8 Determining Process and Measurement Systems Capability

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  S8.1 Fixed Versus Random Factors in the Analysis of Variance
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Learning Objectives

1. Investigate and analyze process capability using control charts, histograms, and probability plots
2. Understand the difference between process capability and process potential
3. Calculate and properly interpret process capability ratios
4. Understand the role of the normal distribution in interpreting most process capability ratios
5. Calculate confidence intervals on process capability ratios
6. Know how to conduct and analyze a measurement systems capability (or gauge R & R) experiment
7. Know how to estimate the components of variability in a measurement system
8. Know how to set specifications on components in a system involving interaction components to ensure that overall system requirements are met
9. Estimate the natural limits of a process from a sample of data from that process

Process Capability

Process capability refers to the **uniformity** of the process. Obviously, the variability of critical-to-quality characteristics in the process is a measure of the uniformity of output. There are two ways to think of this variability:

1. The natural or inherent variability in a critical-to-quality characteristic at a specified time; that is, "instantaneous" variability
2. The variability in a critical-to-quality characteristic over time

Natural tolerance limits are defined as follows:

$$UNTL = \mu + 3\sigma$$

$$LNTL = \mu - 3\sigma$$

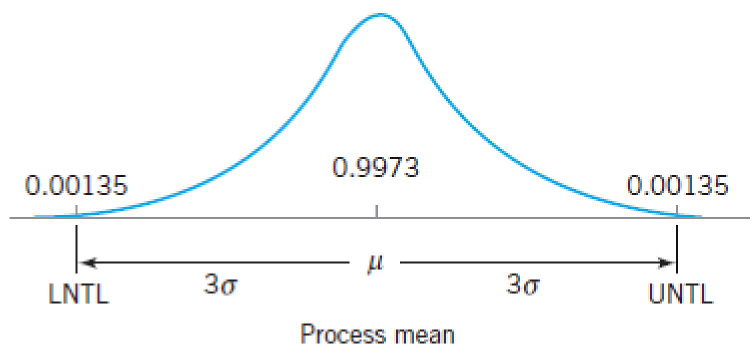

FIGURE 8.1 Upper and lower natural tolerance limits in the normal distribution.

We define **process capability analysis** as an engineering study to estimate process capability. The estimate of process capability may be in the form of a probability distribution having a specified shape, center (mean), and spread (standard deviation). For example, we may determine that the process output is normally distributed with mean $\mu = 1.0$ cm and standard deviation $\sigma = 0.001$ cm. In this sense, a process capability analysis may be performed **without regard to specifications on the quality characteristic.** Alternatively, we may express process capability as a percentage outside of specifications. However, specifications are not *necessary* to process capability analysis.

Uses of process capability data:

1. Predicting how well the process will hold the tolerances
2. Assisting product developers/designers in selecting or modifying a process
3. Assisting in establishing an interval between sampling for process monitoring
4. Specifying performance requirements for new equipment
5. Selecting between competing suppliers and other aspects of supply chain management
6. Planning the sequence of production processes when there is an interactive effect of processes on tolerances
7. Reducing the variability in a manufacturing process

Reasons for Poor Process Capability

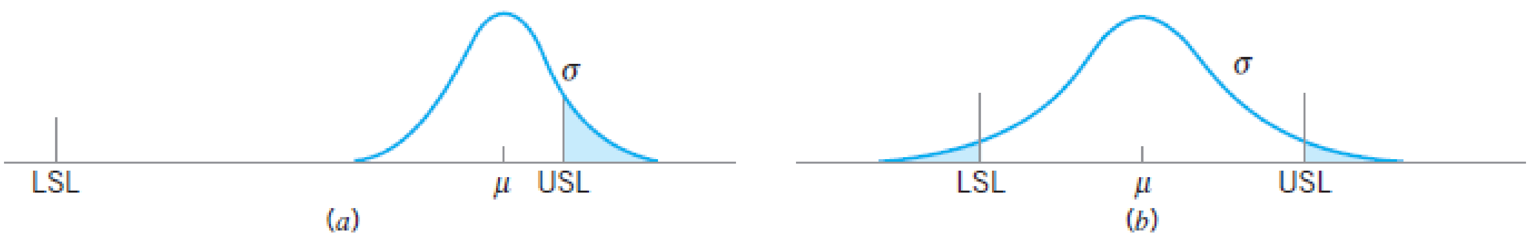

FIGURE 8.3 Some reasons for poor process capability. (*a*) Poor process centering. (*b*) Excess process variability.

Process may have good potential capability

8.2 Process Capability Analysis Using a Histogram or a Probability Plot

8.2.1 Using the Histogram

The histogram can be helpful in estimating process capability. Alternatively, a stem-and-leaf plot may be substituted for the histogram. At least 100 or more observations should be available for the histogram (or the stem-and-leaf plot) to be moderately stable so that a reasonably reliable estimate of process capability may be obtained. If the quality engineer has access to the process and can control the data-collection effort, the following steps should be followed prior to data collection:

1. Choose the machine or machines to be used. If the results based on one (or a few) machines are to be extended to a larger population of machines, the machine selected should be representative of those in the population. Furthermore, if the machine has multiple workstations or heads, it may be important to collect the data so that head-to-head variability can be isolated. This may imply that designed experiments should be used.
2. Select the process operating conditions. Carefully define conditions, such as cutting speeds, feed rates, and temperatures, for future reference. It may be important to study the effects of varying these factors on process capability.
3. Select a representative operator. In some studies, it may be important to estimate *operator* variability. In these cases, the operators should be selected at random from the population of operators.
4. Carefully monitor the data-collection process, and record the time order in which each unit is produced.

The histogram, along with the sample average $\bar{x}$ and sample standard deviation s, provides information about process capability. You may wish to review the guidelines for constructing histograms in Chapter 3.

EXAMPLE 8.1 Estimating Process Capability with a Histogram

Figure 8.2 presents a histogram of the bursting strength of 100 glass containers. The data are shown in Table 8.1. What is the capability of the process?

