Course instructor: Nida Ibrar

Chapter 01: Matrices

Topic: DETERMINANTS

Lecture: 03

- Determinants:

The determinants of two matrices is the difference of the product of entries in the two diagonals.

$|A|$ ; $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ; $|A| = ad - bc$

Example 01: $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}_{2\times 1}$

$|A| = \begin{vmatrix} 2 & -1 \\ 4 & 3 \end{vmatrix} = 2(3) - (-1)(4)$
$= 6 - (-4)$
$= 6 + 4$
$= 10$

$\boxed{|A| = 10}$

B = $\begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix}$
$= 1(8) - (4)(2)$
$= 8 - 8$

$\boxed{|B| = 0}$

- Singular and Non-Singular Matrices:

$|A| = 0$ $\quad$ $|A| \neq 0$ $\quad$ A = Square Matrix

Example:

$A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ B = $\begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -1 \\ 4 & 3 \end{vmatrix}$
$= 2(3) - (-1)(4)$
$= 6 - (-4)$
$= 6 + 4 = 10$

$|A| = 10 \neq 0$

Hence A Matrix is not singular

$|B| = 8 - 8 = 0$

$\boxed{|B| = 0}$

B Matrix is singular

- Adjoint of the 2 x 2 Matrix

A = Square Matrix

Defined as

$$\text{Adj}A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2}$

Moving diag

Exp: $A = \begin{bmatrix} 1 & -3 \\ 4 & -2 \end{bmatrix}$; $|A| = -2 - (-12) = -2 + 12 = 10$

$$\text{Adj}A = \begin{bmatrix} -2 & -(-3) \\ -(4) & 1 \end{bmatrix}$$

$$\text{Adj}A = \begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix}$$

- Inverse of a 2x2 Matrix:

$$\boxed{A^{-1} = \frac{\text{Adj}A}{|A|}}$$

$A_{2\times2}$ = square Matrix

$$A^{-1} = \frac{\begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix}}{10} = \frac{1}{10}\begin{bmatrix} -2 & 3 \\ -4 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2/10 & 3/10 \\ -4/10 & 1/10 \end{bmatrix}$$

$$\boxed{A^{-1}A = I}$$

$$A^{-1}A = \begin{bmatrix} -1/5 & 3/10 \\ -2/5 & 1/10 \end{bmatrix}_{2\times2}\begin{bmatrix} 1 & -3 \\ 4 & -2 \end{bmatrix}_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{5}(1) + \frac{3}{10}(4) & -\frac{1}{5}(-3) + \frac{3}{10}(-2) \\ -\frac{2}{5}(1) + \frac{1}{10}(4) & -\frac{2}{5}(-3) + \frac{1}{10}(-2) \end{bmatrix}$$

$$\begin{bmatrix} -\frac{1}{5} + \frac{6}{5} = \frac{-1+6}{5} = \frac{5}{5} & \frac{3}{5} - \frac{3}{5} \quad 0 \\ -\frac{2}{5} + \frac{2}{5} \quad 0 & \frac{6}{5} - \frac{1}{5} \quad 1 \end{bmatrix}$$

- Transpose of a Matrix

Exchange the column & Row

column ⇒ Row

$A^t$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

Lecture: 03

$A = [2 \quad 3]$

$A^T = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad A^T = [1 \quad 0]$

**Evaluate the following determinants.**

a) $\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$ b) $\begin{vmatrix} 3 & -1 \\ 6 & -2 \end{vmatrix}$ c) $\begin{vmatrix} x & x^2 \\ y & y^2 \end{vmatrix}$

**b) Example Find the value of x when** $\begin{vmatrix} 2x-1 & 2x+1 \\ x+1 & 4x+2 \end{vmatrix} = 0$

$\begin{vmatrix} 2x-1 & 2x+1 \\ x+1 & 4x+2 \end{vmatrix} = 0$

$(2x-1)(4x+2) - (2x+1)(x+1) = 0$

$2x(4x+2) - 1(4x+2) - [2x(x+1) + 1(x+1)] = 0$

$(8x^2 + 4x - 4x - 2) - [2x^2 + 2x + x + 1] = 0$

$8x^2 - 2 - 2x^2 - 3x - 1 = 0$

$8x^2 - 2x^2 - 3x - 3 = 0$

$6x^2 - 3x - 3 = 0$

$3(2x^2 - x - 1) = 0 \qquad 3(x^2 - x - 1) / 3 = \frac{0}{3} = 0$

$\boxed{2x^2 - x - 1 = 0}$

$2x^2 - x - 1 = 0 \qquad 2x^2$

$ax^2 + bx + c = 0$

$2x^2 - 2x + x - 1 = 0$

$2x(x-1) + 1(x-1) = 0$

$(2x+1)(x-1) = 0$

$2x + 1 = 0 \qquad x - 1 = 0$

$2x = -1 \qquad \boxed{x = 1}$

$\boxed{x = -1/2}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 2$
$b = -1$
$c = -1$

c) Find inverse if $\begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix}_{2\times 2}$