15 Lot-by-Lot Acceptance Sampling for Attributes

CHAPTER OUTLINE

The supplemental material is on the textbook Website www.wiley.com/college/montgomery.

Learning Objectives

1. Understand the role of acceptance sampling in modern quality control systems
2. Understand the advantages and disadvantages of sampling
3. Understand the difference between attributes and variables sampling plans, and the major types of acceptance-sampling procedures
4. Know how single-, double-, and sequential-sampling plans are used
5. Understand the importance of random sampling
6. Know how to determine the OC curve for a single-sampling plan for attributes
7. Understand the effects of the sampling plan parameters on sampling plan performance
8. Know how to design single-sampling, double-sampling, and sequential-sampling plans for attributes
9. Know how rectifying inspection is used
10. Understand the structure and use of MIL STD 105E and its civilian counterpart plans
11. Understand the structure and use of the Dodge–Romig system of sampling plans

15.1 The Acceptance Sampling Problem

Typical application of acceptance sampling is for **lot disposition**, sometimes referred to as **lot sentencing**, for receiving inspection activities

- Accepted lots are put into production
- Rejected lots may be returned to supplier or subjected to other **lot-disposition action**

Sampling methods may also be used during various stages of production

Three aspects of sampling are important:

1. It is the purpose of acceptance sampling to sentence lots, not to estimate the lot quality. Most acceptance-sampling plans are not designed for estimation purposes.
2. Acceptance-sampling plans do not provide any *direct* form of quality control. Acceptance sampling simply accepts and rejects lots. Even if all lots are of the same quality, sampling will accept some lots and reject others, the accepted lots being no better than the rejected ones. Process controls are used to control and systematically improve quality, but acceptance sampling is not.
3. The most effective use of acceptance sampling is *not* to "inspect quality into the product," but rather as an audit tool to ensure that the output of a process conforms to requirements.

Situations where acceptance sampling is likely to be useful:

1. When testing is destructive
2. When the cost of 100% inspection is extremely high
3. When 100% inspection is not technologically feasible or would require so much calendar time that production scheduling would be seriously impacted
4. When there are many items to be inspected and the inspection error rate is sufficiently high that 100% inspection might cause a higher percentage of defective units to be passed than would occur with the use of a sampling plan
5. When the supplier has an excellent quality history, and some reduction in inspection from 100% is desired, but the supplier's process capability is sufficiently low as to make no inspection an unsatisfactory alternative
6. When there are potentially serious product liability risks, and although the supplier's process is satisfactory, a program for continuously monitoring the product is necessary

15.1.1 Advantages and Disadvantages of Sampling

When acceptance sampling is contrasted with 100% inspection, it has the following advantages:

1. It is usually less expensive because there is less inspection.
2. There is less handling of the product, hence reduced damage.
3. It is applicable to destructive testing.
4. Fewer personnel are involved in inspection activities.
5. It often greatly reduces the amount of inspection error.
6. The rejection of entire lots as opposed to the simple return of defectives often provides a stronger motivation to the supplier for quality improvements.

Acceptance sampling also has several disadvantages, however. These include the following:

1. There are risks of accepting "bad" lots and rejecting "good" lots.
2. Less information is usually generated about the product or about the process that manufactured the product.
3. Acceptance sampling requires planning and documentation of the acceptance-sampling procedure whereas 100% inspection does not.

We have pointed out that acceptance sampling is a "middle ground" between the extremes of 100% inspection and no inspection. It often provides a methodology for moving between these extremes as sufficient information is obtained on the control of the manufacturing process that produces the product. Although there is no direct control of quality in the application of an acceptance-sampling plan to an isolated lot, when that plan is applied to a stream of lots from a supplier, it becomes a means of providing protection for both the producer of the lot and the consumer. It also provides for an accumulation of quality history regarding the process that produces the lot, and it may provide feedback that is useful in process control, such as determining when process controls at the supplier's plant are not adequate. Finally, it may place economic or psychological pressure on the supplier to improve the production process.

Types of sampling plans

- One major classification is by data type, **variables** and **attributes**

- Another is based on the number of samples required for a decision. These include:
  - Single-sampling plans
  - Double-sampling plans
  - Multiple-sampling plans
  - Sequential-sampling plans
- Single-, double-, multiple-, and sequential sampling plans can be designed to produce equivalent results. Factors to consider include:
  - Administrative efficiency
  - Type of information produced by the plan
  - Average amount of inspection required by plan
  - Impact of the procedure on manufacturing flow

Lot Formation and Random Sampling

- There are a number of important considerations informing lots for inspection, including:
- Lots should be homogeneous.
- Larger lots are preferred over smaller ones.
- Lots should be conformable to materials-handling systems used in both supplier and consumer facilities.

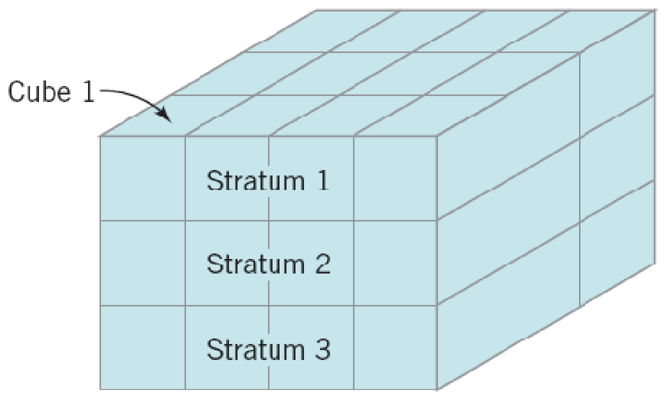


FIGURE 15.1 Stratifying a lot.

15.1.5 Guidelines for Using Acceptance Sampling

- An **acceptance-sampling plan** consists of sample size and acceptance/rejection criteria for lot sentencing
- An **acceptance-sampling scheme** is a set of procedures consisting of acceptance-sampling plans in which lot sizes, sample sizes, and acceptance/rejection criteria along with amount of 100% inspection and sampling are related
- A **sampling system** is a unified collection of one or more schemes

■ TABLE 15.1
Acceptance-Sampling Procedures

Objective	Attributes Procedure	Variables Procedure
Assure quality levels for consumer/producer	Select plan for specific OC curve	Select plan for specific OC curve
Maintain quality at a target	AQL system; MIL STD 105E, ANSI/ASQC Z1.4	AQL system; MIL STD 414, ANSI/ASQC Z1.9
Assure average outgoing quality level	AOQL system; Dodge–Romig plans	AOQL system
Reduce inspection, with small sample sizes, good-quality history	Chain sampling	Narrow-limit gauging
Reduce inspection after good-quality history	Skip-lot sampling; double sampling	Skip-lot sampling; double sampling
Assure quality no worse than target	LTPD plan; Dodge–Romig plans	LTPD plan; hypothesis testing

15.2 Single-Sampling Plans for Attributes

15.2.1 Definition of a Single-Sampling Plan

Suppose that a lot of size N has been submitted for inspection. A **single-sampling plan** is defined by the sample size n and the acceptance number c. Thus, if the lot size is $N = 10{,}000$, then the sampling plan

$$n = 89$$
$$c = 2$$

means that from a lot of size 10,000 a random sample of $n = 89$ units is inspected and the number of nonconforming or defective items d observed. If the number of observed defectives d is less than or equal to $c = 2$, the lot will be accepted. If the number of observed defectives d is greater than 2, the lot will be rejected. Since the quality characteristic inspected is an attribute, each unit in the sample is judged to be either conforming or nonconforming. One or several attributes can be inspected in the same sample; generally, a unit that is nonconforming to specifications on one or more attributes is said to be a defective unit. This procedure is called a single-sampling plan because the lot is sentenced based on the information contained in one sample of size n.