SOLUTION

Analysis of the 100 observations gives

$$\bar{x} = 264.06 \qquad s = 32.02$$

Consequently, the process capability would be estimated as

$$\bar{x} \pm 3s$$

or

$$264.06 \pm 3(32.02) \simeq 264 \pm 96 \text{ psi}$$

Furthermore, the shape of the histogram implies that the distribution of bursting strength is approximately normal. Thus, we can estimate that approximately 99.73% of the bottles manufactured by this process will burst between 168 and 360 psi. Note that we can estimate process capability *independently of the specifications on bursting strength.*

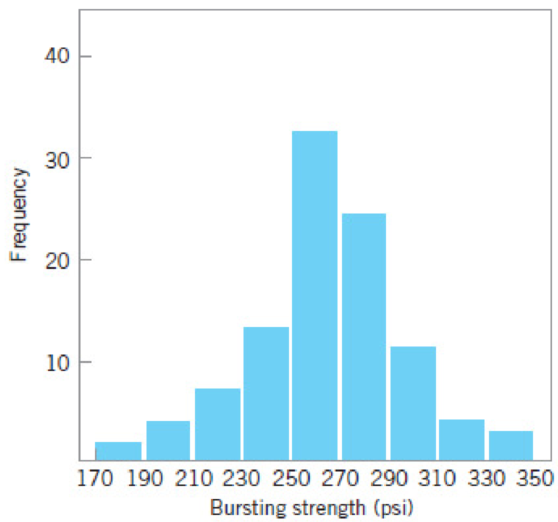

FIGURE 8.2 Histogram for the bursting-strength data.

TABLE 8.1
Bursting Strengths for 100 Glass Containers

265	197	346	280	265	200	221	265	261	278
205	286	317	242	254	235	176	262	248	250
263	274	242	260	281	246	248	271	260	265
307	243	258	321	294	328	263	245	274	270
220	231	276	228	223	296	231	301	337	298
268	267	300	250	260	276	334	280	250	257
260	281	208	299	308	264	280	274	278	210
234	265	187	258	235	269	265	253	254	280
299	214	264	267	283	235	272	287	274	269
215	318	271	293	277	290	283	258	275	251

8.2.2 Probability Plotting

Probability plotting is an alternative to the histogram that can be used to determine the shape, center, and spread of the distribution. It has the advantage that it is unnecessary to divide the range of the variable into class intervals, and it often produces reasonable results for moderately small samples (which the histogram will not). Generally, a probability plot is a graph of the ranked data versus the sample cumulative frequency on special paper with a vertical scale chosen so that the cumulative distribution of the assumed type is a straight line. In Chapter 3 we discussed and illustrated **normal probability plots.** These plots are very useful in process capability studies.

To illustrate the use of a normal probability plot in a process capability study, consider the following 20 observations on glass container bursting strength: 197, 200, 215, 221, 231, 242, 245, 258, 265, 265, 271, 275, 277, 278, 280, 283, 290, 301, 318, and 346. Figure 8.4 is the normal probability plot of strength. Note that the data lie nearly along a straight line, implying that the distribution of bursting strength is normal. Recall from Chapter 4 that the mean of the normal distribution is the fiftieth percentile, which we may estimate from Fig. 8.4 as approximately 265 psi, and the standard deviation of the distribution is the *slope* of the straight line. It is convenient to estimate the standard deviation as the difference between the eighty-fourth and the fiftieth percentiles. For the strength data shown above and using Fig. 8.4, we find that

$$\hat{\sigma} = 84\text{th percentile} - 50\text{th percentile} = 298 - 265 \text{ psi} = 33 \text{ psi}$$

Note that $\hat{\mu} = 265$ psi and $\hat{\sigma} = 33$ psi are not far from the sample average $\bar{x} = 264.06$ and standard deviation $s = 32.02$.

Probability Plotting

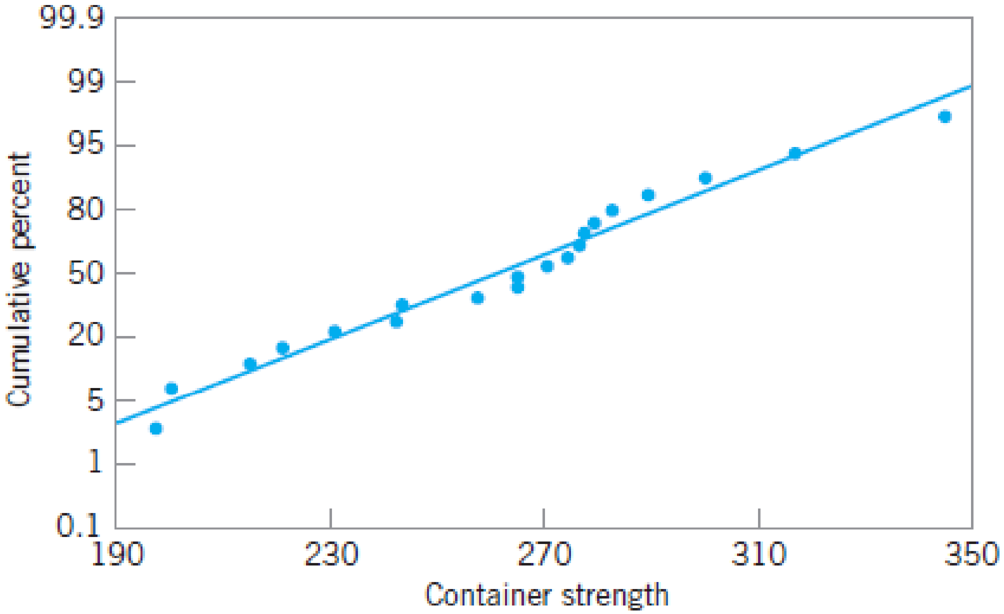

FIGURE 8.4 Normal probability plot of the container-strength data.

Care should be exercised in using probability plots. If the data do not come from the assumed distribution, inferences about process capability drawn from the plot may be seriously in error. Figure 7-5 presents a normal probability plot of times to failure (in hours) of a valve in a chemical plant. From examining this plot, we can see that the distribution of failure time is not normal.

