

Course: Linear Algebra

Course Code: MATH-203

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Class: Bs IT 3rd SS

Department of CS & IT





Introduction to Linear Algebra

BS INFORMATION TECHNOLOGY

What is Algebra?

Algebra is a branch of mathematics dealing with symbols and the rules for manipulating those symbols.

In elementary algebra, those symbols (today Latin and Greek letters) represent quantities without fixed values, known as variables.

What is Algebra?

I have two fields that total 1800 square yards. Yields for each field are $\frac{2}{3}$ gallon of grain per square yard and $\frac{1}{2}$ gallon per square yard. The first field gave 500 more gallons than the second. What are the areas of each field?

What is Linear Algebra?

- It is a branch of Mathematics concerning with **vector space** and **linear mapping** between such spaces.
- Linear algebra is about **linear combinations**. That is using arithmetic on columns of numbers called **vectors** and arrays of numbers called **matrices**, to create new columns and arrays of numbers. Linear algebra is the study of **lines** and **planes**, **vector spaces** and **mappings** that are required for linear transformation.
- It is a study of **system of linear equation**.

What is Linear Algebra?

“A branch of mathematics that is concerned with mathematical structures closed under the operations of addition and scalar multiplication and that includes the theory of system of linear equations, matrices, determinants, vector spaces and linear transformation.”

Why is linear algebra important?

- Linear algebra is vital in multiple areas of science in general. Because linear equations are so easy to solve, practically every area of modern science.
- It converts large number of problems to matrix and thus we solve the matrix.

Linear Algebra in IT field:

Linear algebra in Information Technology can broadly divided into two categories:

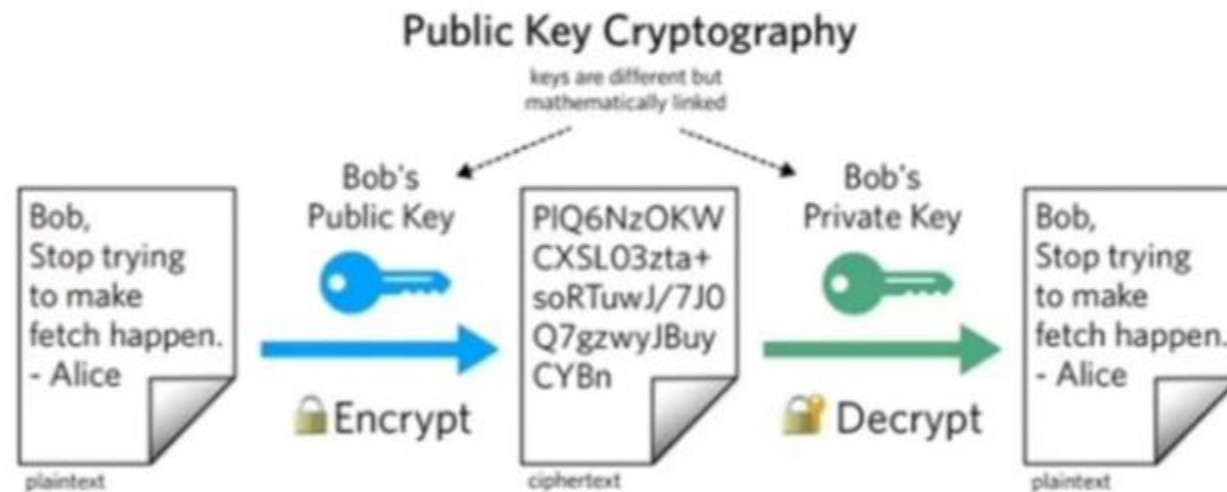
- Linear algebra for spatial quantities, Here you are dealing with 2-, 3-, or 4-dimensional vectors and you are concerned with rotations, projections, and other matrix operations that have some spatial interpretation. This is the kind of linear algebra that comes up, for example, in computer graphic or physics simulations.
- Linear algebra for statistics. Here you are dealing with vectors in high-dimensional spaces that have no particular spatial interpretation, and you are interested in matrix decompositions and so on. This domain includes single processing, statistical machine learning, and compression.

Applications of Linear Algebra in Information Technology

- There are so many applications of linear algebra in IT.
- From simple circuit solving to large web engine algorithms.

