Chapter 1
Introduction

1.1 Occurrence of Plasmas in Nature

It is now believed that the universe is made of 69 % dark energy, 27 % dark matter, and 1 % normal matter. All that we can see in the sky is the part of normal matter that is in the plasma state, emitting radiation. Plasma in physics, not to be confused with blood plasma, is an "ionized" gas in which at least one of the electrons in an atom has been stripped free, leaving a positively charged nucleus, called an ion. Sometimes plasma is called the "fourth state of matter." When a solid is heated, it becomes a liquid. Heating a liquid turns it into a gas. Upon further heating, the gas is ionized into a plasma. Since a plasma is made of ions and electrons, which are charged, electric fields are rampant everywhere, and particles "collide" not just when they bump into one another, but even at a distance where they can feel their electric fields. Hydrodynamics, which describes the flow of water through pipes, say, or the flow around boats in yacht races, or the behavior of airplane wings, is already a complicated subject. Adding the electric fields of a plasma greatly expands the range of possible motions, especially in the presence of magnetic fields.

Plasma usually exists only in a vacuum. Otherwise, air will cool the plasma so that the ions and electrons will recombine into normal neutral atoms. In the laboratory, we need to pump the air out of a vacuum chamber. In the vacuum of space, however, much of the gas is in the plasma state, and we can see it. Stellar interiors and atmospheres, gaseous nebulas, and entire galaxies can be seen because they are in the plasma state. On earth, however, our atmosphere limits our experience with plasmas to a few examples: the flash of a lightning bolt, the soft glow of the Aurora Borealis, the light of a fluorescent tube, or the pixels of a plasma TV. We live in a small part of the universe where plasmas do not occur naturally; otherwise, we would not be alive.

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The reason for this can be seen from the Saha equation, which tells us the amount of ionization to be expected in a gas in thermal equilibrium:

$$\frac{n_i}{n_n} \approx 2.4 \times 10^{21} \frac{T^{3/2}}{n_i} e^{-U_i/KT} \tag{1.1}$$

Here n_i and n_n are, respectively, the density (number per m^3) of ionized atoms and of neutral atoms, T is the gas temperature in °K, K is Boltzmann's constant, and U_i is the ionization energy of the gas—that is, the number of joules required to remove the outermost electron from an atom. (The mks or International System of units will be used in this book.) For ordinary air at room temperature, we may take $n_n \approx 3 \times 10^{25}\ m^{-3}$ (see Problem 1.1), $T \approx 300°$ K, and $U_i = 14.5$ eV (for nitrogen), where 1 eV $= 1.6 \times 10^{-19}$ J. The fractional ionization $n_i/(n_n + n_i) \approx n_i/n_n$ predicted by Eq. (1.1) is ridiculously low:

$$\frac{n_i}{n_n} \approx 10^{-122}$$

As the temperature is raised, the degree of ionization remains low until U_i is only a few times KT. Then n_i/n_n rises abruptly, and the gas is in a plasma state. Further increase in temperature makes n_n less than n_i, and the plasma eventually becomes fully ionized. This is the reason plasmas exist in astronomical bodies with temperatures of millions of degrees, but not on the earth. Life could not easily coexist with a plasma—at least, plasma of the type we are talking about. The natural occurrence of plasmas at high temperatures is the reason for the designation "the fourth state of matter."

Although we do not intend to emphasize the Saha equation, we should point out its physical meaning. Atoms in a gas have a spread of thermal energies, and an atom is ionized when, by chance, it suffers a collision of high enough energy to knock out an electron. In a cold gas, such energetic collisions occur infrequently, since an atom must be accelerated to much higher than the average energy by a series of "favorable" collisions. The exponential factor in Eq. (1.1) expresses the fact that the number of fast atoms falls exponentially with U_i/KT. Once an atom is ionized, it remains charged until it meets an electron; it then very likely recombines with the electron to become neutral again. The recombination rate clearly depends on the density of electrons, which we can take as equal to n_i. The equilibrium ion fraction, therefore, should decrease with n_i; and this is the reason for the factor n_i^{-1} on the right-hand side of Eq. (1.1). The plasma in the interstellar medium owes its existence to the low value of n_i (about 1 per cm^3), and hence the low recombination rate.