An obvious disadvantage of probability plotting is that it is not an objective procedure. It is possible for two analysts to arrive at different conclusions using the same data. For this reason, it is often desirable to supplement probability plots with more formal statistically based goodness-of-fit tests. A good introduction to these tests is in Shapiro (1980). Augmenting the interpretation of a normal probability plot with the Shapiro–Wilk test for normality can make the procedure much more powerful and objective.

- The distribution may not be normal; other types of probability plots can be useful in determining the appropriate distribution.

8.3 Process Capability Ratios

8.3.1 Use and Interpretation of C_p

It is frequently convenient to have a simple, quantitative way to express process capability. One way to do so is through the **process capability ratio (PCR)** C_p first introduced in Chapter 6. Recall that

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} \tag{8.4}$$

where USL and LSL are the upper and lower specification limits, respectively. C_p and other process capability ratios are used extensively in industry. They are also widely *misused*. We will point out some of the more common abuses of process capability ratios. An excellent recent book on process capability ratios that is highly recommended is Kotz and Lovelace (1998). There is also extensive technical literature on process capability analysis and process capability ratios. The review paper by Kotz and Johnson (2002) and the bibliography (papers) by Spiring, Leong, Cheng, and Yeung (2003) and Yum and Kim (2011) are excellent sources.

In a practical application, the process standard deviation σ is almost always unknown and must be replaced by an estimate σ. To estimate σ we typically use either the *sample standard deviation s* or $\overline{R}/d_2$ (when variables control charts are used in the capability study). This results in an estimate of C_p—say,

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} \tag{8.5}$$

To illustrate the calculation of C_p, recall the semiconductor hard-bake process first analyzed in Example 6.1 using $\bar{x}$ and R charts. The specifications on flow width are USL = 1.00 microns and LSL = 2.00 microns, and from the R chart we estimated $\sigma = \bar{R}/d_2 = 0.1398$. Thus, our estimate of the PCR C_p is

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{2.00 - 1.00}{6(0.1398)} = 1.192$$

In Chapter 6, we assumed that flow width is approximately normally distributed (a reasonable assumption, based on the histogram in Fig. 8.7) and the cumulative normal distribution table in the Appendix was used to estimate that the process produces approximately 350 ppm (parts per million) defective.

The PCR C_p in equation 8.4 has a useful practical interpretation—namely,

$$P = \left(\frac{1}{C_p}\right)100 \tag{8.6}$$

This is the percentage of the specification band used up by the process.

For the hard bake process:

$$P = \left(\frac{1}{1.192}\right)100 = 83.89$$

One-Sided PCR

$$C_{pu} = \frac{\text{USL} - \mu}{3\sigma} \quad \text{(upper specification only)} \qquad (8.7)$$

$$C_{pl} = \frac{\mu - \text{LSL}}{3\sigma} \quad \text{(lower specification only)} \qquad (8.8)$$

Interpretation of the PCR

■ **TABLE 8.2**

Values of the Process Capability Ratio (C_p) and Associated Process Fallout for a Normally Distributed Process (in Defective ppm) That Is in Statistical Control

PCR	Process Fallout (in defective ppm) One-Sided Specifications	Process Fallout (in defective ppm) Two-Sided Specifications
0.25	226,628	453,255
0.50	66,807	133,614
0.60	35,931	71,861
0.70	17,865	35,729
0.80	8,198	16,395
0.90	3,467	6,934
1.00	1,350	2,700
1.10	484	967
1.20	159	318
1.30	48	96
1.40	14	27
1.50	4	7
1.60	1	2
1.70	0.17	0.34
1.80	0.03	0.06
2.00	0.0009	0.0018

Assumptions for Interpretation of Numbers in Table 8.2

1. The quality characteristic has a normal distribution.
2. The process is in statistical control.
3. In the case of two-sided specifications, the process mean is centered between the lower and upper specification limits.

- Violation of these assumptions can lead to big trouble in using the data in Table 8.2.

■ **TABLE 8.3**

Recommended Minimum Values of the Process Capability Ratio

	Two-Sided Specifications	One-Sided Specifications
Existing processes	1.33	1.25
New processes	1.50	1.45
Safety, strength, or critical parameter, existing process	1.50	1.45
Safety, strength, or critical parameter, new process	1.67	1.60

- C_p does not take process centering into account
- It is a measure of **potential** capability, not **actual** capability