Applications of Linear Algebra

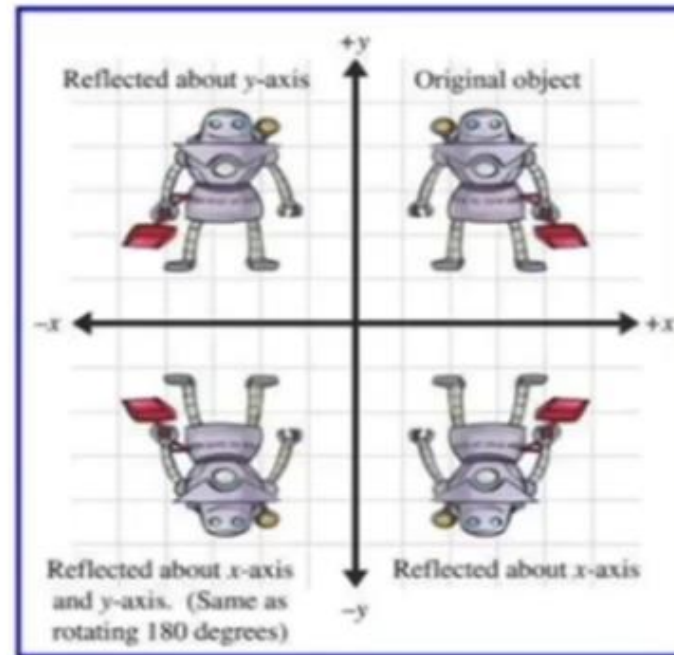
Cryptography



Applications of Linear Algebra

Computer Graphics

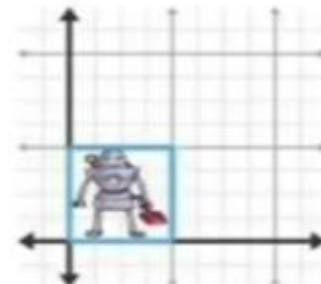
Reflection



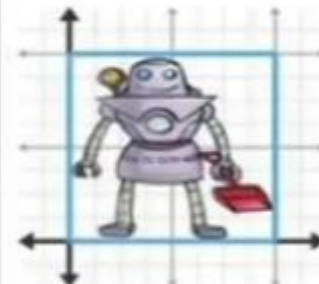
Applications of Linear Algebra

Computer Graphics

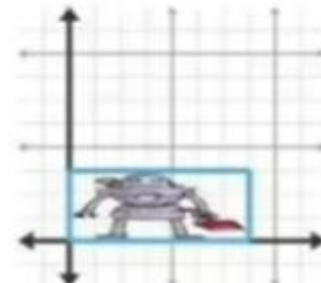
Contraction, Extraction



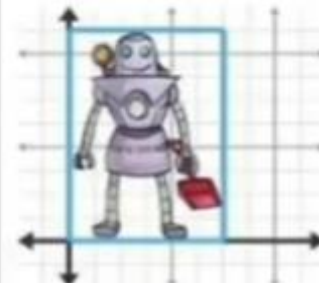
$k_x=1, k_y=1$ (Unscaled)