1.2 Definition of Plasma

Any ionized gas cannot be called a plasma, of course; there is always some small degree of ionization in any gas. A useful definition is as follows:

A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior.

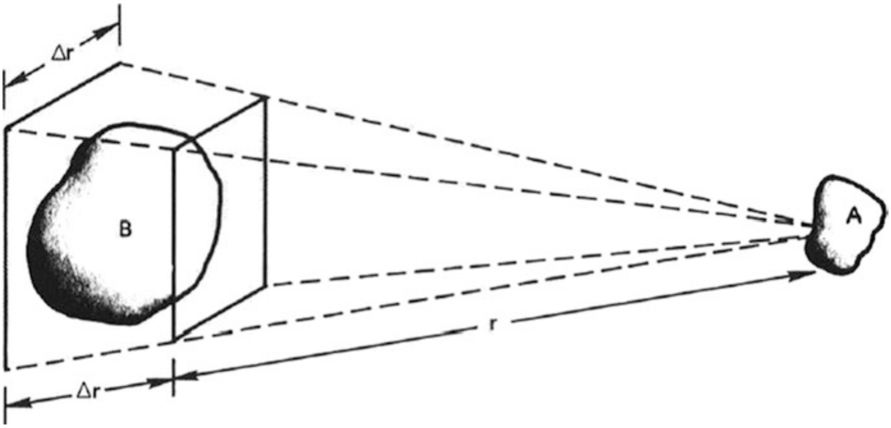


Fig. 1.1 Illustrating the long range of electrostatic forces in a plasma

We must now define "quasineutral" and "collective behavior." The meaning of quasineutrality will be made clear in Sect. 1.4. What is meant by "collective behavior" is as follows.

Consider the forces acting on a molecule of, say, ordinary air. Since the molecule is neutral, there is no net electromagnetic force on it, and the force of gravity is negligible. The molecule moves undisturbed until it makes a collision with another molecule, and these collisions control the particle's motion. A macroscopic force applied to a neutral gas, such as from a loudspeaker generating sound waves, is transmitted to the individual atoms by collisions. The situation is totally different in a plasma, which has *charged* particles. As these charges move around, they can generate local concentrations of positive or negative charge, which give rise to electric fields. Motion of charges also generates currents, and hence magnetic fields. These fields affect the motion of other charged particles far away.

Let us consider the effect on each other of two slightly charged regions of plasma separated by a distance r (Fig. 1.1). The Coulomb force between A and B diminishes as $1/r^2$. However, for a given solid angle (that is, $\Delta r/r = \text{constant}$), the volume of plasma in B that can affect A increases as r^3. Therefore, elements of plasma exert a force on one another even at large distances. It is this long-ranged Coulomb force that gives the plasma a large repertoire of possible motions and enriches the field of study known as plasma physics. In fact, the most interesting results concern so-called "collisionless" plasmas, in which the long-range electromagnetic forces are so much larger than the forces due to ordinary local collisions that the latter can be neglected altogether. By "collective behavior" we mean motions that depend not only on local conditions but on the state of the plasma in remote regions as well.

The word "plasma" seems to be a misnomer. It comes from the Greek $\pi\lambda\acute{\alpha}\sigma\mu\alpha$, $-\alpha\tau o\varsigma$, $\tau\acute{o}$, which means something molded or fabricated. Because of collective behavior, a plasma does not tend to conform to external influences; rather, it often behaves as if it had a mind of its own.

