MINORS:

The minor of an element in a determinant of order n is the determinant of order $n-1$ obtained by removing the row and the column which contain the given element.

For example, the minor of a_{32} in the 4th order determinant

$n=4$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}_{4\times4} \Rightarrow$$

is obtained by crossing out the row and column containing a_{32} as shown, and writing the resulting determinant of order 3, namely

$n=4$

$n-1 = 4-1 = 3$

$$\begin{vmatrix} a_{11} & a_{13} & a_{14} \\ a_{21} & a_{23} & a_{24} \\ a_{41} & a_{43} & a_{44} \end{vmatrix}_{3\times3}$$

The minor of an element is denoted by capital letters

M_{32} ; P, A

$n=3,4,5$

VALUE OF A DETERMINANT OF ORDER n

The value of a determinant may be obtained in terms of minors as follows:

(1) Choose any row (or column). M_{32}

(2)Multiply each element in the row (or column) by its corresponding minor preceded by $(-1)^{i+j}$ where $i+j$ is the sum of the row number i and column number j. The minor of an element with the attached sign is called the cofactor of the element.

$A_{32} = (-1)^{i+j} M_{32} = (-1)^{3+2} M_{32}$ $\quad M_{32}$ ↓ R C

(3) Add algebraically the products obtained in

c_{ij} $\quad \boxed{A_{ij} = (-1)^{i+j} M_{32}}$

cofactor

For example, let us expand the determinant

Write the minor and corresponding cofactor of the element in the second row, third column for the determinant

$$\begin{vmatrix} 2 & -2 & 3 & 1 \\ 1 & 3 & 2 & 5 \\ 1 & -2 & 5 & -1 \\ 2 & 1 & 3 & -2 \end{vmatrix}$$

4x4

$R = 2$
$C = 3$

$$M_{23} = \begin{vmatrix} 2 & -2 & 1 \\ 1 & -2 & -1 \\ 2 & 1 & -2 \end{vmatrix} = 15$$

$$C_{23} = (-1)^{5} M_{23}$$

$$= -1 \begin{vmatrix} 2 & -2 & 1 \\ 1 & -2 & -1 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= (-1)(15)$$

$$C_{23} = -15$$

$$C_{23} \Rightarrow a_{23}$$

$(-1)^{2}$ even $= (-1)(-1) = 1$

$(-1)^{3}$ odd $= (-1)(-1)(-1)$ R C

$= 1(-1)$

$= -1$

$(-1)^{2+3} = (-1)^{5} = (-1)$

$+ \quad - \quad +$

$$= 2\begin{vmatrix} -2 & -1 \\ 1 & -2 \end{vmatrix} - (-2)\begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix} + (1)\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}$$

$$= 2(4-(-1)) + 2(-2-(-2)) + (1-(-4))$$

$$= 2(4+1) + 2(\cancel{-2}+\cancel{2}) + (1+4)$$

$$= 2(5) + 0 + 5$$

$$= 10 + 5$$

$$= 15$$

4th row

Write the minor and corresponding cofactor of the element in ~~the second row, third column for~~ the determinant

$$\begin{vmatrix} 3 & -2 & 4 & 2 \\ 2 & 1 & 5 & -3 \\ 1 & 5 & -2 & 2 \\ -3 & -2 & -4 & 1 \end{vmatrix}$$

Sol :- $R = 4$

$-3, -2, -4, 1$

$$M_{41} = \begin{vmatrix} -2 & 4 & 2 \\ 1 & 5 & -3 \\ 5 & -2 & 2 \end{vmatrix} ; \quad C_{41} = (-1)^{4+1=5} M_{41} = (-1)\begin{vmatrix} -2 & 4 & 2 \\ 1 & 5 & -3 \\ 5 & -2 & 2 \end{vmatrix}$$

$$M_{42} = \begin{vmatrix} 3 & 4 & 2 \\ 2 & 5 & -3 \\ 1 & -2 & 2 \end{vmatrix} ; \quad C_{42} = (-1)^{4+2} M_{42} = (1)\begin{vmatrix} 3 & 4 & 2 \\ 2 & 5 & -3 \\ 1 & -2 & 2 \end{vmatrix}$$

$$M_{43} = \begin{vmatrix} 3 & -2 & 2 \\ 2 & 1 & -3 \\ 1 & 5 & 2 \end{vmatrix} ; \quad C_{43} = (-1)^{4+3} M_{42} = (-1)\begin{vmatrix} 3 & -2 & 2 \\ 2 & 1 & -3 \\ 1 & 5 & 2 \end{vmatrix}$$

$$M_{44} = \begin{vmatrix} 3 & -2 & 4 \\ 2 & 1 & 5 \\ 1 & 5 & 2 \end{vmatrix} ; \quad C_{44} = (-1)^{4+4} M_{44} = (1)\begin{vmatrix} 3 & -2 & 4 \\ 2 & 1 & 5 \\ 1 & 5 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{vmatrix}$$

Sol:-

$$\begin{array}{cccc} & a_{11} & a_{12} & a_{13} \\ |A| = & \multicolumn{3}{c}{\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{vmatrix}} \end{array}$$

Laplace Expansion

$$\boxed{|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}}$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$|A| = a_{11}(-1)^{1+1}M_{11} + a_{12}(-1)^{1+2}M_{12} + a_{13}(-1)^{1+3}M_{13}$$

$$= a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$$

$$= 1\begin{vmatrix} 3 & 1 \\ -3 & 2 \end{vmatrix} - (-2)\begin{vmatrix} -2 & 1 \\ 4 & 2 \end{vmatrix} + (3)\begin{vmatrix} -2 & 3 \\ 4 & -3 \end{vmatrix}$$

$$= (6-(-3)) + 2(-4-4) + 3(6-12)$$

$$= (6+3) + 2(-4-4) + 3(6-12)$$

$$= 9 - 16 - 18 = -25$$

$$|A| = -25$$