$k_x=2, k_y=2$



$k_x=1.75, k_y=0.75$

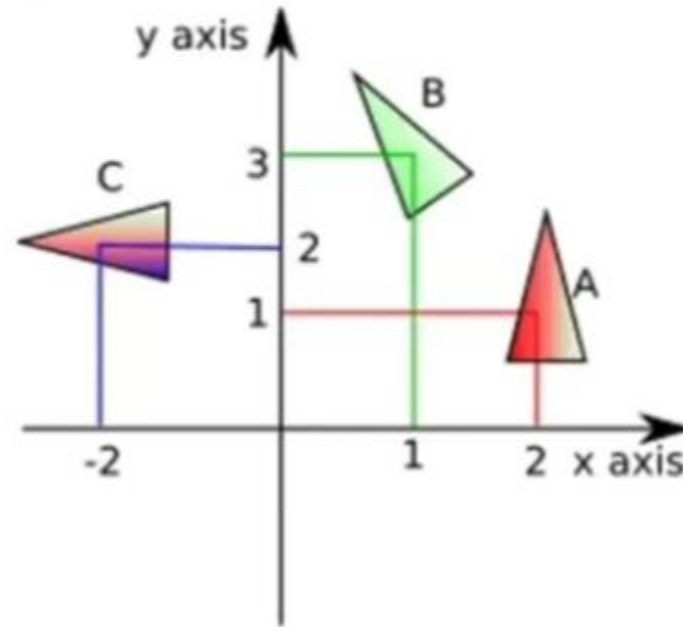


$k_x=1.5, k_y=2.25$

Applications of Linear Algebra

Computer Graphics

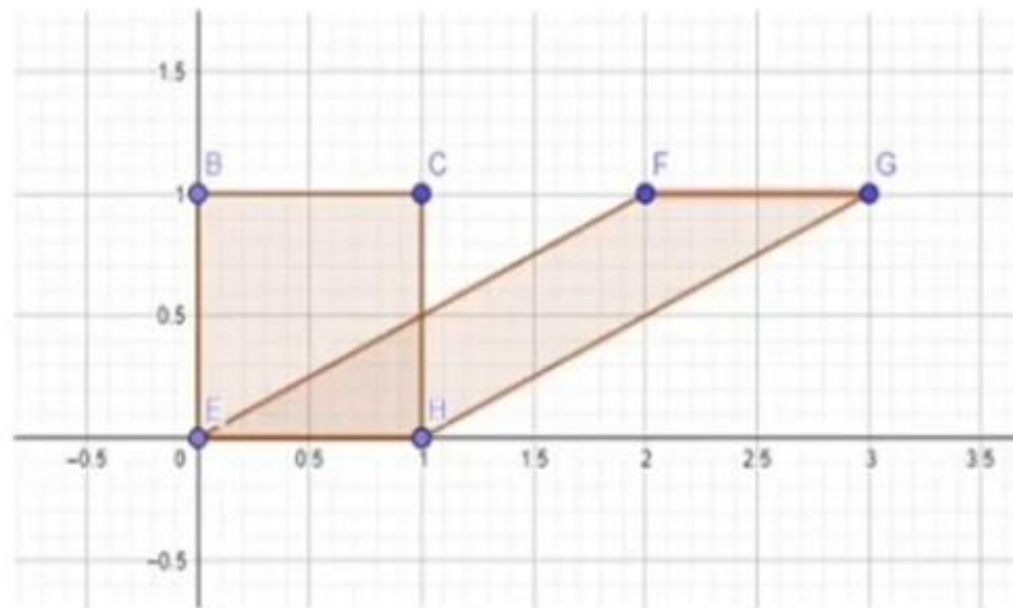
Rotation



Applications of Linear Algebra

Computer Graphics

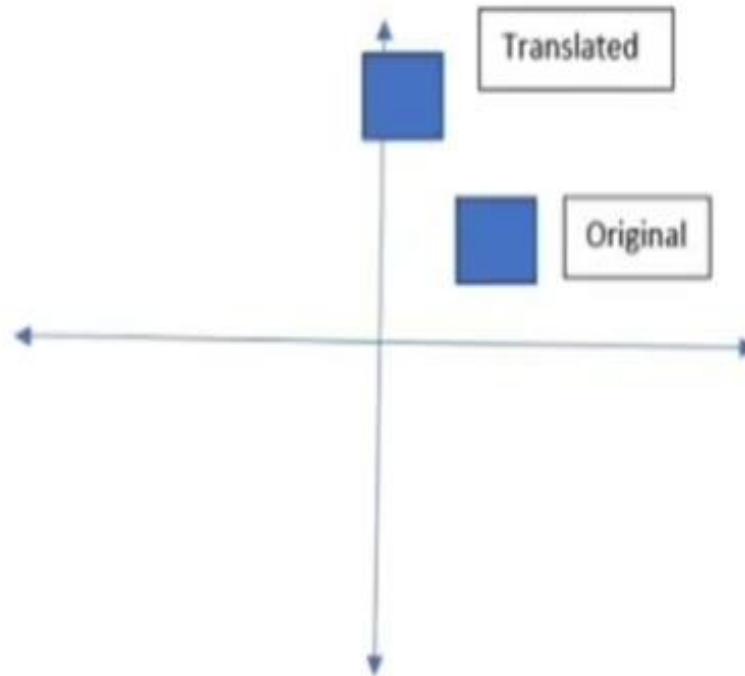
Shear



Applications of Linear Algebra

Computer Graphics

Translation



Applications of Linear Algebra

Computer Graphics

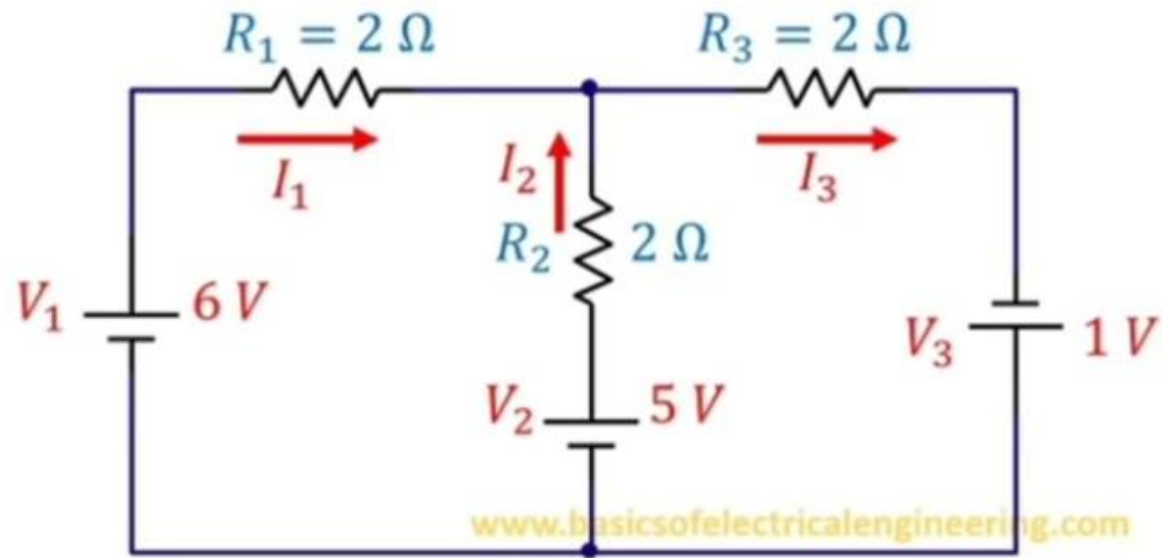
Conversion of dimension

2D to 3D