1.3 Concept of Temperature

Before proceeding further, it is well to review and extend our physical notions of "temperature." A gas in thermal equilibrium has particles of all velocities, and the most probable distribution of these velocities is known as the Maxwellian distribution. For simplicity, consider a gas in which the particles can move only in one dimension. (This is not entirely frivolous; a strong magnetic field, for instance, can constrain electrons to move only along the field lines.) The one-dimensional Maxwellian distribution is given by

$$f(u) = A\exp\left(-\tfrac{1}{2}mu^2/KT\right) \tag{1.2}$$

where $f\,du$ is the number of particles per m^3 with velocity between u and $u + du$, $\frac{1}{2}mu^2$ is the kinetic energy, and K is Boltzmann's constant,

$$K = 1.38 \times 10^{-23}\,\mathrm{J}/^\circ\mathrm{K}$$

Note that a capital K is used here, since lower-case k is reserved for the propagation constant of waves. The density n, or number of particles per m^3, is given by (see Fig. 1.2)

$$n = \int_{-\infty}^{\infty} f(u)du \tag{1.3}$$

The constant A is related to the density n by (see Problem 1.2)

$$A = n\left(\frac{m}{2\pi KT}\right)^{1/2} \tag{1.4}$$

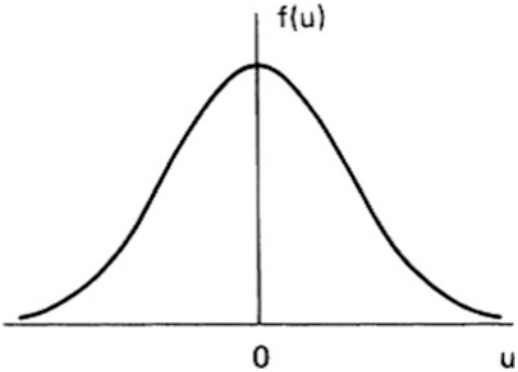


Fig. 1.2 A Maxwellian velocity distribution

The width of the distribution is characterized by the constant T, which we call the temperature. To see the exact meaning of T, we can compute the average kinetic energy of particles in this distribution:

$$E_{\mathrm{av}} = \frac{\int_{-\infty}^{\infty} \frac{1}{2} m u^2 f(u) du}{\int_{-\infty}^{\infty} f(u) du} \tag{1.5}$$

Defining

$$v_{th} = (2KT/m)^{1/2} \quad \text{and} \quad y = u/v_{\mathrm{th}} \tag{1.6}$$

we can write Eq. (1.2) as

$$f(u) = A \exp\left(-u^2/v_{\mathrm{th}}^2\right)$$

and Eq. (1.5) as

$$E_{\mathrm{av}} = \frac{\frac{1}{2} m A v_{\mathrm{th}}^3 \int_{-\infty}^{\infty} \left[\exp\left(-y^2\right)\right] y^2 dy}{A v_{\mathrm{th}} \int_{-\infty}^{\infty} \exp\left(-y^2\right) dy}$$

The integral in the numerator is integrable by parts:

$$\begin{aligned} \int_{-\infty}^{\infty} y \cdot \left[\exp\left(-y^2\right)\right] y dy &= \left[-\tfrac{1}{2}[\exp(-y^2)] y\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} -\tfrac{1}{2} \exp\left(-y^2\right) dy \\ &= \tfrac{1}{2} \int_{-\infty}^{\infty} \exp\left(-y^2\right) dy \end{aligned}$$

Canceling the integrals, we have

$$E_{\mathrm{av}} = \frac{\frac{1}{2} m A v_{\mathrm{th}}^3 \frac{1}{2}}{A v_{\mathrm{th}}} = \tfrac{1}{4} m v_{\mathrm{th}}^2 = \tfrac{1}{2} KT \tag{1.7}$$

Thus the average kinetic energy is $\frac{1}{2} KT$.

It is easy to extend this result to three dimensions. Maxwell's distribution is then

$$f(u, v, w) = A_3 \exp\left[-\tfrac{1}{2} m(u^2 + v^2 + w^2)/KT\right] \tag{1.8}$$

where

$$A_3 = n\left(\frac{m}{2\pi KT}\right)^{3/2} \tag{1.9}$$

The average kinetic energy is

$$E_{\mathrm{av}} = \frac{\iiint_{-\infty}^{\infty} A_3 \frac{1}{2} m(u^2 + v^2 + w^2) \exp\left[-\frac{1}{2} m(u^2 + v^2 + w^2)/KT\right] du\, dv\, dw}{\iiint_{-\infty}^{\infty} A_3 \exp\left[-\frac{1}{2} m(u^2 + v^2 + w^2)/KT\right] du\, dv\, dw}$$

