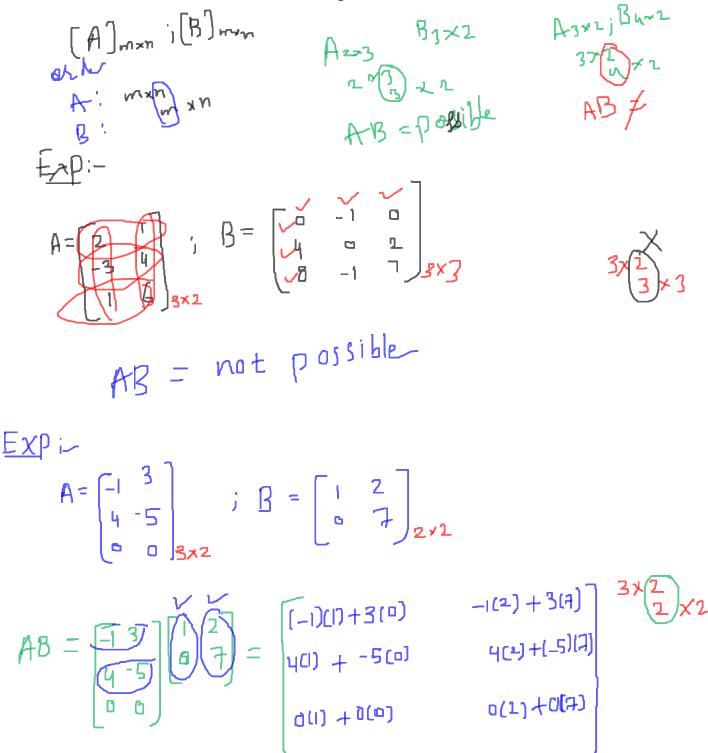
<u>Multiplication of two matrixes</u>:

If A and B are two matrices, the product of AB is defined as

Number of columns of A is equal to number of row B

$[A]_{m \times n} \; ; [B]_{m \times n}$

order

$A: \quad m \times \boxed{n} \times n$

$B:$

Exp:-

$A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}_{3 \times 2}$; $B = \begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix}_{3 \times 3}$

$B_{3 \times 2}$

$A_{2 \times 3}$

$2 \times \boxed{3} \times 2$

$AB = possible$

$A_{3 \times 2} ; B_{4 \times 2}$

$3 \times \boxed{2} \times 2$

$AB \neq$

$3 \times \boxed{2}$ ✗
$\boxed{3} \times 3$

AB = not possible

Exp:-

$A = \begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 0 \end{bmatrix}_{3 \times 2}$; $B = \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}_{2 \times 2}$

$AB = \begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} (-1)(1)+3(0) & -1(2)+3(7) \\ 4(1) + -5(0) & 4(2)+(-5)(7) \\ 0(1) + 0(0) & 0(1)+0(7) \end{bmatrix}$

$3 \times \boxed{2}$
$\boxed{2} \times 2$

$$AB = \begin{bmatrix} -1+0 & -2+21 \\ 4+0 & 8-35 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} -1 & 19 \\ 4 & -27 \\ 0 & 0 \end{bmatrix}$$

Exp :

$$A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix}_{2\times 3} \quad ; \quad B = \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix}_{2\times 3}$$

a) A+B   b) A-B   c) 3A   d) 5A - 2B

$$A+B = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 2+2 & 1+(-3) & 1+4 \\ -1+(-3) & -1+(1) & 4+(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 & 5 \\ -4 & 0 & 2 \end{bmatrix}$$

k=3

$$C) 3A = 3\begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 3\times 2 & 3\times 1 & 3\times(1) \\ 3\times(-1) & 3\times(-1) & 3\times 4 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 3 \\ -3 & -3 & 12 \end{bmatrix}$$

$$d) \; 5A-2B = 5\begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} - 2\begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 10 & 5 & 5 \\ -5 & -5 & 20 \end{bmatrix} - \begin{bmatrix} 4 & -6 & 8 \\ -6 & 2 & -4 \end{bmatrix}$$

5-6 = -1

$$= \begin{bmatrix} +10-4 & 5-(-6) & 5-8 \\ -5-(-6) & -5-2 & 26-(-4) \end{bmatrix} = \begin{bmatrix} 6 & 11 & -3 \\ 1 & -7 & 24 \end{bmatrix}$$

-2+4 = +