Conversion



Applications of Linear Algebra

Applications in Circuit Analysis



Applications of Linear Algebra

Traffic Flow



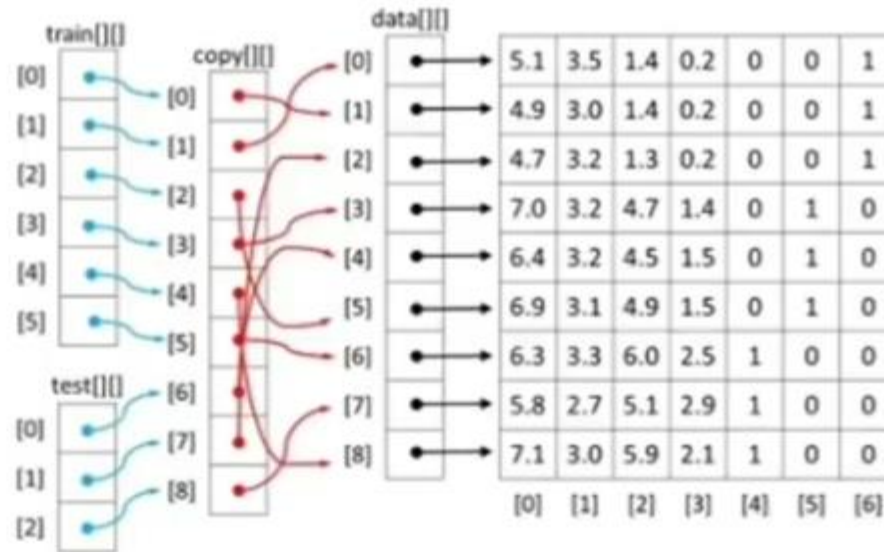
Applications of Linear Algebra

Image Processing



Applications of Linear Algebra

Machine Learning



Applications of linear algebra in real world:

- Electrical circuit
- Variable cost
- Traffic flow
- Chemical mixing

Electrical circuits:

➤ Electrical circuit is nothing but Just a combination of Capacitor Transistor etc.