15.2.2 The OC Curve

An important measure of the performance of an acceptance-sampling plan is the **operating-characteristic (OC) curve.** This curve plots the probability of accepting the lot versus the lot fraction defective. Thus, the OC curve displays the discriminatory power of the sampling plan. That is, it shows the probability that a lot submitted with a certain fraction defective will be either accepted or rejected. The OC curve of the sampling plan $n = 89$, $c = 2$ is shown in

Fig. 15.2. It is easy to demonstrate how the points on this curve are obtained. Suppose that the lot size N is large (theoretically infinite). Under this condition, the distribution of the number of defectives d in a random sample of n items is binomial with parameters n and p, where p is the fraction of defective items in the lot. An equivalent way to conceptualize this is to draw lots of N items at random from a theoretically infinite process, and then to draw random samples of n from these lots. Sampling from the lot in this manner is the equivalent of sampling directly from the process. The probability of observing exactly d defectives is

$$P\{d \text{ defectives}\} = f(d) = \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d} \qquad (15.1)$$

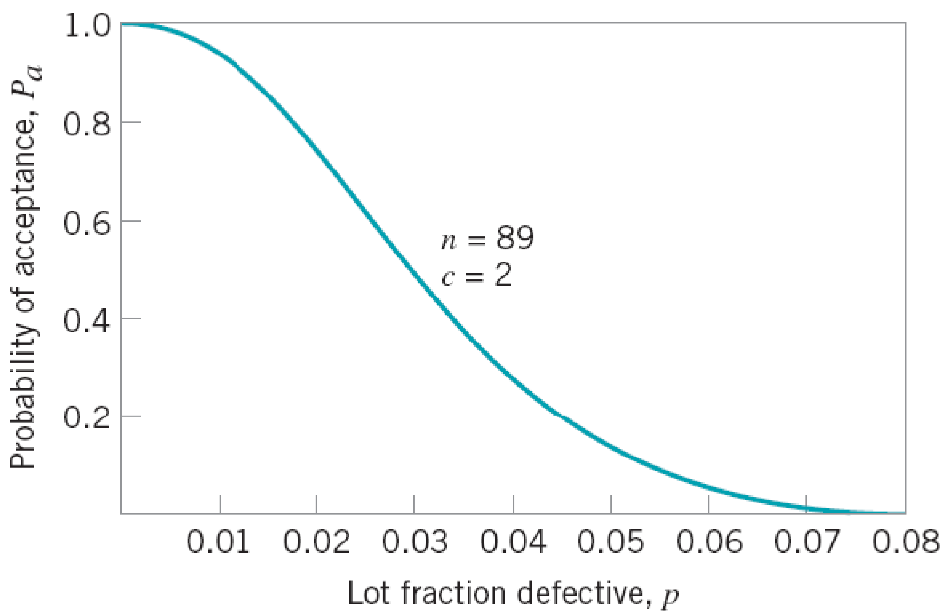
The probability of acceptance is simply the probability that d is less than or equal to c, or

$$P_a = P\{d \le c\} = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d} \qquad (15.2)$$

■ **TABLE 15.2**

Probabilities of Acceptance for the Single-Sampling Plan $n = 89$, $c = 2$

Fraction Defective, p	Probability of Acceptance, P_a
0.005	0.9897
0.010	0.9397
0.020	0.7366
0.030	0.4985
0.040	0.3042
0.050	0.1721
0.060	0.0919
0.070	0.0468
0.080	0.0230
0.090	0.0109



■ **FIGURE 15.2** OC curve of the single-sampling plan $n = 89$, $c = 2$.

Effect of *n* and *c* on OC curves:

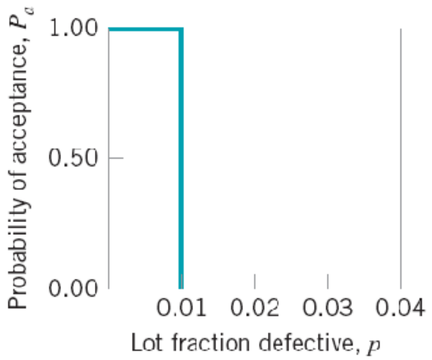


FIGURE 15.3 Ideal OC curve.

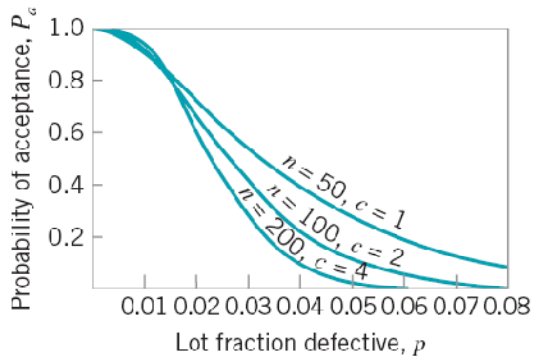


FIGURE 15.4 OC curves for different sample sizes.

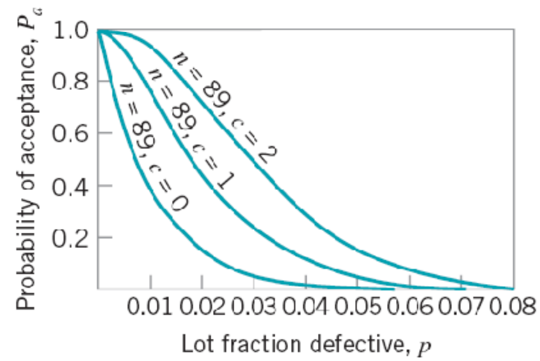


FIGURE 15.5 The effect of changing the acceptance number on the OC curve.

Specific Points on the OC Curve

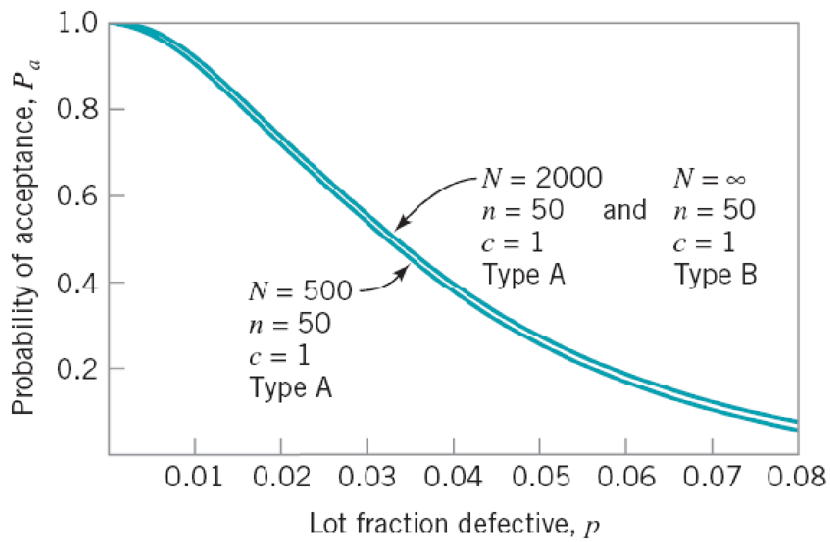
- The poorest quality level for the supplier's process that a consumer would consider to be acceptable as a process average is called the **acceptable quality level (AQL)**
    - AQL is a property of the supplier's manufacturing process, not a property of the sampling plan

- The protection obtained for individual lots of poor quality is established by the **lot tolerance percent defective (LTPD)**
    - Also called **rejectable quality level (RQL)** and the **limiting quality level (LQL)**
    - LTPD is a level of lot quality specified by the consumer, not a characteristic of the sampling plan

- Sampling plans can be designed to have specified performance at the AQL and the LTPD points

Type-A and Type-B OC Curves. The OC curves that were constructed in the previous examples are called **type-B OC curves.** In the construction of the OC curve it was assumed that the samples came from a large lot or that we were sampling from a stream of lots selected at random from a process. In this situation, the **binomial distribution** is the exact probability distribution for calculating the probability of lot acceptance. Such an OC curve is referred to as a type-B OC curve.

The **type-A OC curve** is used to calculate probabilities of acceptance for an isolated lot of finite size. Suppose that the lot size is N, the sample size is n, and the acceptance number is c. The exact sampling distribution of the number of defective items in the sample is the **hypergeometric distribution.**



■ **FIGURE 15.6** Type-A and type-B OC curves.

