Chapter: 02: Quadratic Equation

EQUATIONS:

An equation is a statement of equality between two expressions called members.

(1) $5x = 9$ L.H.S = R.H.S
(2) $ax + b = 0$ exp = exp

(1) CONDITIONAL EQUATION OR SIMPLY AN EQUATION:

An equation which is true for only certain values of the variables (or unknowns) involved is called a conditional equation or simply an equation

$ax + by = c$
$x = ?$
$y = ?$

Exp $5x = 6$ (1)
$\boxed{x = 6/5}$

$5(6/5) = 6$
$6 = 6$
L.H.S = R.H.S

(2) LINEAR EQUATION:

a.5

A linear equation in the variable x can be written as $\boxed{ax + b = 0}$; $\boxed{a \neq 0}$; a, b = constant

Exp:- $\boxed{5x = 0}$
Sol: linear Eqs
$a = 5$; $b = 0$
$a \neq 5$

Exp: $2x - 3 = 0$
Sol: $a = 2$; $b = -3$
$a \neq 0$
linear Eqs

$a = 0$
$0(x) + b = 0$
$\boxed{b = 0}$

$0x + 5 = 6$
$a = 0$
not linear

(3) QUADRATIC EQUATION

degree

A quadratic equation in the variable x has the form $\boxed{ax^2 + bx + c = 0}$ where a, b, and c are constants and a≠0. It is also called $\boxed{2^{nd} \text{ degree polynomial}}$

$a = 0$
$0(x^2) + bx + c = 0$
$\boxed{bx + c = 0}$ linear

$ax^2 + bx + c = 0$

Examples

i. $x^2 - 7x + 10 = 0$; Quadratic ; $a = 1$, $b = -7$; $c = 10$
ii. $6x^2 + x - 15 = 0$; $a = 6$, $b = 1$, $c = -15$; Quadratic Eqs.
iii. $4x^2 + 5x + 3 = 0$; $a = 4$, $b = 5$; $c = 3$; Quadratic Eqs

$ax^2 + bx + c = 0$

iv. $3x^2 - x = 0$; $a = 3$, $b = -1$; $c = 0$; Quadratic Eqs

v. $x^2 = 4$; $x^2 - 4 = 0$; $a = 1$; $b = 0$; $c = -4$; Quadratic

<u>AN INCOMPLETE QUADRATIC EQUATION:</u>

If $\boxed{b = 0}$ and $\boxed{c = 0}$ in quadratic equation is called incomplete quadratic equation.

$\boxed{b = 0}$; $\boxed{c = 0}$; ~~pure~~ quadratic Eq.

Example:

i. $3x^2 = 0 \Rightarrow a = 3$; $\boxed{b = 0}$; $\boxed{c = 0}$ incomplete quadratic eq!

ii. $7x^2 - 2x = 0 \Rightarrow a = 7$; $b = -2$; $c = 0$

iii.

iv.

<u>ZEROS OR ROOTS OF THE EQUATION:</u>

To solve $\boxed{ax^2 + bx + c = 0}$ is to find the value of x which satisfy the equation, these value of x is called $zero\ or\ root\ of\ the\ equation$. Or the solution of an equation is called root

x_1, x_2, x_3

Example:

1. $x^2 - 9 = 0$

$x_1 = ?$

$x_2 = ?$

$x^2 = 9 \rightarrow (1)$

$\sqrt{x^2} = \sqrt{9}$

$\boxed{|x| = \sqrt{9} \quad x = \pm 3}$

$x_1 = 3$ ✓ [illegible]

$x_2 = -3$ ✓

$x = \pm 3$

$x = 3$

$(3)^2 = 9$

$9 = 9$

L.H.S = R.H.S

$x = -3$

$(-3)^2 = 9 \Rightarrow 9 = 9$

<u>METHODS OF SOLVING QUADRATIC EQUATIONS</u>

There are three basic technique to solving a quadratic equation

1) By factorization
2) By completing square
3) By quadratic formula

(1) SOLUTION BY FACTORIZATION:

2nd

It involves factoring the polynomial $ax^2 + bx + c = 0$, it makes use of the fact that if $\boxed{ab=0}$ then $\boxed{a=0}$ or $\boxed{b=0}$

$\underset{a}{(x-2)}\underset{b}{(x-3)} = 0$

$a = 0$ or $b = 0$

$(x-2) = 0$; $(x-3) = 0$

$\boxed{x = 2}$; $\boxed{x = 3}$

Expo1:- $x^2 - 7x + 10 = 0$

Sol:- $\boxed{x^2 - 7x + 10} = 0$

M	Add/sub
$10x^2$	$-7x$
$= -5x \times -2x$	$\boxed{-5x - 2x}$
$10x^2$	$-7x$

$x \cdot x = x^2$

(1) $x^2 - 5x - 2x + 10 = 0$

(2) $x(x-5) - 2(x-5) = 0$

(3) $\underset{a}{(x-2)}\underset{b}{(x-5)} = 0$

if $ab = 0$; $a = 0$, $b = 0$

(4) $x - 2 = 0$; $(x-5) = 0$

$\boxed{x_1 = 2}$; $\boxed{x_2 = 5}$

Root $= \{2; 5\}$

Check:-

$x^2 - 7x + 10 = 0$

$x = 2$

$(2)^2 - 7(2) + 10 = 0$

$4 - 14 + 10 = 0$

$14 - 14 = 0$

$0 = 0$

$x = 5$

$(5)^2 - 7(5) + 10 = 0$

$25 - 35 + 10 = 0$

$35 - 35 = 0$

$0 = 0$

(b) $7x^2 - 5x = 0$

$\underset{a}{x}\underset{b}{(7x - 5)} = 0$

$x = 0$; $7x - 5 = 0$

$7x = 5$

$\boxed{x = 5/7}$

Root $= \{0, 5/7\}$

For $x = 0$

$7(0)^2 - 5(0) = 0$

$0 = 0$

For $x = 5/7$

$7 \cdot \left(\frac{5}{7}\right)^2 - 5\left(5/7\right) = 0$

$\frac{25}{7} - \frac{25}{7} = 0$

$0 = 0$

(1a)

For completing square :-

① $(a+b)^2 = a^2+b^2+2ab$ ✓

② $(a-b)^2 = a^2+b^2-2ab$

Exp:- $x^2+4x-437=0$

sol:-

① $x^2+4x-437=0$

$$x^2+\boxed{4x} = 437$$

② $x^2+4x+4 = 437+4$

$$(x+2)^2 = 441$$

$$(x+2)^2 = x^2+(2)^2+2.x(2)$$
$$= x^2+4+4x$$

③ $\sqrt{(x+2)^2} = \sqrt{441}$

$$x+2 = \pm 21$$

④ $x+2=21$ or $x+2=-21$

$$x = 21-2 \qquad x = -21-2$$

$$\boxed{x = 19} \qquad \boxed{x=-23}$$

Root $= \{19, -23\}$

② $x^2-6x-2=0$

① $x^2-6x = 2$

② $x^2-6x+(3)^2 = 2+(3)^2$

$$(a-b)^2 = a^2+b^2-2ab$$
$$= x^2+3^2-2x3$$
$$(x-3)^2 = x^2+9-6x$$

$$(x-3)^2 = 2+9$$

$$(x-3)^2 = 11$$

$$\sqrt{(x-3)^2} = \sqrt{11}$$

$$(x-3) = \pm\sqrt{11}$$

$x-3 = \sqrt{11}$ or $x-3 = -\sqrt{11}$

$$\boxed{x = \sqrt{11}+3}$$

$$\boxed{x = -\sqrt{11}+3}$$