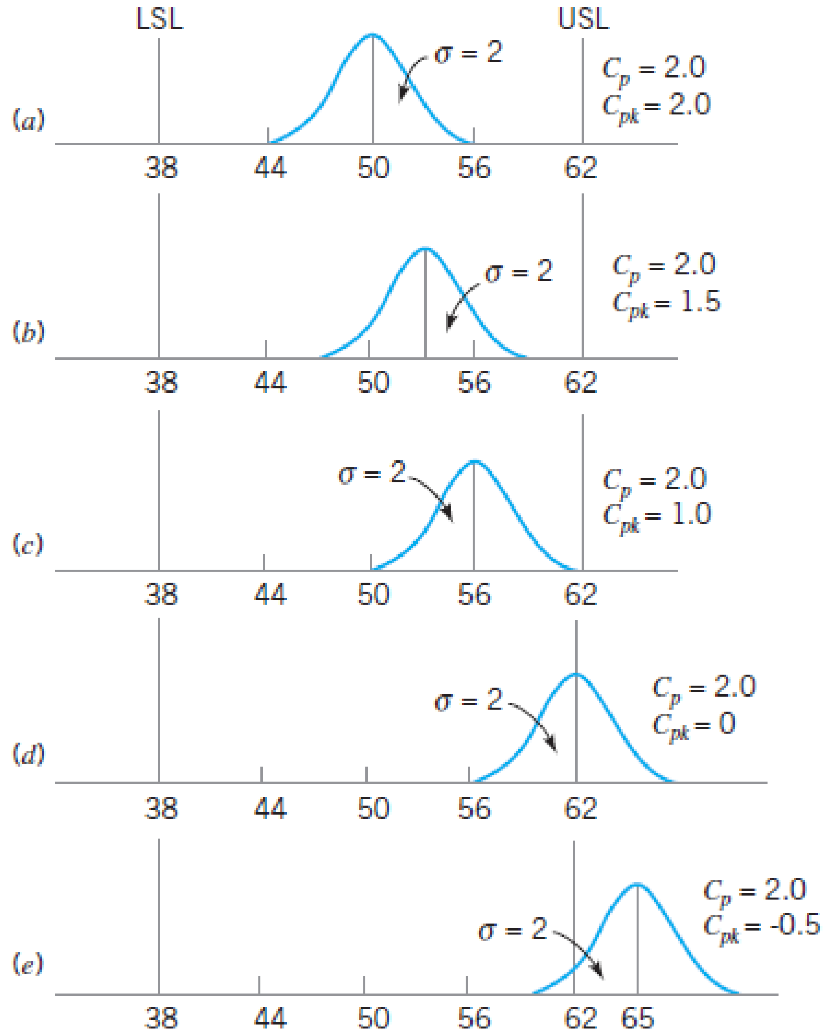

FIGURE 8.8 Relationship of C_p and C_{pk}.

A Measure of **Actual** Capability

$$C_{pk} = \min\left(C_{pu}, C_{pl}\right) \tag{8.9}$$

Note that C_{pk} is simply the one-sided PCR for the specification limit nearest to the process average. For the process shown in Figure 8.8*b*, we would have

$$\begin{aligned}
C_{pk} &= \min\left(C_{pu}, C_{pl}\right) \\
&= \min\left(C_{pu} = \frac{\text{USL} - \mu}{3\sigma}, C_{pl} = \frac{\mu - \text{LSL}}{3\sigma}\right) \\
&= \min\left(C_{pu} = \frac{62 - 53}{3(2)} = 1.5, C_{pl} = \frac{53 - 38}{3(2)} = 2.5\right) \\
&= 1.5
\end{aligned}$$

Generally, if $C_p = C_{pk}$, the process is centered at the midpoint of the specifications, and when $C_{pk} < C_p$ the process is off center.

Normality and Process Capability Ratios

- The assumption of normality is critical to the usual interpretation of these ratios (such as Table 8.2)
- For non-normal data, options are
  1. Transform non-normal data to normal
  2. Extend the usual definitions of PCRs to handle non-normal data
  3. Modify the definitions of PCRs for general families of distributions

8.3.5 Confidence Intervals and Tests on Process Capability Ratios

Confidence Intervals on Process Capability Ratios. Much of the industrial use of process capability ratios focuses on computing and interpreting the **point estimate** of the desired quantity. It is easy to forget that $\hat{C}_p$ or $\hat{C}_{pk}$ (for examples) are simply point estimates, and, as such, are subject to statistical fluctuation. An alternative that should become standard practice is to report **confidence intervals for process capability ratios.**

It is easy to find a confidence interval for the "first generation" ratio C_p. If we replace σ by s in the equation for C_p, we produce the usual point estimator $\hat{C}_p$. If the quality characteristic follows a normal distribution, then a $100(1 - \alpha)\%$ CI on C_p is obtained from

$$\frac{\text{USL} - \text{LSL}}{6s}\sqrt{\frac{\chi^2_{1-\alpha/2,n-1}}{n-1}} \le C_p \le \frac{\text{USL} - \text{LSL}}{6s}\sqrt{\frac{\chi^2_{\alpha/2,n-1}}{n-1}} \qquad (8.19)$$

or

$$\hat{C}_p\sqrt{\frac{\chi^2_{1-\alpha/2,n-1}}{n-1}} \le C_p \le \hat{C}_p\sqrt{\frac{\chi^2_{\alpha/2,n-1}}{n-1}} \qquad (8.20)$$

where $\chi^2_{1-\alpha/2,n-1}$ and $\chi^2_{\alpha/2,n-1}$ are the lower $\alpha/2$ and upper $\alpha/2$ percentage points of the chi-square distribution with $n - 1$ degrees of freedom. These percentage points are tabulated in Appendix Table III.

EXAMPLE 8.4 A Confidence Interval in C_p

Suppose that a stable process has upper and lower specifications at USL = 62 and LSL = 38. A sample of size $n = 20$ from this process reveals that the process mean is centered approximately at the midpoint of the specification interval and that the sample standard deviation $s = 1.75$. Find a 95% CI on C_p.

SOLUTION

A point estimate of C_p is

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6s} = \frac{62 - 38}{6(1.75)} = 2.29$$

The 95% confidence interval on C_p is found from equation 8.20 as follows:

$$\hat{C}_p \sqrt{\frac{\chi^2_{1-0.025,n-1}}{n-1}} \le C_p \le \hat{C}_p \sqrt{\frac{\chi^2_{0.025,n-1}}{n-1}}$$

$$2.29\sqrt{\frac{8.91}{19}} \le C_p \le 2.29\sqrt{\frac{32.85}{19}}$$

$$1.57 \le C_p \le 3.01$$

where $\chi^2_{0.975,19} = 8.91$ and $\chi^2_{0.025,19} = 32.85$ were taken from Appendix Table III.

For more complicated ratios such as C_{pk} and C_{pm}, various authors have developed approximate confidence intervals; for example, see Zhang, Stenback, and Wardrop (1990), Bissell (1990), Kushler and Hurley (1992), and Pearn et al. (1992). If the quality characteristic is normally distributed, then an approximate 100(1 – α)% CI on C_{pk} is given as follows.

$$\hat{C}_{pk}\left[1 - Z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right] \le C_{pk} \le \hat{C}_{pk}\left[1 + Z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right] \quad (8.21)$$

Kotz and Lovelace (1998) give an extensive summary of confidence intervals for various PCRs.

EXAMPLE 8.5 A Confidence Interval on C_{pk}

A sample of size $n = 20$ from a stable process is used to estimate C_{pk}, with the result that $\hat{C}_{pk} = 1.33$. Find an approximate 95% CI on C_{pk}.