Other Aspects of OC Curve Behavior

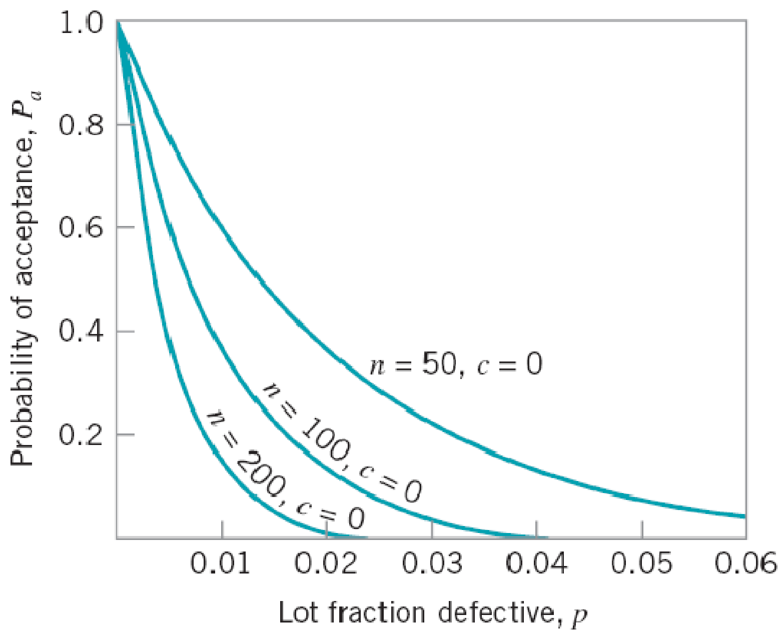


FIGURE 15.7 OC curves for single-sampling plan with $c = 0$.

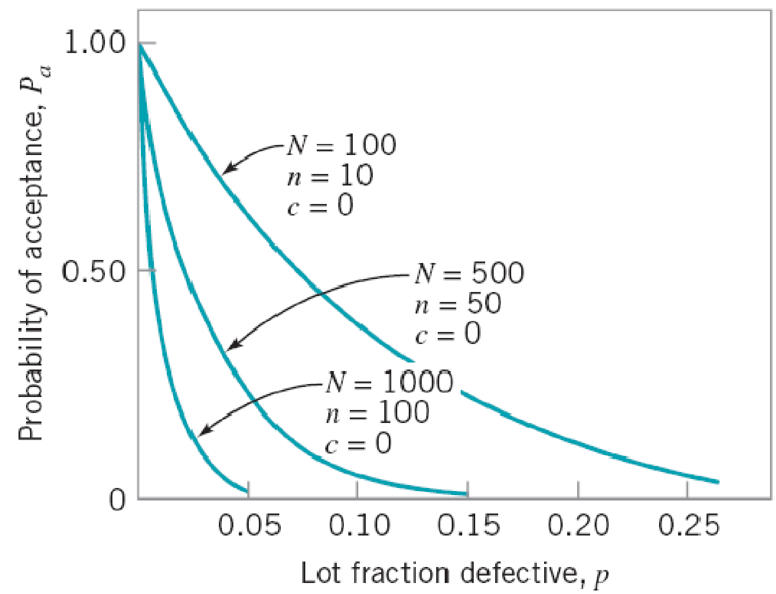


FIGURE 15.8 OC curves for sampling plans where sample size n is 10% of the lot size.

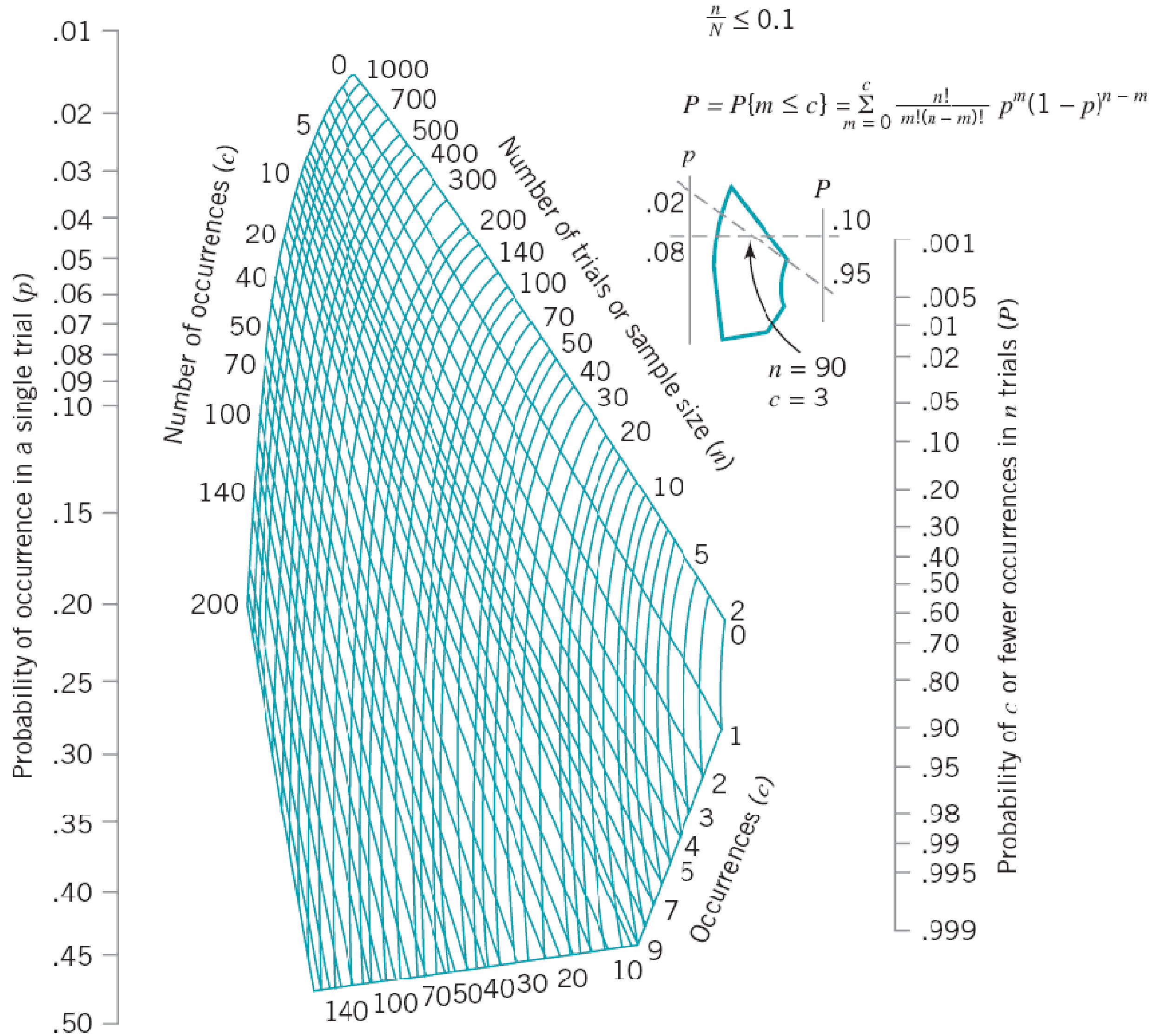
15.2.3 Designing a Single-Sampling Plan with a Specified OC Curve

A common approach to the design of an acceptance-sampling plan is to require that the OC curve pass through two designated points. Note that one point is not enough to fully specify the sampling plan; however, two points are sufficient. In general, it does not matter which two points are specified.