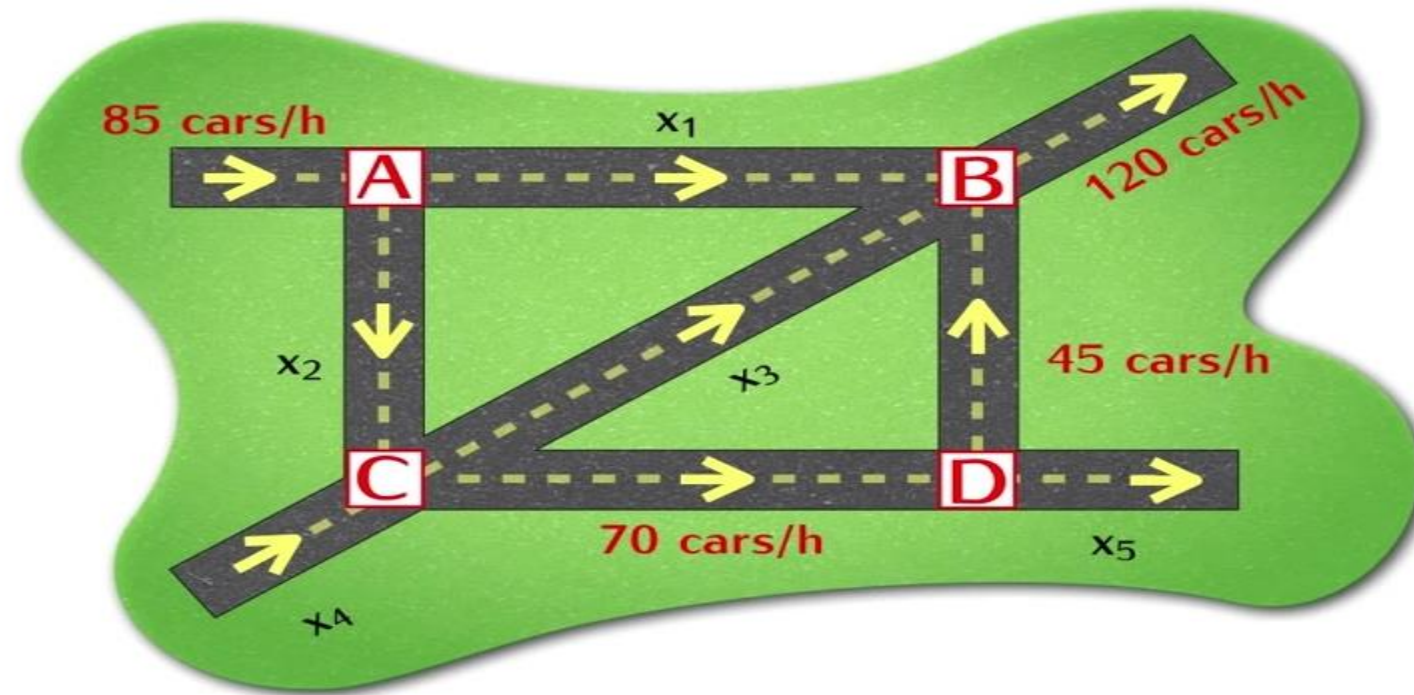
➤ An electrical circuit is a path in Which electrons from a voltage Or current source flow.



Linear algebra in electrical circuits:

Linear algebra most apparently uses by electrical engineers. When there is system of linear algebra arises the concept of linear algebra various electrical circuits solution like Kirchhoff's law, ohms law are conceptually arise linear algebra. To solve various linear equations we need to introduce the concept of linear algebra.

