$a_{12} = a_{21}$

PROPERTIES OF DETERMINANTS

1. Interchanging corresponding rows and columns of a determinant does not change the value of the determinant.

$a_{12} = a_{21}$
$a_{13} = a_{31}$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

L.H.S = R.H.S

Exp :-

$$\begin{vmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} + 0\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix}$$

$$= 1(-1-1) - 2(1-2) + 0$$

$$= -2 - 2(-1) + 0 = -2 + 2 = 0$$

$$= 1\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} - (-1)\begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 1(-1-1) + 1(-2-0) + 2(2-0)$$

$$= -2 - 2 + 4$$

$$= -4 + 4$$

$$= 0$$

L.H.S = R.H.S

2. If each element in a row (or column) is zero, the value of the determinant is zero.

$$\begin{vmatrix} a_{11} & 0 & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & 0 & a_{33} \end{vmatrix} = 0$$

or

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

3. Interchanging any two rows (or columns) reverses the sign of the determinant.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix}$$

L.H.S = R.H.S

Exp :-

$$\begin{vmatrix} 1 & -1 & 0 \\ -1 & 1 & 2 \\ 0 & 1 & -1 \end{vmatrix} = -\begin{vmatrix} 0 & 1 & -1 \\ -1 & 1 & 2 \\ 0 & 1 & -1 \end{vmatrix}$$

Exp (2)

$$\begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 0 & -2 \end{vmatrix} = 0$$

$$1\begin{vmatrix} 0 & 1 \\ 0 & -2 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix}$$

$$0 - 0 + 0$$

$$= 0$$

same

4. If two rows (or columns) of a determinant are identical, the value of the determinant is zero.

$$\begin{vmatrix} a_{11} & a_{12} & a_{11} \\ a_{21} & a_{22} & a_{21} \\ a_{31} & a_{32} & a_{31} \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 2 & 1 \\ -1 & 3 & -1 \\ 0 & 1 & 0 \end{vmatrix} = 0$$

$$1\begin{vmatrix} 3 & -1 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} -1 & -1 \\ 0 & 0 \end{vmatrix} + 1\begin{vmatrix} -1 & 3 \\ 0 & 1 \end{vmatrix}$$

$$1(0+1) - 2(0) + 1(-1-0)$$

$$+1 - 0 + -1 = \cancel{1} \cancel{-1} = 0$$

$$\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

5. If each of the elements in a row (or column) of a determinant is multiplied by the same number p, the value of the determinant is multiplied by p.

$p = 1$

$p = 2$

$$\begin{vmatrix} pa_{11} & a_{12} & a_{13} \\ pa_{21} & a_{22} & a_{23} \\ pa_{31} & a_{32} & a_{33} \end{vmatrix} = p\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

1. Evaluate the determinants :-

(1)
$$\begin{vmatrix} 5 & -2 & -4 \\ 3 & -1 & -3 \\ -2 & 1 & 2 \end{vmatrix} \quad 3\times 3$$

$$= 5\begin{vmatrix} -1 & -3 \\ 1 & 2 \end{vmatrix} - (-2)\begin{vmatrix} 3 & -3 \\ -2 & 2 \end{vmatrix} + (-4)\begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix}$$

$$= 5(-2-(-3)) + 2(6-6) - 4(3-2)$$

$$= 5(-2+3) + 2(0) - 4(1)$$

$$= 5(1) + 0 - 4$$

$$= 5-4$$

$$= 1$$

(4)
$$\begin{vmatrix} 3a & a-l & a \\ 3a & a+l & a-l \\ 3a & a & a+l \end{vmatrix}$$

P = same

$R_2 - R_1 = R_2$

$$\begin{array}{ccc} a & -l & \\ \cancel{+a} & +l & \\ \hline \end{array}$$

$$= 3a\begin{vmatrix} 1 & a-l & a \\ 1 & a+l & a-l \\ 1 & a & a+l \end{vmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} = 3a\begin{vmatrix} 1 & a-l & a \\ 0 & 2l & -l \\ 0 & l & l \end{vmatrix}$$

$R_3 - R_1 = R_3$

$\cancel{1} + \cancel{a} + \cancel{l} + \cancel{a} - l - \cancel{1} - \cancel{a} + l - \cancel{a}$

$$\boxed{0 \quad 2l \quad -l}$$

$$\begin{array}{ccc} 1 & a & a+l \\ -1 & -a+l & -a \\ \hline 0 & l & l \end{array}$$

$$3a\left[1\begin{vmatrix} 2l & -l \\ l & l \end{vmatrix} - (a-l)\begin{vmatrix} 0 & -l \\ 0 & l \end{vmatrix} + a\begin{vmatrix} 0 & 2l \\ 0 & l \end{vmatrix}\right]$$

$$= 3a\left(2l^2 - (-l^2)\right)$$

$$= 3a\left(2l^2 + l^2\right) = 3a(3l^2)$$

$$= 9al^2$$