Suppose that we wish to construct a sampling plan such that the probability of acceptance is $1 - \alpha$ for lots with fraction defective p_1, and the probability of acceptance is β for lots with fraction defective p_2. Assuming that binomial sampling (with type-B OC curves) is appropriate, we see that the sample size n and acceptance number c are the solution to

$$1 - \alpha = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_1^d \left(1 - p_1\right)^{n-d}$$

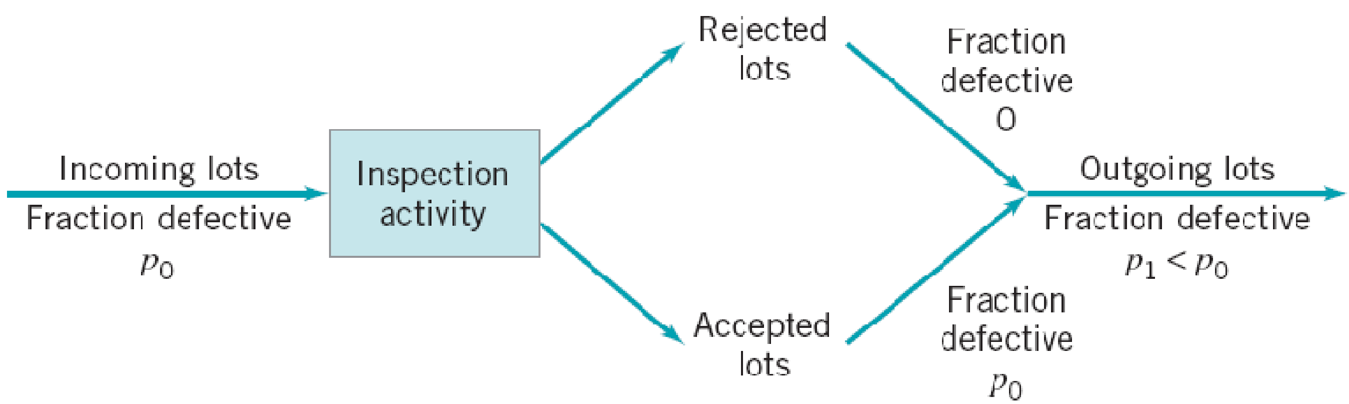
$$\beta = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p_2^d \left(1 - p_2\right)^{n-d} \qquad (15.3)$$



■ **FIGURE 15.9** Binomial nomograph.

15.2.4 Rectifying Inspection

Acceptance-sampling programs usually require corrective action when lots are rejected. This generally takes the form of 100% inspection or **screening** of rejected lots, with all discovered defective items either removed for subsequent rework or return to the supplier, or replaced from a stock of known good items. Such sampling programs are called **rectifying inspection programs,** because the inspection activity affects the final quality of the outgoing product. This is illustrated in Fig. 15.10. Suppose that incoming lots to the inspection activity have fraction defective p_0. Some of these lots will be accepted, and others will be rejected. The rejected lots will be screened, and their final fraction defective will be zero. However, accepted lots have fraction defective p_0. Consequently, the outgoing lots from the inspection activity are a mixture of lots with fraction defective p_0 and fraction defective zero, so the average fraction defective in the stream of outgoing lots is p_1, which is less than p_0. Thus, a rectifying inspection program serves to "correct" lot quality.



■ FIGURE 15.10 Rectifying inspection.

Average outgoing quality is widely used for the evaluation of a rectifying sampling plan. The average outgoing quality is the quality in the lot that results from the application of rectifying inspection. It is the average value of lot quality that would be obtained over a long sequence of lots from a process with fraction defective p. It is simple to develop a formula for average outgoing quality (AOQ). Assume that the lot size is N and that all discovered defectives are replaced with good units. Then in lots of size N, we have

1. n items in the sample that, after inspection, contain no defectives, because all discovered defectives are replaced
2. $N - n$ items that, if the lot is rejected, also contain no defectives
3. $N - n$ items that, if the lot is accepted, contain $p(N - n)$ defectives

Thus, lots in the outgoing stage of inspection have an expected number of defective units equal to $P_a p(N - n)$, which we may express as an *average fraction defective,* called the **average outgoing quality** or

$$\text{AOQ} = \frac{P_a p(N-n)}{N} \tag{15.4}$$

To illustrate the use of equation (15.4), suppose that $N = 10{,}000$, $n = 89$, and $c = 2$, and that the incoming lots are of quality $p = 0.01$. Now at $p = 0.01$, we have $P_a = 0.9397$, and the AOQ is

$$\begin{aligned}\text{AOQ} &= \frac{P_a p(N-n)}{N} \\ &= \frac{(0.9397)(0.01)(10{,}000-89)}{10{,}000} \\ &= 0.0093\end{aligned}$$

That is, the average outgoing quality is 0.93% defective. Note that as the lot size N becomes large relative to the sample size n, we may write equation (15.4) as

$$\text{AOQ} \simeq P_a p \qquad (15.5)$$

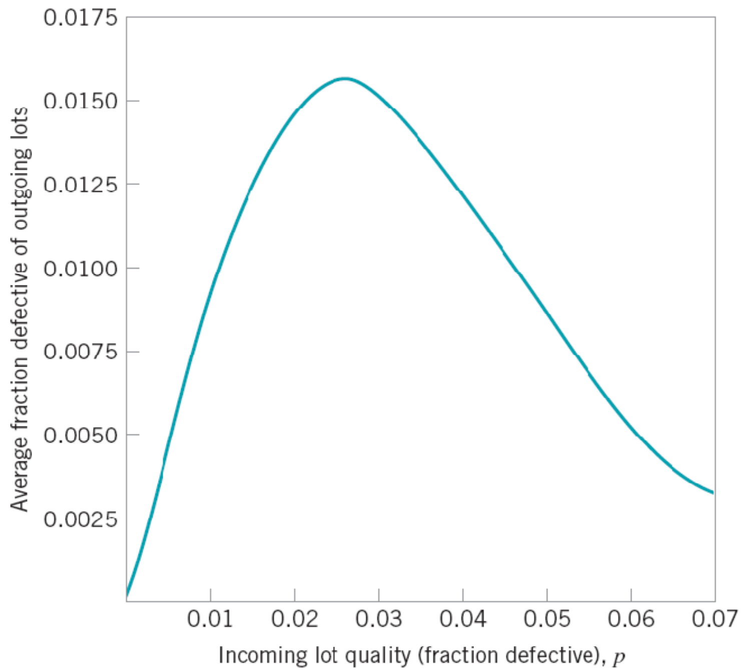


FIGURE 15.11 Average outgoing quality curve for $n = 89$, $c = 2$.

AOQL is the maximum point on the curve

Another important measure relative to rectifying inspection is the total amount of inspection required by the sampling program. If the lots contain no defective items, no lots will be rejected, and the amount of inspection per lot will be the sample size n. If the items are all defective, every lot will be submitted to 100% inspection, and the amount of inspection per lot will be the lot size N. If the lot quality is $0 < p < 1$, the average amount of inspection per lot will vary between the sample size n and the lot size N. If the lot is of quality p and the probability of lot acceptance is P_a, then the **average total inspection** per lot will be

$$\text{ATI} = n + (1 - P_a)(N - n) \tag{15.6}$$

To illustrate the use of equation (15.6), consider our previous example with $N = 10{,}000$, $n = 89$, $c = 2$, and $p = 0.01$. Then, since $P_a = 0.9397$, we have

$$\begin{aligned} \text{ATI} &= n + (1 - P_a)(N - n) \\ &= 89 + (1 - 0.9397)(10{,}000 - 89) \\ &= 687 \end{aligned}$$