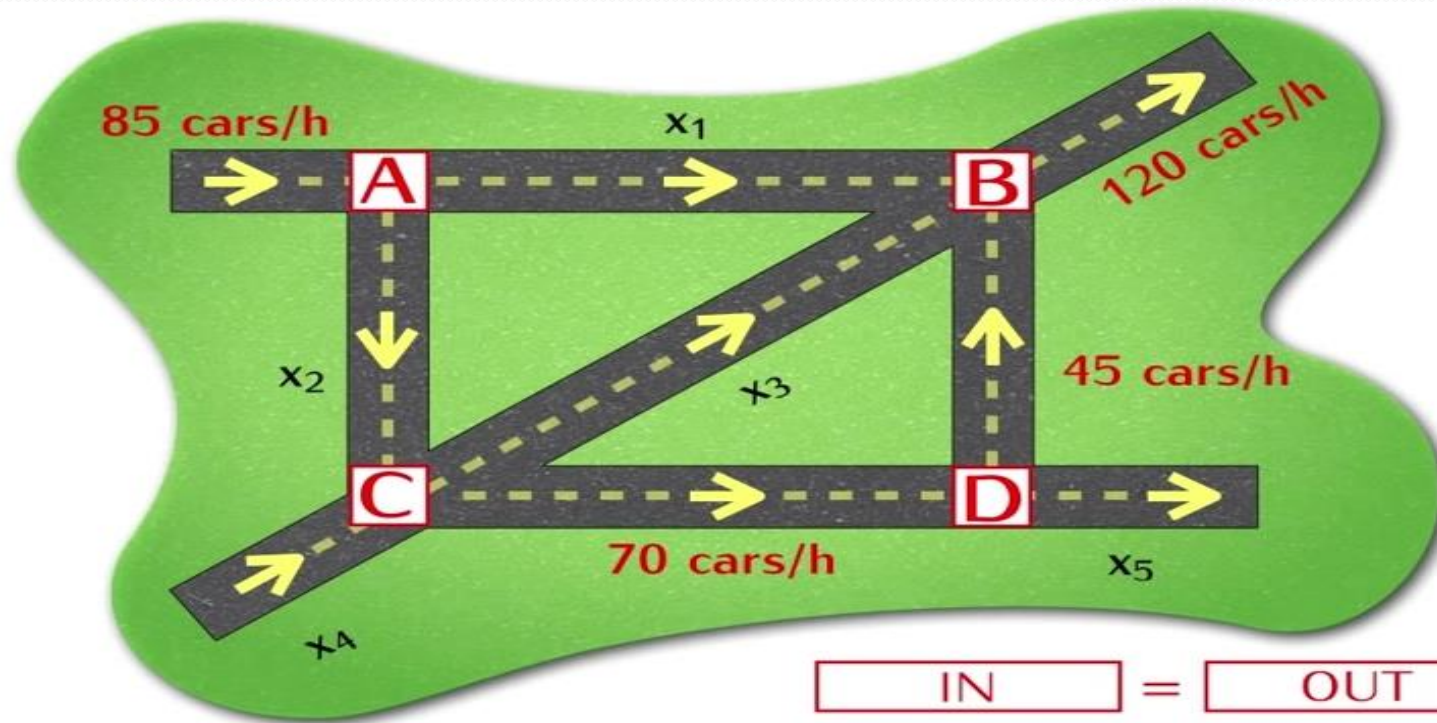
How linear algebra used in traffic flow:



Problem. Find the flow rate of cars on each segment of streets.

Observations:

- flow into an intersection = flow out of that intersection
- total flow in = total flow out



IN	=	OUT
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total:

$$85 + x_4 = 120 + x_5$$

@ A:

$$85 = x_1 + x_2$$

@ B:

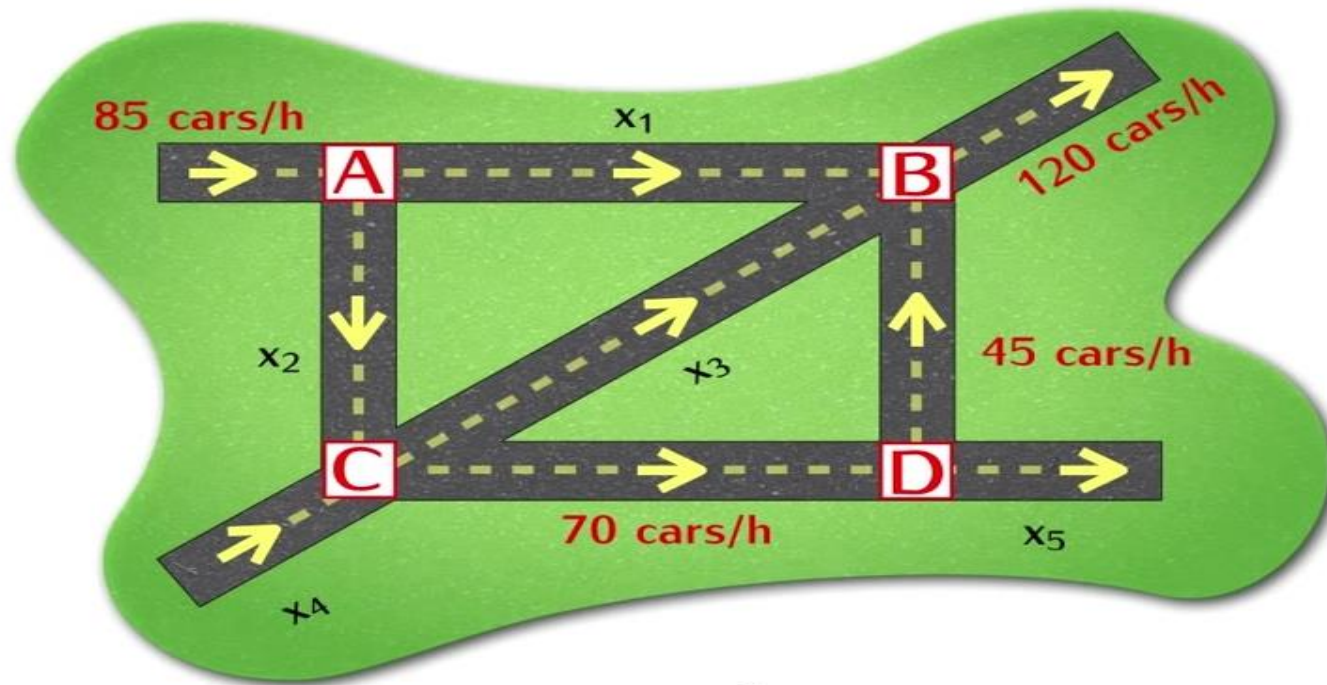
$$x_1 + x_3 + 45 = 120$$

@ C:

$$x_2 + x_4 = 70 + x_3$$

@ D:

$$70 = 45 + x_5$$



$$\begin{cases} x_4 - x_5 = 35 \\ x_1 + x_2 = 85 \\ x_1 + x_3 = 75 \\ x_2 - x_3 + x_4 = 70 \\ x_5 = 25 \end{cases}$$

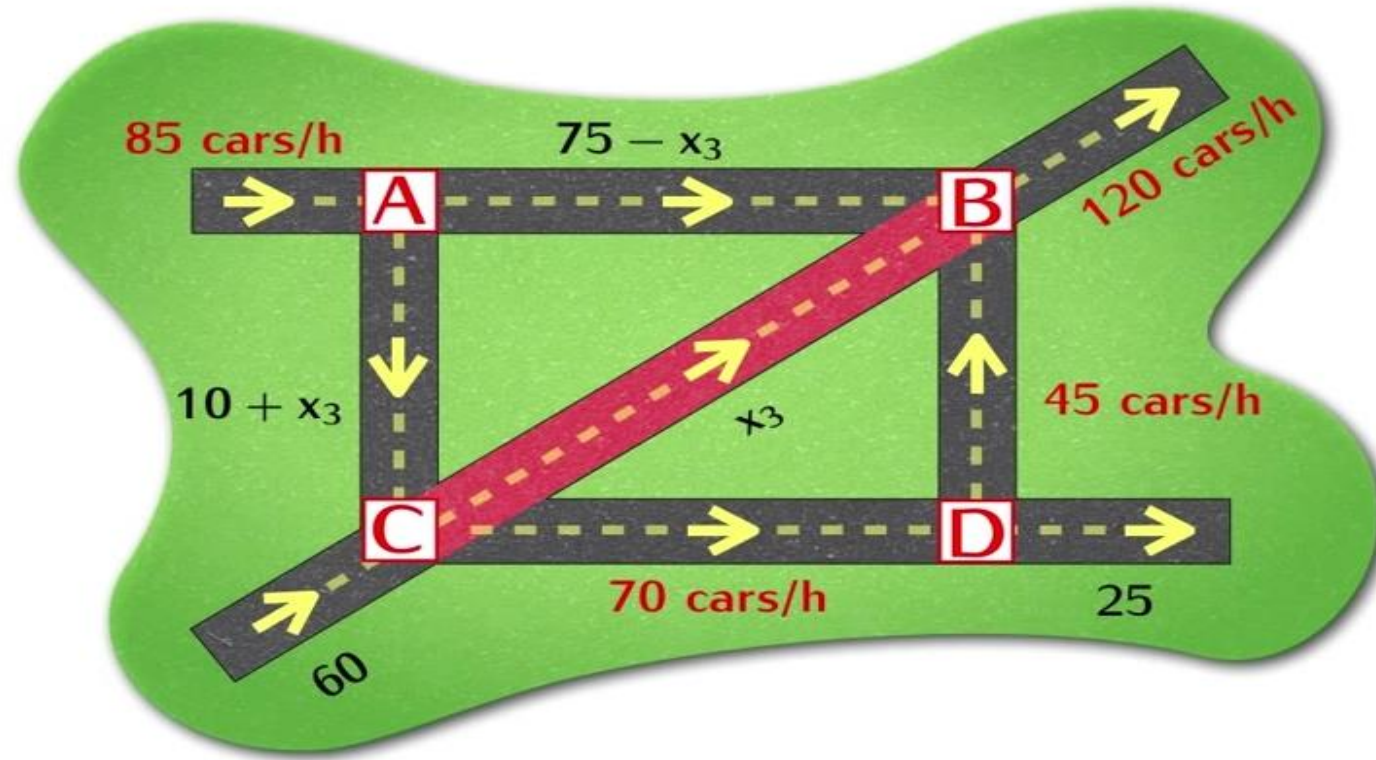
$$\begin{cases} x_4 - x_5 = 35 \\ x_1 + x_2 = 85 \\ x_1 + x_3 = 75 \\ x_2 - x_3 + x_4 = 70 \\ x_5 = 25 \end{cases}$$