SOLUTION

Using equation 8.21, an approximate 95% CI on C_{pk} is

$$\hat{C}_{pk}\left[1 - Z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right] \le C_{pk} \le \hat{C}_{pk}\left[1 + Z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^2} + \frac{1}{2(n-1)}}\right]$$

$$1.33\left[1 - 1.96\sqrt{\frac{1}{9(20)(1.33)^2} + \frac{1}{2(19)}}\right] \le C_{pk} \le 1.33\left[1 + 1.96\sqrt{\frac{1}{9(20)(1.33)^2} + \frac{1}{2(19)}}\right]$$

or

$$0.88 \le C_{pk} \le 1.78$$

This is an extremely wide confidence interval. Based on the sample data, the ratio C_{pk} could be less than 1 (a very bad situation), or it could be as large as 1.78 (a reasonably good situation). Thus, we have learned very little about actual process capability, because C_{pk} is very imprecisely estimated. The reason for this, of course, is that a very small sample ($n = 20$) has been used.

Process Capability Analysis using Control Charts

■ **TABLE 8.5**

Glass Container Strength Data (psi)

Sample	Data					$\bar{x}$	R
1	265	205	263	307	220	252.0	102
2	268	260	234	299	215	255.2	84
3	197	286	274	243	231	246.2	89
4	267	281	265	214	318	269.0	104
5	346	317	242	258	276	287.8	104
6	300	208	187	264	271	246.0	113
7	280	242	260	321	228	266.2	93
8	250	299	258	267	293	273.4	49
9	265	254	281	294	223	263.4	71
10	260	308	235	283	277	272.6	73
11	200	235	246	328	296	261.0	128
12	276	264	269	235	290	266.8	55
13	221	176	248	263	231	227.8	87
14	334	280	265	272	283	286.8	69
15	265	262	271	245	301	268.8	56
16	280	274	253	287	258	270.4	34
17	261	248	260	274	337	276.0	89
18	250	278	254	274	275	266.2	28
19	278	250	265	270	298	272.2	48
20	257	210	280	269	251	253.4	70
						$\bar{\bar{x}} = 264.06$	$\bar{R} = 77.3$

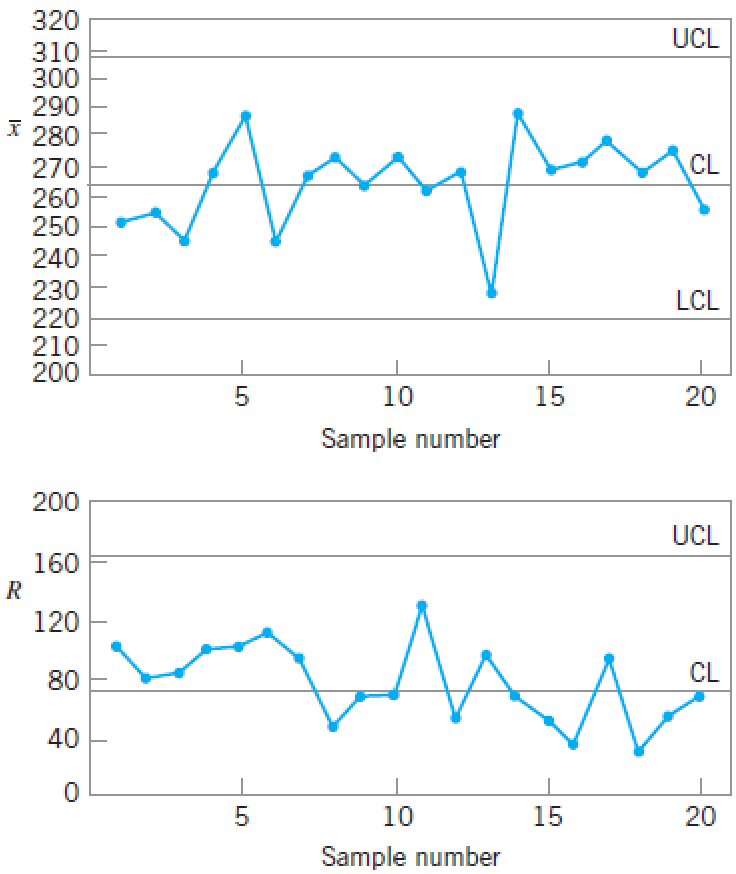

FIGURE 8.12 $\bar{x}$ and R charts for the bottle-strength data.

$$\hat{\mu} = \bar{\bar{x}} = 264.06$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{77.3}{2.326} = 33.23$$

Since LSL = 200

$$\hat{C}_{pl} = \frac{\mu - \text{LSL}}{3\hat{\sigma}} = \frac{264.06 - 200}{3(33.23)} = 0.64$$

7.8 Gauge and Measurement Systems Capability Studies

- Determine how much of the observed variability is due to the gauge or measurement system
- Isolate the components of variability in the measurement system
- Assess whether the gauge is capable (suitable for the intended application)

To introduce some of the basic ideas of measurement systems analysis (MSA), consider a simple but reasonable model for measurement system capability studies

$$y = x + \varepsilon \tag{8.23}$$

where y is the total observed measurement, x is the true value of the measurement on a unit of product, and ε is the measurement error. We will assume that x and ε are normally and independently distributed random variables with means μ and 0 and variances (σ_P^2) and $(\sigma_{\text{Gauge}}^2)$, respectively. The variance of the total observed measurement, y, is then

$$\sigma_{\text{Total}}^2 = \sigma_P^2 + \sigma_{\text{Gauge}}^2 \tag{8.24}$$

Control charts and other statistical methods can be used to separate these components of variance, as well as to give an assessment of gauge capability.

EXAMPLE 8.7 Measuring Gauge Capability

An instrument is to be used as part of a proposed SPC implementation. The quality-improvement team involved in designing the SPC system would like to get an assessment of gauge capability. Twenty units of the product are obtained, and the process operator who will actually take the measurements for the control chart uses the instrument to measure each unit of product twice. The data are shown in Table 8.6.