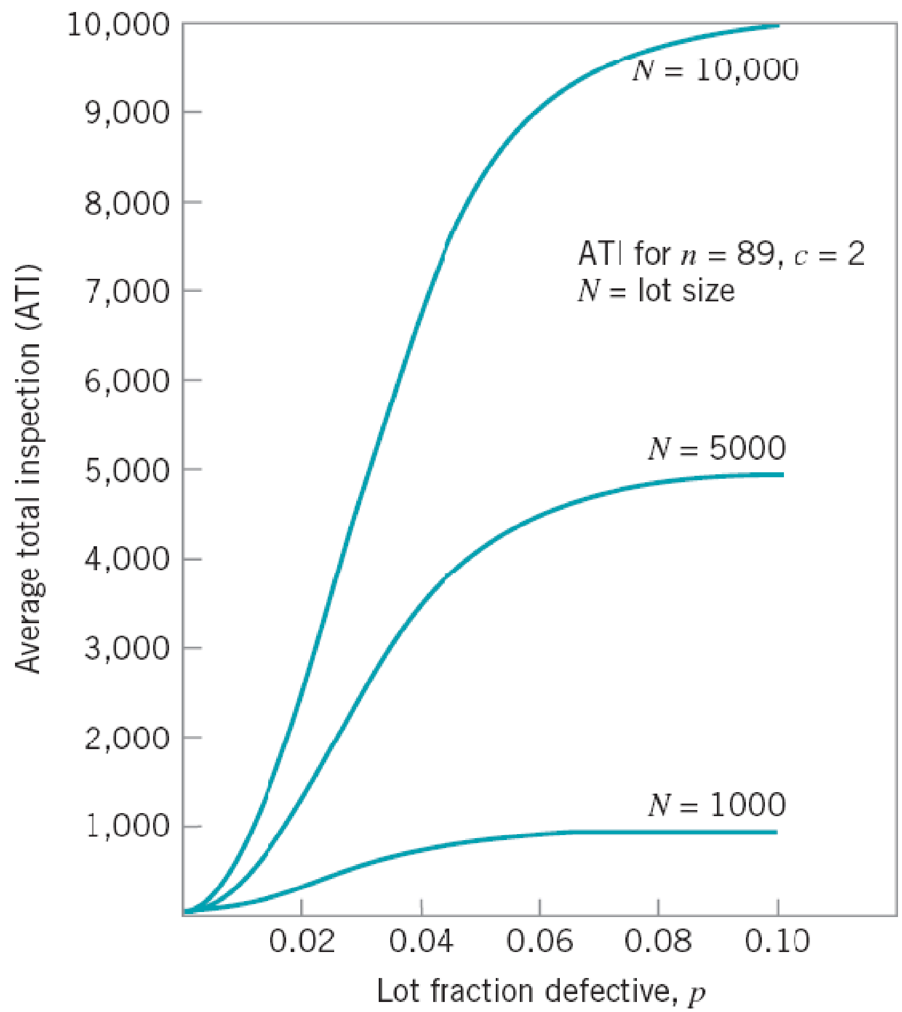


FIGURE 15.12 Average total inspection (ATI) curves for sampling plan $n = 89$, $c = 2$, for lot sizes of 1000, 5000, and 10,000.

The AOQL of a rectifying inspection plan is a very important characteristic. It is possible to design rectifying inspection programs that have specified values of AOQL. However, specification of the AOQL is not sufficient to determine a unique sampling plan. Therefore, it is relatively common practice to choose the sampling plan that has a specified AOQL and, in addition, yields a minimum ATI at a particular level of lot quality. The level of lot quality usually chosen is the most likely level of incoming lot quality, which is generally called the *process average*. The procedure for generating these plans is relatively straightforward and is illustrated in Duncan (1986). Generally, it is unnecessary to go through this procedure, because tables of sampling plans that minimize ATI for a given AOQL and a specified process average p have been developed by Dodge and Romig. We describe the use of these tables in Section 15.5.

It is also possible to design a rectifying inspection program that gives a specified level of protection at the LTPD point and that minimizes the average total inspection for a specified process average p. The Dodge–Romig sampling inspection tables also provide these LTPD plans. Section 15.5 discusses the use of the Dodge–Romig tables to find plans that offer specified LTPD protection.

15.3 Double, Multiple and Sequential Sampling

15.3.1 Double-Sampling Plans

A double-sampling plan is a procedure in which, under certain circumstances, a second sample is required before the lot can be sentenced. A double-sampling plan is defined by four parameters:[2]

$$n_1 = \text{sample size on the first sample}$$

$$c_1 = \text{acceptance number of the first sample}$$

$$n_2 = \text{sample size on the second sample}$$

$$c_2 = \text{acceptance number for both sample}$$

As an example, suppose $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, and $c_2 = 3$. Thus, a random sample of $n_1 = 50$ items is selected from the lot, and the number of defectives in the sample, d_1, is observed. If $d_1 \leq c_1 = 1$, the lot is accepted on the first sample. If $d_1 > c_2 = 3$, the lot is rejected on the first sample. If $c_1 < d_1 \leq c_2$, a second random sample of size $n_2 = 100$ is drawn from the lot, and the number of defectives in this second sample, d_2, is observed. Now the combined number of observed defectives from both the first and second sample, $d_1 + d_2$, is used to determine the lot sentence. If $d_1 + d_2 \leq c_2 = 3$, the lot is accepted. However, if $d_1 + d_2 > c_2 = 3$, the lot is rejected. The operation of this double-sampling plan is illustrated graphically in Fig. 15.13.

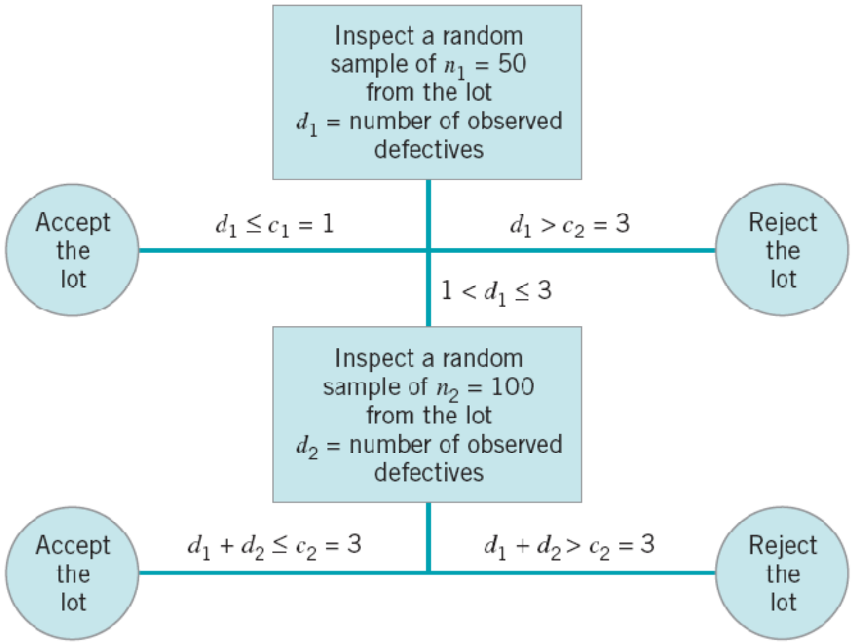


FIGURE 15.13 Operation of the double-sampling plan, $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, $c_2 = 3$.

Advantage of a double-sampling plan over single sampling is that it may reduce total amount of required inspection

Suppose first sample in a double-sampling plan is smaller than for a single-sampling plan

- If lot is accepted or reject on first sample, cost of inspection is lower
- Also possible to reject a lot without completing inspection of second sample

The OC Curve:

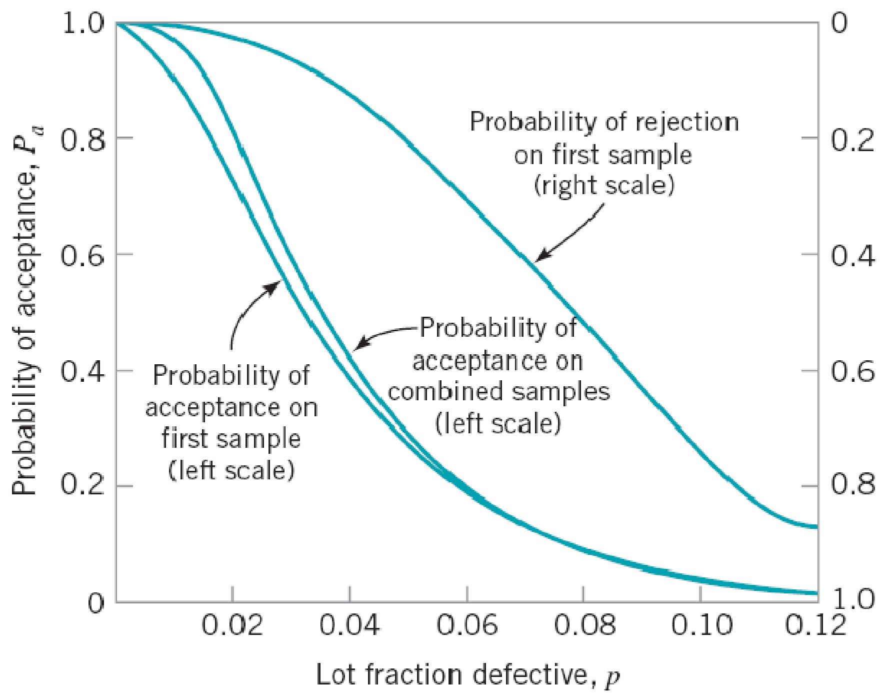


FIGURE 15.14 OC curves for the double-sampling plan, $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, $c_2 = 3$.