We note that this expression is symmetric in u, v, and w, since a Maxwellian distribution is isotropic. Consequently, each of the three terms in the numerator is the same as the others. We need only to evaluate the first term and multiply by three:

$$E_{\mathrm{av}} = \frac{3A_3 \int \frac{1}{2} mu^2 \exp\left(-\frac{1}{2} mu^2/KT\right) du \iint \exp\left[-\frac{1}{2} m(v^2 + w^2)/KT\right] dv\, dw}{A_3 \int \exp\left(-\frac{1}{2} mu^2/KT\right) du \iint \exp\left[-\frac{1}{2} m(v^2 + w^2)/KT\right] dv\, dw}$$

Using our previous result, we have

$$E_{\mathrm{av}} = \tfrac{3}{2} KT \tag{1.10}$$

The general result is that E_{ay} equals $\frac{1}{2}KT$ per degree of freedom.

Since T and E_{av} are so closely related, it is customary in plasma physics to give temperatures in units of energy. To avoid confusion on the number of dimensions involved, it is not E_{av} but the energy corresponding to KT that is used to denote the temperature. For $KT = 1$ eV $= 1.6 \times 10^{-19}$ J, we have

$$T = \frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = 11{,}600$$

Thus the conversion factor is

$$1\,\mathrm{eV} = 11{,}600\ {}^{\circ}\mathrm{K} \tag{1.11}$$

By a 2-eV plasma we mean that $KT = 2$ eV, or $E_{\mathrm{av}} = 3$ eV in three dimensions.

It is interesting that a plasma can have several temperatures at the same time. It often happens that the ions and the electrons have separate Maxwellian distributions with different temperatures T_i and T_e. This can come about because the collision rate among ions or among electrons themselves is larger than the rate of collisions between an ion and an electron. Then each species can be in its own thermal equilibrium, but the plasma may not last long enough for the two temperatures to equalize. When there is a magnetic field **B**, even a single species, say ions, can have two temperatures. This is because the forces acting on an ion along **B** are different from those acting perpendicular to **B** (due to the Lorentz force). The components of velocity perpendicular to **B** and parallel to **B** may then belong to different Maxwellian distributions with temperatures $T_{\perp}$ and $T_{\|}$.

Before leaving our review of the notion of temperature, we should dispel the popular misconception that high temperature necessarily means a lot of heat. People are usually amazed to learn that the electron temperature inside a fluorescent light bulb is about 20,000° K. "My, it doesn't feel that hot!" Of course, the heat capacity must also be taken into account. The density of electrons inside a fluorescent tube is much less than that of a gas at atmospheric pressure, and the total amount of heat transferred to the wall by electrons striking it at their thermal velocities is not that great. Everyone has had the experience of a cigarette ash dropped innocuously on his hand. Although the temperature is high enough to cause a burn, the total amount of heat involved is not. Many laboratory plasmas have temperatures of the order of 1,000,000° K (100 eV), but at densities of only 10^{18}–10^{19} per m^3, the heating of the walls is not a serious consideration.

Problems

1.1. Compute the density (in units of m^{-3}) of an ideal gas under the following conditions:

(a) At 0 °C and 760 Torr pressure (1 Torr = 1 mmHg). This is called the Loschmidt number.

(b) In a vacuum of 10^{-3} Torr at room temperature (20 °C). This number is a useful one for the experimentalist to know by heart (10^{-3} Torr = 1 μ).

1.2. Derive the constant A for a normalized one-dimensional Maxwellian distribution

$$\hat{f}(u) = A\exp\left(-mu^2/2KT\right)$$

such that

$$\int_{-\infty}^{\infty} \hat{f}(u)du = 1$$

Hint: To save writing, replace $(2KT/m)^{1/2}$ by v_{th} (Eq. 1.6).