SOLUTION

Figure 8.14 shows the $\bar{x}$ and R charts for these data. Note that the $\bar{x}$ chart exhibits many out-of-control points. This is to be expected, because in this situation the $\bar{x}$ chart has an interpretation that is somewhat different from the usual interpretation. The $\bar{x}$ chart in this example shows the **discriminating power** of the instrument—literally, the ability of the gauge to distinguish between units of product. The R chart directly shows the magnitude of measurement error, or the gauge capability. The R values represent the difference between measurements made on the same unit using the same instrument. In this example, the R chart is in control. This indicates that the operator is having no difficulty in making consistent measurements. Out-of-control points on the R chart could indicate that the operator is having difficulty using the instrument.

The standard deviation of measurement error, σ_{Gauge}, can be estimated as follows:

$$\hat{\sigma}_{\text{Gauge}} = \frac{\bar{R}}{d_2} = \frac{1.0}{1.128} = 0.887$$

The distribution of measurement error is usually well approximated by the normal. Thus, $\hat{\sigma}_{\text{Guage}}$ is a good estimate of gauge capability.

TABLE 8.6

Parts Measurement Data

Part Number	Measurements 1	Measurements 2	$\bar{x}$	R
1	21	20	20.5	1
2	24	23	23.5	1
3	20	21	20.5	1
4	27	27	27.0	0
5	19	18	18.5	1
6	23	21	22.0	2
7	22	21	21.5	1
8	19	17	18.0	2
9	24	23	23.5	1
10	25	23	24.0	2
11	21	20	20.5	1
12	18	19	18.5	1
13	23	25	24.0	2
14	24	24	24.0	0
15	29	30	29.5	1
16	26	26	26.0	0
17	20	20	20.0	0
18	19	21	20.0	2
19	25	26	25.5	1
20	19	19	19.0	0
			$\bar{\bar{x}} = 22.3$	$\bar{R} = 1.0$

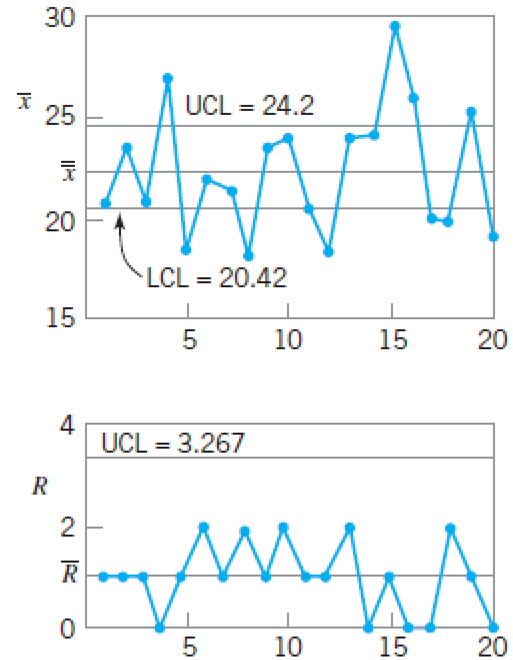

FIGURE 8.14 Control charts for the gauge capability analysis in Example 8.7.

The *P/T* ratio:

$$P/T = \frac{k\hat{\sigma}_{\text{Guage}}}{\text{USL} - \text{LSL}} \tag{8.25}$$

In equation 8.25, popular choices for the constant k are $k = 5.15$ and $k = 6$. The value $k = 5.15$ corresponds to the limiting value of the number of standard deviations between bounds of a 95% tolerance interval that contains at least 99% of a normal population, and $k = 6$ corresponds to the number of standard deviations between the usual natural tolerance limits of a normal population.

The part used in Example 8.7 has USL = 60 and LSL = 5. Therefore, taking $k = 6$ in equation 8.25, an estimate of the P/T ratio is

$$P/T = \frac{6(0.887)}{60 - 5} = \frac{5.32}{55} = 0.097$$

Values of the estimated ratio P/T of 0.1 or less often are taken to imply adequate gauge capability. This is based on the generally used rule that requires a measurement device to be calibrated in units one-tenth as large as the accuracy required in the final measurement. However, we should use **caution** in accepting this general rule of thumb in all cases. A gauge must be sufficiently capable to measure product accurately enough and precisely enough so that the analyst can make the correct decision. This may not necessarily require that $P/T \leq 0.1$.

Estimating the Variance Components

$$\hat{\sigma}^2_{\text{Total}} = s^2 = (3.17)^2 = 10.05$$

Since from equation (8.24) we have

$$\sigma^2_{\text{Total}} = \sigma^2_P + \sigma^2_{\text{Gauge}}$$

and because we have an estimate of $\hat{\sigma}^2_{\text{Gauge}} = (0.887)^2 = 0.79$, we can obtain an estimate of σ^2_P as

$$\hat{\sigma}^2_P = \hat{\sigma}^2_{\text{Total}} - \hat{\sigma}^2_{\text{Gauge}} = 10.05 - 0.79 = 9.26$$

Therefore, an estimate of the standard deviation of the product characteristic is

$$\hat{\sigma}_p = \sqrt{9.26} = 3.04$$

There are other measures of gauge capability that have been proposed. One of these is the ratio of process (part) variability to total variability:

$$\rho_P = \frac{\sigma_P^2}{\sigma_{\text{Total}}^2} \tag{8.26}$$

and another is the ratio of measurement system variability to total variability:

$$\rho_M = \frac{\sigma_{\text{Gauge}}^2}{\sigma_{\text{Total}}^2} \tag{8.27}$$

Obviously, $\rho_P = 1 - \rho_M$. For the situation in Example 8.7 we can calculate an estimate of ρ_M as follows:

$$\hat{\rho}_M = \frac{\hat{\sigma}_{\text{Gauge}}^2}{\hat{\sigma}_{\text{Total}}^2} = \frac{0.79}{10.05} = 0.0786$$

Thus the variance of the measuring instrument contributes about 7.86% of the total observed variance of the measurements.