See textbook for calculations

The Average Sample Number Curve. The average sample number curve of a double-sampling plan is also usually of interest to the quality engineer. In single-sampling, the size of the sample inspected from the lot is always constant, whereas in double-sampling, the size of the sample selected depends on whether or not the second sample is necessary. The probability of drawing a second sample varies with the fraction defective in the incoming lot. With complete inspection of the second sample, the average sample size in double-sampling is equal to the size of the first sample times the probability that there will only be one sample, plus the size of the combined samples times the probability that a second sample will be necessary. Therefore, a general formula for the average sample number in double-sampling, if we assume complete inspection of the second sample, is

$$\begin{aligned} \text{ASN} &= n_1 P_{\text{I}} + (n_1 + n_2)(1 - P_{\text{I}}) \\ &= n_1 + n_2(1 - P_{\text{I}}) \end{aligned} \tag{15.7}$$

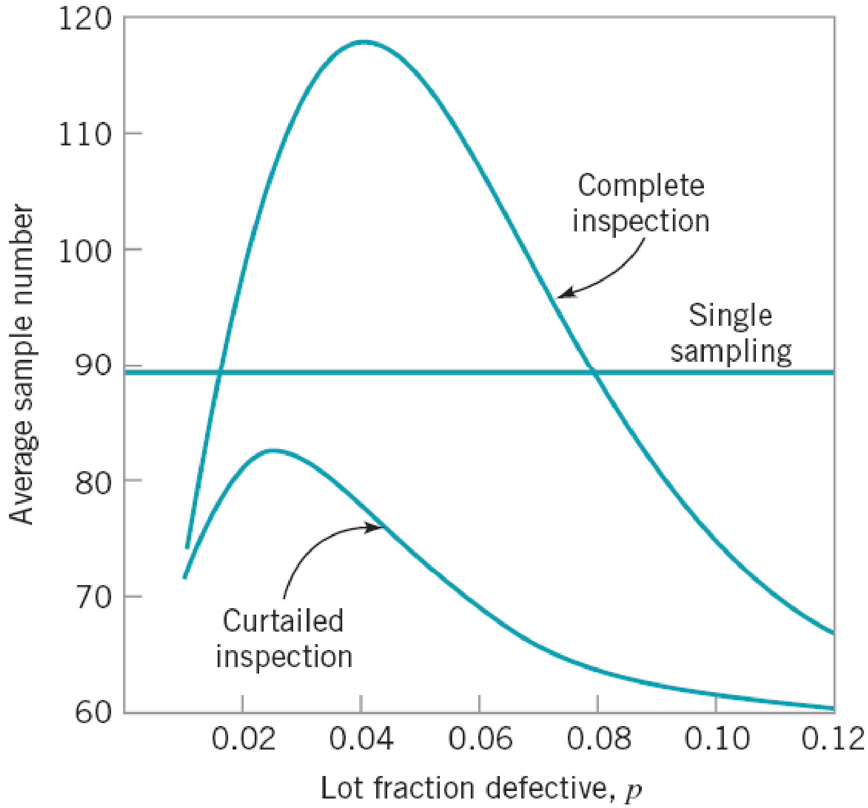
where P_{I} is the probability of making a lot-dispositioning decision on the *first* sample. This is

$$P_{\text{I}} = P\{\text{lot is accepted on the first sample}\} + P\{\text{lot is rejected on the first sample}\}$$

If equation (15.7) is evaluated for various values of lot fraction defective p, the plot of ASN versus p is called an **average sample number curve.**

The ASN with curtailment:

$$\text{ASN} = n_1 + \sum_{j=c_1+1}^{c_2} P(n_1, j)\left[n_2 P_L(n_2, c_2 - j) + \frac{c_2 - j + 1}{p} P_M(n_2 + 1, c_2 - j + 2)\right] \qquad (15.8)$$



■ FIGURE 15.15 Average sample number curves for single and double sampling.

Rectifying Inspection. When rectifying inspection is performed with double-sampling, the AOQ curve is given by

$$\text{AOQ} = \frac{\left[P_a^{\text{I}}\left(N - n_1\right) + P_a^{\text{II}}\left(N - n_1 - n_2\right)\right]p}{N} \tag{15.9}$$

assuming that all defective items discovered, either in sampling or 100% inspection, are replaced with good ones. The average total inspection curve is given by

$$\text{ATI} = n_1 P_a^{\text{I}} + \left(n_1 + n_2\right)P_a^{\text{II}} + N\left(1 - P_a\right) \tag{15.10}$$

Remember that $P_a = P_a^{\text{I}} + P_a^{\text{II}}$ is the probability of final lot acceptance and that the acceptance probabilities depend on the level of lot or process quality p.

15.3.2 Multiple-Sampling Plans

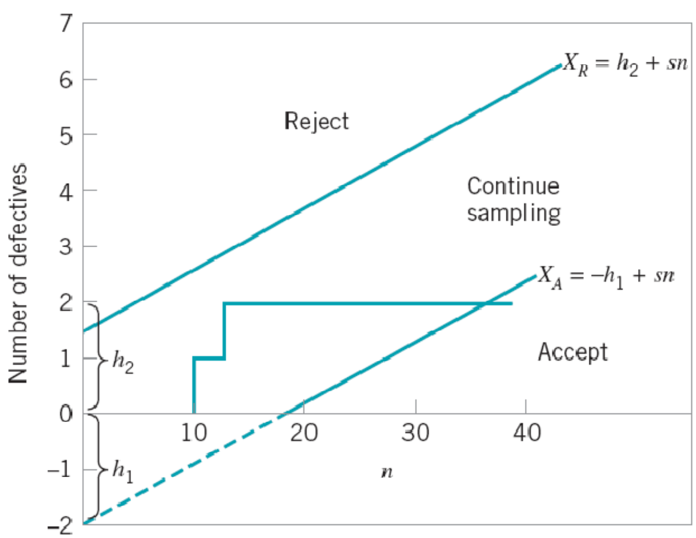
A multiple-sampling plan is an extension of double-sampling in that more than two samples can be required to sentence a lot. An example of a multiple-sampling plan with five stages follows.

Cumulative-Sample Size	Acceptance Number	Rejection Number
20	0	3
40	1	4
60	3	5
80	5	7
100	8	9

This plan will operate as follows: If, at the completion of any stage of sampling, the number of defective items is less than or equal to the acceptance number, the lot is accepted. If, during any stage, the number of defective items equals or exceeds the rejection number, the lot is rejected; otherwise the next sample is taken. The multiple-sampling procedure continues until the fifth sample is taken, at which time a lot disposition decision must be made. The first sample is usually inspected 100%, although subsequent samples are usually subject to curtailment.