1.2a. (Advanced problem). Find A for a two-dimensional distribution which integrates to unity. Extra credit for a solution in cylindrical coordinates.

$$\hat{f}(u, v) = A\exp\left[-m\left(u^2 + v^2\right)/2KT\right]$$

1.4 Debye Shielding

A fundamental characteristic of the behavior of plasma is its ability to shield out electric potentials that are applied to it. Suppose we tried to put an electric field inside a plasma by inserting two charged balls connected to a battery (Fig. 1.3). The balls would attract particles of the opposite charge, and almost immediately a cloud of ions would surround the negative ball and a cloud of electrons would surround

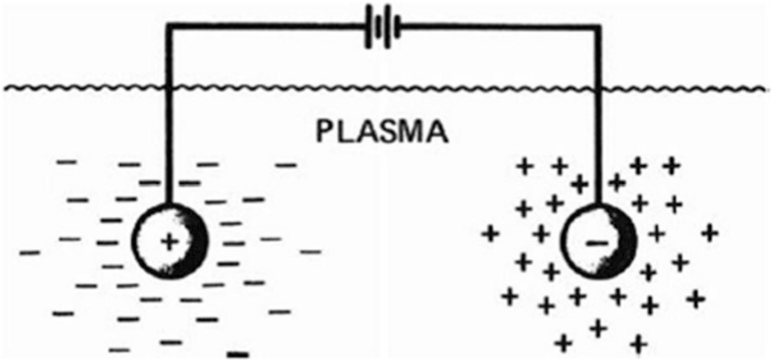
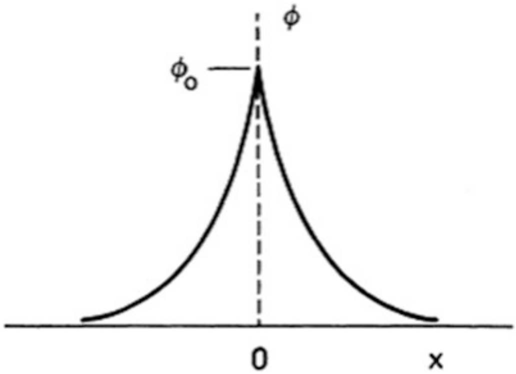


Fig. 1.3 Debye shielding

Fig. 1.4 Potential distribution near a grid in a plasma



the positive ball. (We assume that a layer of dielectric keeps the plasma from actually recombining on the surface, or that the battery is large enough to maintain the potential in spite of this.) If the plasma were cold and there were no thermal motions, there would be just as many charges in the cloud as in the ball, the shielding would be perfect, and no electric field would be present in the body of the plasma outside of the clouds. On the other hand, if the temperature is finite, those particles that are at the edge of the cloud, where the electric field is weak, have enough thermal energy to escape from the electrostatic potential well. The "edge" of the cloud then occurs at the radius where the potential energy is approximately equal to the thermal energy KT of the particles, and the shielding is not complete. Potentials of the order of KT/e can leak into the plasma and cause finite electric fields to exist there.

Let us compute the approximate thickness of such a charge cloud. Imagine that the potential ϕ on the plane x = 0 is held at a value ϕ_0 by a perfectly transparent grid (Fig. 1.4). We wish to compute ϕ(x). For simplicity, we assume that the ion–electron mass ratio M/m is infinite, so that the ions do not move but form a uniform background of positive charge. To be more precise, we can say that M/m is large enough that the inertia of the ions prevents them from moving significantly on the time scale of the experiment. Poisson's equation in one dimension is

$$\varepsilon_0 \nabla^2 \phi = \varepsilon_0 \frac{d^2\phi}{dx^2} = -e(n_i - n_e) \quad (Z = 1) \tag{1.12}$$

If the density far away is n_∞, we have

$$n_i = n_\infty$$

In the presence of a potential energy $q\phi$, the electron distribution function is

$$f(u) = A\exp\left[-\left(\tfrac{1}{2}mu^2 + q\phi\right)/KT_e\right] \tag{1.13}$$