Another measure of measurement system adequacy is defined by the AIAG (1995) [note that there is also on updated edition of this manual, AIAG (2002)] as the **signal-to-noise ratio (*SNR*)**:

$$SNR = \sqrt{\frac{2\rho_P}{1-\rho_P}} \tag{8.28}$$

AIAG defined the *SNR* as the number of distinct levels or categories that can be reliably obtained from the measurements. A value of five or greater is recommended, and a value of less than two indicates inadequate gauge capability. For Example 8.7 we have $\hat{\rho}_M = 0.0786$, and using $\hat{\rho}_P = 1 - \hat{\rho}_M$ we find that $\hat{\rho}_P = 1 - \hat{\rho}_M = 1 - 0.0786 = 0.9214$, so an estimate of the *SNR* in equation (8.28) is

$$SNR = \sqrt{\frac{2\hat{\rho}_P}{1-\hat{\rho}_P}} = \sqrt{\frac{2(0.9214)}{1-0.9214}} = 4.84$$

The gauge is not capable by this criterion

Discrimination Ratio

$$DR = \frac{1 + \rho_P}{1 - \rho_P} \tag{8.29}$$

Some authors have suggested that for a gauge to be capable the *DR* must exceed four. This is a very arbitrary requirement. For the situation in Example 8.7, we would calculate an estimate of the discrimination ratio as

$$\widehat{DR} = \frac{1 + \rho_P}{1 - \rho_P} = \frac{1 + 0.9214}{1 - 0.9214} = 24.45$$

Clearly by this measure, the gauge is capable.

Accuracy and Precision

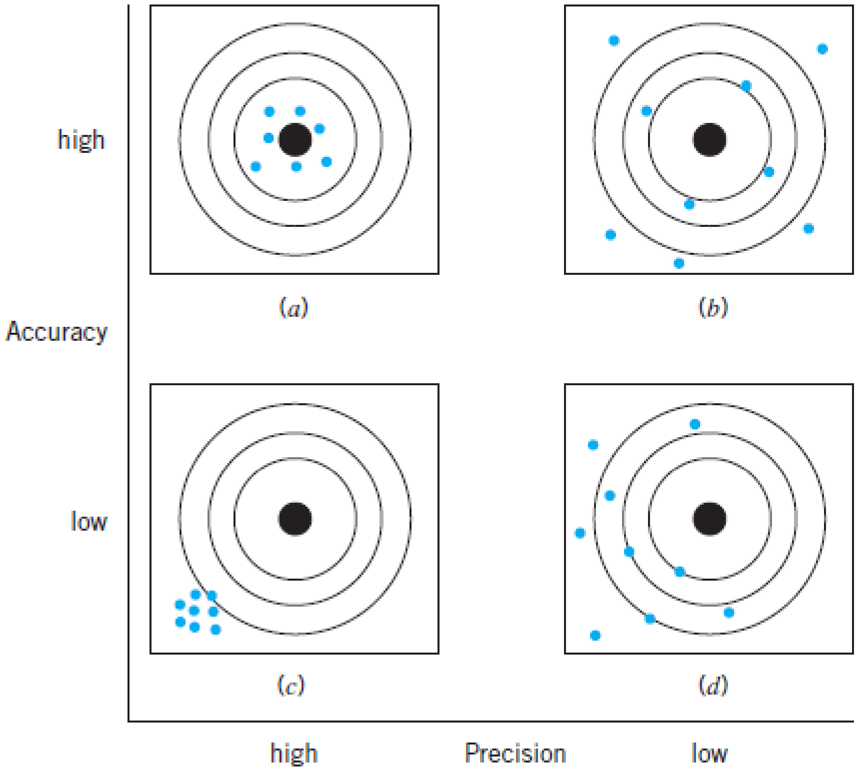

FIGURE 8.15 The concepts of accuracy and precision. (*a*) The gauge is accurate and precise. (*b*) The gauge is accurate but not precise. (*c*) The gauge is not accurate but it is precise. (*d*) The gauge is neither accurate nor precise.

We have focused only on precision

Gauge R&R Studies

It is also possible to design measurement systems capability studies to investigate two components of measurement error, commonly called the **repeatability** and the **reproducibility** of the gauge. We define reproducibility as the variability due to different operators using the gauge (or different time periods, or different environments, or in general, different conditions) and repeatability as reflecting the basic inherent precision of the gauge itself. That is,

$$\sigma^2_{\text{Measurement Error}} = \sigma^2_{\text{Gauge}} = \sigma^2_{\text{Repeatability}} + \sigma^2_{\text{Reproducibility}} \qquad (8.30)$$

The experiment used to measure the components of σ^2_{Gauge} is usually called a gauge R & R study, for the two components of σ^2_{Gauge}. We now show how to analyze **gauge R & R experiments.**

Gauge R&R Studies Are Usually Conducted with a Factorial Experiment

If there are a randomly selected parts and b randomly selected operators, and each operator measures every part n times, then the measurements (i = part, j = operator, k = measurement) could be represented by the model

$$y_{ijk} = \mu + P_i + O_j + (PO)_{ij} + \varepsilon_{ijk} \qquad \begin{cases} i = 1, 2, \ldots, p \\ j = 1, 2, \ldots, o \\ k = 1, 2, \ldots, n \end{cases}$$

where the model parameters P_i, O_j, $(PO)_{ij}$, and ε_{ijk} are all independent random variables that represent the effects of parts, operators, the interaction or joint effects of parts and operators, and random error. This is a **random effects model analysis of variance (ANOVA)**. It is also sometimes called the standard model for a gauge R & R experiment. We assume that the random variables P_i, O_j, $(PO)_{ij}$, and ε_{ijk} are normally distributed with mean zero and variances given by $V(P_i) = \sigma_P^2$, $V(O_j) = \sigma_O^2$, $V[(PO)_{ij}] = \sigma_{PO}^2$, and $V(\varepsilon_{ijk}) = \sigma^2$. Therefore, the variance of any observation is

$$V(y_{ijk}) = \sigma_P^2 + \sigma_O^2 + \sigma_{PO}^2 + \sigma^2 \qquad (8.31)$$

and σ_P^2, σ_O^2, σ_{PO}^2, and σ^2 are the **variance components.** We want to estimate the variance components.