15.3.3 Sequential-Sampling Plans

Sequential-sampling is an extension of the double-sampling and multiple-sampling concept. In sequential-sampling, we take a sequence of samples from the lot and allow the number of samples to be determined entirely by the results of the sampling process. In practice, sequential-sampling can theoretically continue indefinitely, until the lot is inspected 100%. In practice, sequential-sampling plans are usually truncated after the number inspected is equal to three times the number that would have been inspected using a corresponding single-sampling plan. If the sample size selected at each stage is greater than one, the process is usually called *group* sequential-sampling. If the sample size inspected at each stage is one, the procedure is usually called **item-by-item sequential-sampling.**



■ FIGURE 15.16 Graphical performance of sequential sampling.

$$X_A = -h_1 + sn \qquad \text{(acceptance line)} \tag{15.11a}$$

$$X_R = h_2 + sn \qquad \text{(rejection line)} \tag{15.11b}$$

where

$$h_1 = \left(\log \frac{1-\alpha}{\beta}\right) \Big/ k \tag{15.12}$$

$$h_2 = \left(\log \frac{1-\beta}{\alpha}\right) \Big/ k \tag{15.13}$$

$$k = \log \frac{p_2(1-p_1)}{p_1(1-p_2)} \tag{15.14}$$

$$s = \log\left[(1-p_1)/(1-p_2)\right] \big/ k \tag{15.15}$$

To illustrate the use of these equations, suppose we wish to find a sequential-sampling plan for which $p_1 = 0.01$, $\alpha = 0.05$, $p_2 = 0.06$, and $\beta = 0.10$. Thus,

$$k = \log \frac{p_2(1-p_1)}{p_1(1-p_2)}$$

$$= \log \frac{(0.06)(0.99)}{(0.01)(0.94)} = 0.80066$$

$$h_1 = \left(\log \frac{1-\alpha}{\beta} \right) \Big/ k$$

$$= \left(\log \frac{0.95}{0.10} \right) \Big/ 0.80066 = 1.22$$

$$h_2 = \left(\log \frac{1-\beta}{\alpha} \right) \Big/ k$$

$$= \left(\log \frac{0.90}{0.05} \right) \Big/ 0.80066 = 1.57$$

$$s = \log\left[(1-p_1)/(1-p_2)\right] \Big/ k$$

$$= \left[\log(0.99/0.94)\right] \Big/ 0.80066 = 0.028$$

Instead of using a graph to determine the lot disposition, the sequential-sampling plan can be displayed in a table such as Table 15.3. The entries in the table are found by substituting values of n into the equations for the acceptance and rejection lines and calculating acceptance and rejection numbers. For example, the calculations for $n = 45$ are

$$X_A = -1.22 + 0.028n$$
$$= -1.22 + 0.028(45) = 0.04 \qquad \text{(accept)}$$
$$X_R = 1.57 + 0.028n$$
$$= 1.57 + 0.028(45) = 2.83 \qquad \text{(reject)}$$

Refer to Table 15.3 in the textbook (p. 654) for an example

The OC Curve and ASN Curve for Sequential Sampling. The OC curve for sequential-sampling can be easily obtained. Two points on the curve are $(p_1, 1 - \alpha)$ and (p_2, β). A third point, near the middle of the curve, is $p = s$ and $P_a = h_2/(h_1 + h_2)$.

The average sample number taken under sequential-sampling is

$$\text{ASN} = P_a\left(\frac{A}{C}\right) + (1 - P_a)\frac{B}{C} \tag{15.16}$$

where

$$A = \log \frac{\beta}{1-\alpha}$$

$$B = \log \frac{1-\beta}{}$$

and

$$C = p\log\left(\frac{p_2}{p_1}\right) + (1-p)\log\left(\frac{1-p_2}{1-p_1}\right)$$

Rectifying Inspection. The average outgoing quality (AOQ) for sequential-sampling is given approximately by

$$\text{AOQ} \simeq P_a p \tag{15.17}$$

The average total inspection is also easily obtained. Note that the amount of sampling is A/C when a lot is accepted and N when it is rejected. Therefore, the average total inspection is

$$\text{ATI} = P_a\left(\frac{A}{C}\right) + (1 - P_a)N \tag{15.18}$$

15.4 Military Standard 105E (ANSI/ASQC Z1.4, ISO 2859)

15.4.1 Description of the Standard

Sampling procedure for inspection by attributes developed during World War II and is the most widely used acceptance-sampling system for attributes in the world today

A collection of sampling schemes; therefore an acceptance-sampling system

Provides for three types of sampling: single, double, and multiple

Primary focal point is the acceptable quality level (AQL)

- Different AQLs may be designated for different types of defects: critical, major, and minor

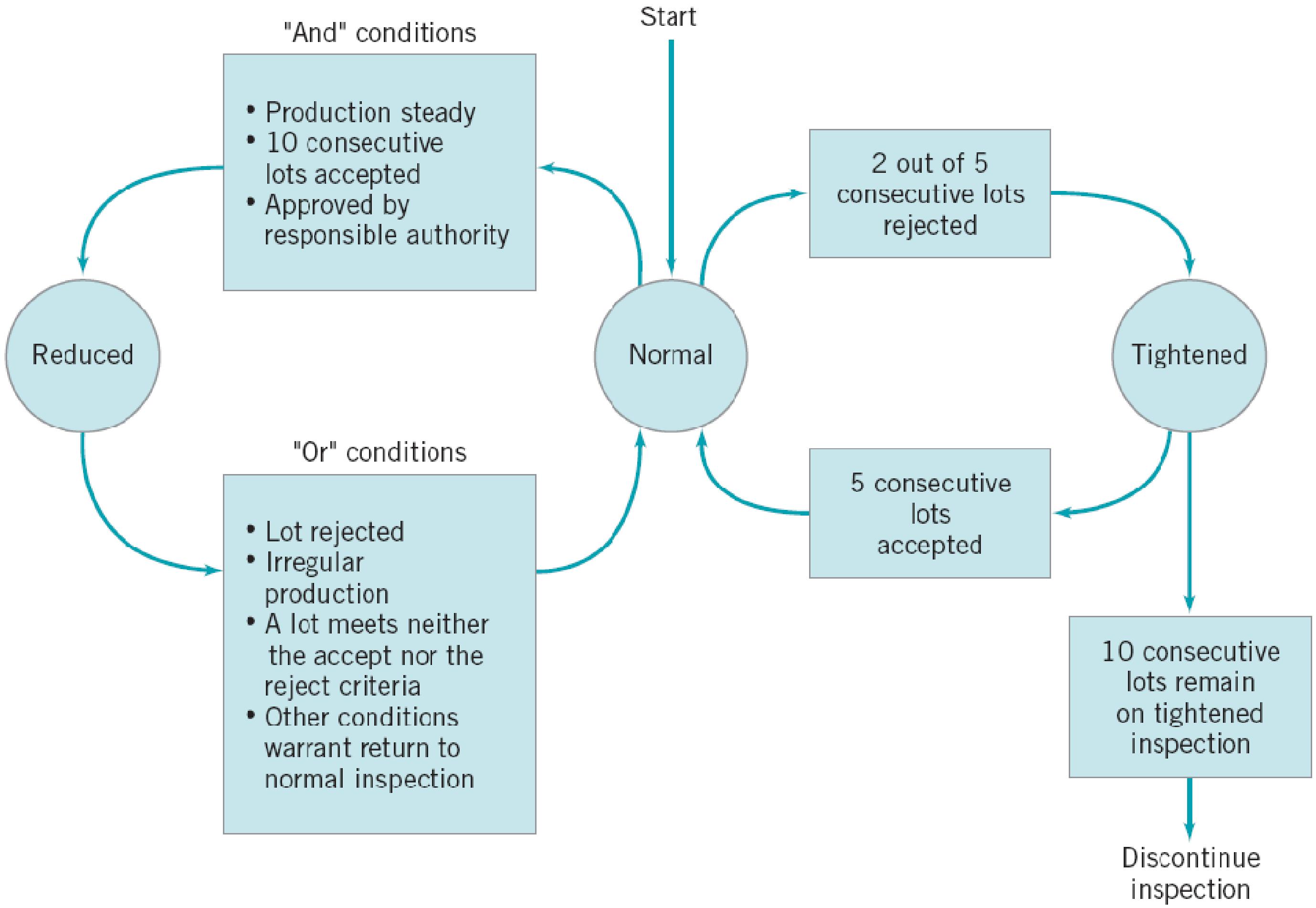
- Generally specified in contract or by authority responsible for sampling

Sample size is determined by lot size and by choice of inspection level

For a specified AQL and inspection level and a given lot size, MIL STD 105E provides a normal sampling plan that is to be used as long as the supplier is producing the product at AQL quality or better. It also provides a procedure for switching to tightened and reduced inspection whenever there is an indication that the supplier's quality has changed. The switching procedures between normal, tightened, and reduced inspection are illustrated graphically in Fig. 15.17 and are described next.