It would not be worthwhile to prove this here. What this equation says is intuitively obvious: There are fewer particles at places where the potential energy is large, since not all particles have enough energy to get there. Integrating $f(u)$ over u, setting $q = -e$, and noting that $n_e(\phi \to 0) = n_\infty$, we find

$$n_e = n_\infty \exp(e\phi/KT_e)$$

This equation will be derived with more physical insight in Sect. 3.5. Substituting for n_i and n_e in Eq. (1.12), we have

$$\varepsilon_0 \frac{d^2\phi}{dx^2} = en_\infty\left(e^{e\phi/KT_e} - 1\right)$$

In the region where $|e\phi/KT_e| \ll 1$, we can expand the exponential in a Taylor series:

$$\varepsilon_0 \frac{d^2\phi}{dx^2} = en_\infty\left[\frac{e\phi}{KT_e} + \frac{1}{2}\left(\frac{e\phi}{KT_e}\right)^2 + \cdots\right] \tag{1.14}$$

No simplification is possible for the region near the grid, where $|e\phi/KT_e|$ may be large. Fortunately, this region does not contribute much to the thickness of the cloud (called a sheath), because the potential falls very rapidly there. Keeping only the linear terms in Eq. (1.*13*), we have

$$\varepsilon_0 \frac{d^2\phi}{dx^2} = \frac{n_\infty e^2}{KT_e}\phi \tag{1.15}$$

Defining

$$\boxed{\lambda_D \equiv \left(\frac{\varepsilon_0 KT_e}{ne^2}\right)^{1/2}} \tag{1.16}$$

where n stands for n_∞, and KT_e is in joules. KT_e is often given in eV, in which case, we will write it also as T_{eV}.

We can write the solution of Eq. (1.14) as

$$\phi = \phi_0 \exp(-|x|/\lambda_D) \tag{1.17}$$

The quantity λ_D, called the Debye length, is a measure of the shielding distance or thickness of the sheath.

Note that as the density is increased, λ_D decreases, as one would expect, since each layer of plasma contains more electrons. Furthermore, λ_D increases with increasing KT_e. Without thermal agitation, the charge cloud would collapse to an infinitely thin layer. Finally, it is the *electron* temperature which is used in the definition of λ_D because the electrons, being more mobile than the ions, generally do the shielding by moving so as to create a surplus or deficit of negative charge. Only in special situations is this not true (see Problem 1.5).

The following are useful forms of Eq. (1.16):

$$\begin{aligned} \lambda_D &= 69(T_e/n)^{1/2}\text{m}, \qquad T_e \text{ in } ^\circ\text{K} \\ \lambda_D &= 7430(KT_e/n)^{1/2}\text{m}, \qquad KT_e \text{ in eV} \end{aligned} \tag{1.18}$$

We are now in a position to define "quasineutrality." If the dimensions L of a system are much larger than λ_D, then whenever local concentrations of charge arise or external potentials are introduced into the system, these are shielded out in a distance short compared with L, leaving the bulk of the plasma free of large electric potentials or fields. Outside of the sheath on the wall or on an obstacle, $\nabla^2\phi$ is very small, and n_i is equal to n_e, typically to better than one part in 10^6. It takes only a small charge imbalance to give rise to potentials of the order of KT/e. The plasma is "quasineutral"; that is, neutral enough so that one can take $n_i \simeq n_e \simeq n$, where n is a common density called the *plasma density*, but not so neutral that all the interesting electromagnetic forces vanish.

A criterion for an ionized gas to be a plasma is that it be dense enough that λ_D is much smaller than L.

The phenomenon of Debye shielding also occurs—in modified form—in single-species systems, such as the electron streams in klystrons and magnetrons or the proton beam in a cyclotron. In such cases, any local bunching of particles causes a large unshielded electric field unless the density is extremely low (which it often is). An externally imposed potential—from a wire probe, for instance—would be shielded out by an adjustment of the density near the electrode. Single-species systems, or unneutralized plasmas, are not strictly plasmas; but the mathematical tools of plasma physics can be used to study such systems.

Debye shielding can be foiled if electrons are so fast that they do not collide with one another enough to maintain a thermal distribution. We shall see later that electron collisions are infrequent if the electrons are very hot. In that case, some electrons, attracted by the positive charge of the ion, come in at an angle so fast that they orbit the ion like a satellite around a planet. How this works will be clear in the discussion of Langmuir probes in a later chapter. Some like to call this effect *anti*-shielding.