■ **TABLE 8.7**

Thermal Impedance Data (°C/$W \times 100$) for the Gauge R & R Experiment

Part Number	Inspector 1			Inspector 2			Inspector 3		
	Test 1	Test 2	Test 3	Test 1	Test 2	Test 3	Test 1	Test 2	Test 3
1	37	38	37	41	41	40	41	42	41
2	42	41	43	42	42	42	43	42	43
3	30	31	31	31	31	31	29	30	28
4	42	43	42	43	43	43	42	42	42
5	28	30	29	29	30	29	31	29	29
6	42	42	43	45	45	45	44	46	45
7	25	26	27	28	28	30	29	27	27
8	40	40	40	43	42	42	43	43	41
9	25	25	25	27	29	28	26	26	26
10	35	34	34	35	35	34	35	34	35

This is a two-factor factorial experiment

ANOVA methods are used to analyze the data and yo estimate the variance components

$$SS_{\text{Total}} = SS_{\text{Parts}} + SS_{\text{Operators}} + SS_{P \times O} + SS_{\text{Error}} \tag{8.32}$$

$$MS_P = \frac{SS_{\text{Parts}}}{p-1}$$

$$MS_O = \frac{SS_{\text{Operators}}}{o-1}$$

$$MS_{PO} = \frac{SS_{P \times O}}{(p-1)(o-1)}$$

$$MS_E = \frac{SS_{\text{Error}}}{po(n-1)}$$

$$E(MS_P) = \sigma^2 + n\sigma_{PO}^2 + bn\sigma_P^2$$

$$E(MS_O) = \sigma^2 + n\sigma_{PO}^2 + an\sigma_O^2$$

$$E(MS_{PO}) = \sigma^2 + n\sigma_{PO}^2$$

$$E(MS_E) = \sigma^2$$

$$\hat{\sigma}^2 = MS_E$$

$$\hat{\sigma}^2_{PO} = \frac{MS_{PO} - MS_E}{n}$$

$$\hat{\sigma}^2_O = \frac{MS_O - MS_{PO}}{pn}$$

$$\hat{\sigma}^2_P = \frac{MS_P - MS_{PO}}{on}$$

TABLE 8.8

ANOVA: Thermal Impedance versus Part Number, Operator

Factor	Type	Levels	Values						
Part Num	random	10	1	2	3	4	5	6	7
			8	9	10				
Operator	random	3	1	2	3				

Analysis of Variance for Thermal

Source	DF	SS	MS	F	P
Part Num	9	3,935.96	437.33	162.27	0.000
Operator	2	39.27	19.63	7.28	0.005
Part Num*Operator	18	48.51	2.70	5.27	0.000
Error	60	30.67	0.51		
Total	89	4,054.40			

Source	Variance component	Error term	Expected Mean Square for Each Term (using unrestricted model)
1 Part Num	48.2926	3	(4) + 3(3) + 9(1)
2 Operator	0.5646	3	(4) + 3(3) + 30(2)
3 Part Num*Operator	0.7280	4	(4) + 3(3)
4 Error	0.5111		(4)

$$\hat{\sigma}_P^2 = \frac{437.33 - 2.70}{(3)(3)} = 48.29$$

$$\hat{\sigma}_O^2 = \frac{19.63 - 2.70}{(10)(3)} = 0.56$$

$$\hat{\sigma}_{PO}^2 = \frac{2.70 - 0.51}{3} = 0.73$$

$$\sigma^2 = 0.51$$

- Negative estimates of a variance component would lead to filling a reduced model, such as, for example:

$$y_{ijk} = \mu + P_i + O_j + \varepsilon_{ijk}$$

Typically we think of σ^2 as the **repeatability** variance component, and the gauge **reproducibility** as the sum of the operator and the part × operator variance components,

$$\sigma^2_{\text{Reproducibility}} = \sigma^2_O + \sigma^2_{PO}$$

Therefore

$$\sigma^2_{\text{Gauge}} = \sigma^2_{\text{Reproducibility}} + \sigma^2_{\text{Repeatability}}$$

For this Example

$$\hat{\sigma}^2_{\text{Gauge}} = \hat{\sigma}^2 + \hat{\sigma}^2_O + \hat{\sigma}^2_{PO}$$
$$= 0.51 + 0.56 + 0.73$$
$$= 1.80$$

The lower and upper specifications on this power module are $LSL = 18$ and $USL = 58$. Therefore the P/T ratio for the gauge is estimated as

$$\widehat{P/T} = \frac{6\hat{\sigma}_{\text{Gauge}}}{\text{USL} - \text{LSL}} = \frac{6(1.34)}{58 - 18} = 0.27$$

By the standard measures of gauge capability, this gauge would not be considered capable because the estimate of the P/T ratio exceeds 0.10.

Important Terms and Concepts

ANOVA approach to a gauge R & R experiment
Components of gauge error
Components of measurement error
Confidence intervals for gauge R & R studies
Confidence intervals on process capability ratios
Consumer's risk or missed fault for a gauge
Control charts and process capability analysis
Delta method

Discrimination ratio (*DR*) for a gauge
Estimating variance components
Factorial experiment
Gauge R & R experiment
Graphical methods for process capability analysis
Measurement systems capability analysis
Natural tolerance limits for a normal distribution
Natural tolerance limits of a process
Nonparametric tolerance limits
Normal distribution and process capability ratios
One-sided process-capability ratios
PCR C_p
PCR C_{pk}
PCR C_{pm}
Precision and accuracy of a gauge
Precision-to-tolerance (P/T) ratio
Process capability
Process capability analysis
Process performance indices P_p and P_{pk}
Producer's risk or false failure for a gauge
Product characterization
Random effects model ANOVA
Signal-to-noise ratio (*SNR*) for a gauge
Tolerance stack-up problems
Transmission of error formula