1. **Normal to tightened.** When normal inspection is in effect, tightened inspection is instituted when two out of five consecutive lots have been rejected on original submission.
2. **Tightened to normal.** When tightened inspection is in effect, normal inspection is instituted when five consecutive lots or batches are accepted on original inspection.
3. **Normal to reduced.** When normal inspection is in effect, reduced inspection is instituted provided all four of the following conditions are satisfied.
   - **a.** The preceding ten lots have been on normal inspection, and none of the lots has been rejected on original inspection.
   - **b.** The total number of defectives in the samples from the preceding ten lots is less than or equal to the applicable limit number specified in the standard.
   - **c.** Production is at a steady rate; that is, no difficulty such as machine breakdowns, material shortages, or other problems have recently occurred.
   - **d.** Reduced inspection is considered desirable by the authority responsible for sampling.

4. **Reduced to normal.** When reduced inspection is in effect, normal inspection is instituted provided any of the following four conditions are satisfied.
   - **a.** A lot or batch is rejected.
   - **b.** When the sampling procedure terminates with neither acceptance nor rejection criteria having been met, the lot or batch is accepted, but normal inspection is reinstituted starting with the next lot.
   - **c.** Production is irregular or delayed.
   - **d.** Other conditions warrant that normal inspection be instituted.
5. **Discontinuance of inspection.** In the event that 10 consecutive lots remain on tightened inspection, inspection under the provision of MIL STD 105E should be terminated, and action should be taken at the supplier level to improve the quality of submitted lots.



■ **FIGURE 15.17** Switching rules for normal, tightened, and reduced inspection, MIL STD 105E.

15.4.2 Procedure

A step-by-step procedure for using MIL STD 105E is as follows:

1. Choose the AQL.
2. Choose the inspection level.
3. Determine the lot size.
4. Find the appropriate sample size code letter from Table 15.4.
5. Determine the appropriate type of sampling plan to use (single, double, multiple).
6. Enter the appropriate table to find the type of plan to be used.
7. Determine the corresponding normal and reduced inspection plans to be used when required.

Table 15.4 presents the sample size code letters for MIL STD 105E. Tables 15.5, 15.6, and 15.7 present the single-sampling plans for normal inspection, tightened inspection, and reduced inspection, respectively. The standard also contains tables for double-sampling plans and multiple-sampling plans for normal, tightened, and reduced inspection.

■ TABLE 15.4

Sample Size Code Letters (MIL STD 105E, Table 1)

	Special Inspection Levels				General Inspection Levels		
Lot or Batch Size	**S-1**	**S-2**	**S-3**	**S-4**	**I**	**II**	**III**
2 to 8	A	A	A	A	A	A	B
9 to 15	A	A	A	A	A	B	C
16 to 25	A	A	B	B	B	C	D
26 to 50	A	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1200	C	C	E	F	G	J	K
1201 to 3200	C	D	E	G	H	K	L
3201 to 10000	C	D	F	G	J	L	M
10001 to 35000	C	D	F	H	K	M	N
35001 to 150000	D	E	G	J	L	N	P
150001 to 500000	D	E	G	J	M	P	Q
500001 and over	D	E	H	K	N	Q	R

Refer to Tables 15.5, -6 and -7 in the textbook on pp. 658 – 660.

15.4.3 Discussion

- Several points about MIL STD 105E should be emphasized:

- MIL STD 105E is AQL-oriented
- Not all possible sample sizes are possible (2,3,5,8,13,20,32,50, etc.)
- Sample sizes are related to lot sizes
- Switching rules are subject to criticism for both misswitching between inspection plans and discontinuation even though there has been no actual quality deterioration
- But a **flagrant** and **common** abuse of MIL STD 105E is failure to use the switching rules at all

15.5 The Dodge–Romig Sampling Plans

H. F. Dodge and H. G. Romig developed a set of sampling inspection tables for lot-by-lot inspection of product by attributes using two types of sampling plans: plans for lot tolerance percent defective (LTPD) protection and plans that provide a specified average outgoing quality limit (AOQL). For each of these approaches to sampling plan design, there are tables for single- and double-sampling.

Sampling plans that emphasize LTPD protection, such as the Dodge–Romig plans, are often preferred to AQL-oriented sampling plans, such as those in MIL STD 105E, particularly for critical components and parts. Many manufacturers believe that they have relied too much on AQLs in the past, and they are now emphasizing other measures of performance, such as defective parts per million (ppm). Consider the following:

AQL	Defective Parts per Million
10%	100,000
1%	10,000
0.1%	1,000
0.01%	100
0.001%	10
0.0001%	1

Thus, even very small AQLs imply large numbers of defective ppm. In complex products, the effect of this can be devastating. For example, suppose that a printed circuit board contains 100 elements, each manufactured by a process operating at 0.5% defective. If the AQLs for these elements are 0.5% and if all elements on the printed circuit board must operate for the card to function properly, then the probability that a board works is

$$P(\text{function properly}) = (0.995)^{100} = 0.6058$$

Thus, there is an obvious need for sampling plans that emphasize LTPD protection, even when the process average fallout is low. The Dodge–Romig plans are often useful in these situations.

AOQL Plans:

- Dodge-Romig (1959) tables give AOQL sampling plans for specified AOQL values
- Six classes of values for process average are specified for various lot sizes
- Tables are available for both single and double sampling
- Designed so that average total inspection at a given AOQL and process average is approximately a minimum
- Refer to Table 15.8 for an example

LTPD Plans:

- Dodge-Romig LTPD tables are designed so that the probability of lot acceptance at the LTPD is 0.1
- Tables are provided for various LTPD values
- Six classes of values for process average are specified for various lot sizes
- Refer to Table 14-9 for an example

Important Terms and Concepts

100% inspection
Acceptable quality level (AQL)
Acceptance-sampling plan
ANSI/ASQC Z1.4, ISO 2859
AOQL plans
Attributes data
Average outgoing quality
Average outgoing quality limit
Average sample number curve
Average total inspection
Dodge–Romig sampling plans
Double-sampling plan
Ideal OC curve
Lot disposition actions
Lot sentencing
Lot tolerance percent defective (LTPD)
LTPD plans
MIL STD 105E
Multiple-sampling plan
Normal, tightened, and reduced inspection
Operating-characteristic (OC) curve
Random sampling
Rectifying inspection
Sample size code letters
Sequential-sampling plan
Single-sampling plan
Switching rules in MIL STD 105E
Type-A and Type-B OC curves
Variables data

Learning Objectives

1. Understand the role of acceptance sampling in modern quality control systems
2. Understand the advantages and disadvantages of sampling
3. Understand the difference between attributes and variables sampling plans, and the major types of acceptance-sampling procedures
4. Know how single-, double-, and sequential-sampling plans are used
5. Understand the importance of random sampling
6. Know how to determine the OC curve for a single-sampling plan for attributes
7. Understand the effects of the sampling plan parameters on sampling plan performance
8. Know how to design single-sampling, double-sampling, and sequential-sampling plans for attributes
9. Know how rectifying inspection is used
10. Understand the structure and use of MIL STD 105E and its civilian counterpart plans
11. Understand the structure and use of the Dodge–Romig system of sampling plans