System of Eqs
family of Eqs

$$\left[\begin{array}{l} a x_1 + b x_2 = c \\ d x_1 + f x_2 = g \end{array}\right.$$

unknown $\left[\begin{array}{l} x_1 = ? \\ x_2 = ? \end{array}\right.$

$$\left[\begin{array}{l} 2x + 1y = 5 \\ 1x - 1y = 1 \end{array}\right]$$

$$Ax = B$$

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

- System of linear Equation

$Ax = B \longrightarrow$ Eq

Square Matrix

$$\begin{bmatrix} a & b \\ d & f \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c \\ g \end{bmatrix}$$

a) ELIMINATION METHOD

b) SUBSTITUTION METHOD

b) CRAMER'S RULE

one variable eliminate

## ELIMINATION METHOD

[1]

$$3x_1 - x_2 = 1 \rightarrow (1)$$

$$x_1 + x_2 = 3 \rightarrow (2)$$

Exp =

$$x_1 + 2x_2 = 4 \longrightarrow (1)$$

$$2x_1 + 4x_2 = 12 \longrightarrow (2)$$

eq (1) + eq (2)

$$\begin{array}{rl} 3x_1 - \cancel{x_2} & = 1 \\ + \; x_1 + \cancel{x_2} & = 3 \\ \hline 4x_1 & = 4 \end{array}$$

$$x_1 = \frac{\cancel{4}\,1}{\cancel{4}}$$

$$\boxed{x_1 = 1}$$

eq (2)

$$x_1 + x_2 = 3$$

put $x_1 = 1$

$$1 + x_2 = 3$$

$$x_2 = 3 - 1$$

$$\boxed{x_2 = 2}$$

$$2 \times (x_1 + 2x_2) = 4 \times 2$$

$$\begin{array}{rl} \cancel{2x_1} + \cancel{4x_2} & = 8 \\ \underline{+}\cancel{2x_1} \; \underline{+}\cancel{4x_2} & = \underline{+}12 \\ \hline 0 & = -4 \end{array}$$

has not solution

system of linear eq {1, 2}

Exp:-

$$2x_1 + 5x_2 = 2 \rightarrow (1)$$

$$x_1 + 2x_2 = 4 \rightarrow (2)$$

2 x eq (2)

$$2(x_1 + 2x_2) = 2 \times 4$$

$$2x_1 + 4x_2 = 8 \rightarrow (3)$$

eq (1) - eq (2)

$$\begin{array}{rl} \cancel{2x_1} + 5x_2 & = 2 \\ \underline{+}\cancel{2x_1} \; \underline{+} 4x_2 & = \underline{+}8 \\ \hline \end{array}$$

$$\boxed{x_2 = -6}$$

eq (2)

$$x_1 + 2x_2 = 4$$

$$x_1 + 2(-6) = 4$$

$$x_1 - 12 = 4$$

$$x_1 = 4 + 12$$

$$\boxed{x_1 = 16}$$

② Substitution Method

Exp
$3x_1 - x_2 = 1 \rightarrow ①$
$x_1 + x_2 = 3 \rightarrow ②$

$\Rightarrow$ eq (2) $x_1 + x_2 = 3$

$\boxed{x_1 = 3 - x_2}$

$\Rightarrow$ eq (1)

$3x_1 - x_2 = 1$

$3(3 - x_2) - x_2 = 1$

$9 - 3x_2 - x_2 = 1$

$-4x_2 = 1 - 9$

$-4x_2 = -8$

$x_2 = \cancel{8}^{2} / \cancel{4}$

$\boxed{x_2 = 2}$

eq (2) $x_1 + 2 = 3$

$x_1 = 3 - 2$

$\boxed{x_1 = 1}$

$\{1, 2\}$ = solution

③ Cramer's Rule :-

$x_1, x_2 = ?$
$x, y = ?$

$\boxed{x = \frac{D_x}{D}}$ ; $\boxed{y = \frac{D_y}{D}}$

$\boxed{x_1 = \frac{D_{x_1}}{D}}$ ; $\boxed{x_2 = \frac{D_{x_2}}{D}}$

2)
$x_1 + 2x_2 = 4 \rightarrow ①$
$2x_1 + 4x_2 = 16 \rightarrow ②$

eq (1)
$x_1 + 2x_2 = 4$

$\boxed{x_1 = 4 - 2x_2}$

eq (1)

$4 - \cancel{2x_2} + \cancel{2x_2} = 4$

$4 = 4$

has no solution

Exp :- $2x + 3y = 8$
$x - 2y = -3$

Cramer's Rule:-
$x = ?$ $y = ?$

$Ax = B$

$$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$$

solve the system

$$2x + 3y = 8$$
$$x - 2y = -3$$

$x = \frac{D_x}{D}$

$y = \frac{D_y}{D}$

$B = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$

Sol :-

$A = \begin{array}{c|c} & \begin{array}{cc} x & y \end{array} \\ \begin{array}{c} x \\ y \end{array} & \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} \end{array}$ ;

$D = |A| = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = 2(-2) - 3(1)$

$= -4 - 3$

$= -7$

$\boxed{D = -7}$

$D_x = \begin{vmatrix} 8 & 3 \\ -3 & -2 \end{vmatrix} =$ ;

$D_y = \begin{vmatrix} 2 & 8 \\ 1 & -3 \end{vmatrix}$

$D_x = 8(-2) - (3)(-3)$

$= -16 - (-9)$

$= -16 + 9$

$\boxed{D_x = -7}$

$D_y = 2(-3) - 8(1)$

$-6 \quad -8$

$\boxed{D_y = -14}$

C.R

$x = \frac{D_x}{D} = \frac{\cancel{-7}}{\cancel{-7}} = 1$

$y = \frac{D_y}{D} = \frac{\cancel{-14}\ 2}{\cancel{-7}} = 2$

$\{x, y\} = \{1, 2\}$

**DETERMINANTS OF THIRD ORDER**

consisting of nine numbers arranged in three rows and three columns is called a determinant of third order. By definition, the value of this determinant is given by

is called the expansion of the determinant