$$\begin{cases} x_1 = 75 - x_3 \\ x_2 = 10 + x_3 \\ x_3 = \textit{free} \\ x_4 = 60 \\ x_5 = 25 \end{cases}$$

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 1 & -1 & 35 \\ 1 & 1 & 0 & 0 & 0 & 85 \\ 1 & 0 & 1 & 0 & 0 & 75 \\ 0 & 1 & -1 & 1 & 0 & 70 \\ 0 & 0 & 0 & 0 & 1 & 25 \end{array} \right]$$

*row
reduction*

$$\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 0 & 75 \\ 0 & 1 & -1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & 60 \\ 0 & 0 & 0 & 0 & 1 & 25 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$



$$\left\{ \begin{array}{l} x_1 = 75 - x_3 \\ x_2 = 10 + x_3 \\ x_3 = \textit{free} \\ x_4 = 60 \\ x_5 = 25 \end{array} \right.$$

Variable cost:

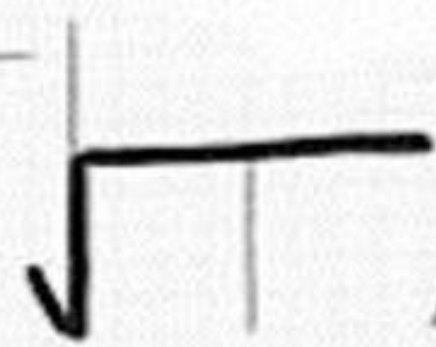
Variable cost are expenses that fluctuate proportionality with the quantity of output.

How linear algebra is used in variable cost:

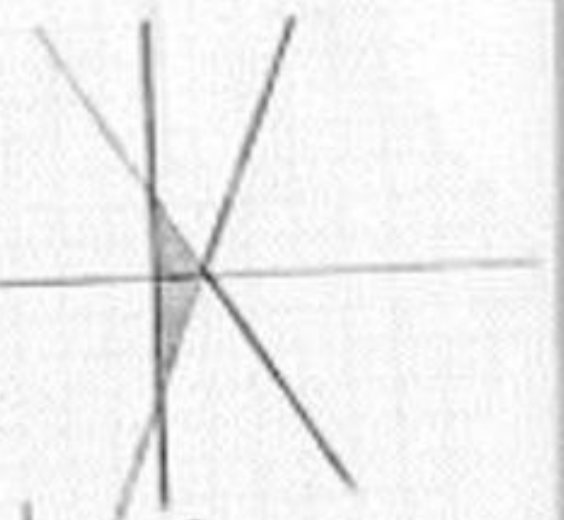
Imagine that you are taking a taxi while on vacation. You know that taxi service charges 9 dollar to pick your family up from your hotel and another 0.15 dollar per mile for the trip. Without knowing how many miles it will be to each destination, you can set up a linear equation that can be used to find the cost of any taxi trip you take on your trip. By using “x” to represent the number of miles to your destination and “y” to represent the cost of that taxi ride, the linear equation would be

$$Y = 0.15x + 9$$





π



$y \neq 0$