1.5 The Plasma Parameter

The picture of Debye shielding that we have given above is valid only if there are enough particles in the charge cloud. Clearly, if there are only one or two particles in the sheath region, Debye shielding would not be a statistically valid concept. Using Eq. (1.17), we can compute the number N_D of particles in a "Debye sphere":

$$N_D = n\tfrac{4}{3}\pi\lambda_D^3 = 1.38\times10^6 T^{3/2}/n^{1/2}\quad (T \text{ in } ^\circ\text{K}) \tag{1.19}$$

In addition to $\lambda_D \ll L$, "collective behavior" requires

$$N_D \ggg 1 \tag{1.20}$$

1.6 Criteria for Plasmas

We have given two conditions that an ionized gas must satisfy to be called a plasma. A third condition has to do with collisions. The weakly ionized gas in an airplane's jet exhaust, for example, does not qualify as a plasma because the charged particles collide so frequently with neutral atoms that their motion is controlled by ordinary hydrodynamic forces rather than by electromagnetic forces. If ω is the frequency of typical plasma oscillations and τ is the mean time between collisions with neutral atoms, we require $\omega\tau > 1$ for the gas to behave like a plasma rather than a neutral gas.

The three conditions a plasma must satisfy are therefore:

1. $\lambda_D \ll L$.
2. $N_D \ggg 1$.
3. $\omega\tau > 1$.

Problems

1.3. Calculate n vs. KT_e curves for five values of λ_D from 10^{-8} to 1, and three values of N_D from 10^3 to 10^9. On a log-log plot of n_e vs. KT_e with n_e from 10^6 to 10^{28} m^{-3} and KT_e from 10^{-2} to 10^5 eV, draw lines of constant λ_D (solid) and N_D (dashed). On this graph, place the following points (n in m^{-3}, KT in eV):

   1. Typical fusion reactor: $n = 10^{20}$, $KT = 30{,}000$.
   2. Typical fusion experiments: $n = 10^{19}$, $KT = 100$ (torus); $n = 10^{23}$, $KT = 1000$ (pinch).
   3. Typical ionosphere: $n = 10^{11}$, $KT = 0.05$.
   4. Typical radiofrequency plasma: $n = 10^{17}$, $KT = 1.5$
   5. Typical flame: $n = 10^{14}$, $KT = 0.1$.
   6. Typical laser plasma; $n = 10^{25}$, $KT = 100$.
   7. Interplanetary space: $n = 10^6$, $KT = 0.01$.

Convince yourself that these are plasmas.

1.4. Compute the pressure, in atmospheres and in tons/ft^2, exerted by a thermonuclear plasma on its container. Assume $KT_e = KT_i = 20$ keV, $n = 10^{21}$ m^{-3}, and $p = nKT$, where $T = T_i + T_e$.

1.5. In a strictly steady state situation, both the ions and the electrons will follow the Boltzmann relation

$$n_j = n_0 \exp\left(-q_i\phi/KT_j\right)$$

For the case of an infinite, transparent grid charged to a potential ϕ, show that the shielding distance is then given approximately by

$$\lambda_D^{-2} = \frac{ne^2}{\epsilon_0}\left(\frac{1}{KT_e} + \frac{1}{KT_i}\right)$$

Show that λ_D is determined by the temperature of the colder species.

1.6. An alternative derivation of λ_D will give further insight to its meaning. Consider two infinite parallel plates at $x = \pm d$, set at potential $\phi = 0$. The space between them is uniformly filled by a gas of density n of particles of charge q.

(a) Using Poisson's equation, show that the potential distribution between the plates is

$$\phi = \frac{nq}{2\varepsilon_0}\left(d^2 - x^2\right)$$

(b) Show that for $d > \lambda_D$, the energy needed to transport a particle from a plate to the midplane is greater than the average kinetic energy of the particles.

1.7. Compute λ_D and N_D for the following cases:

(a) A glow discharge, with $n = 10^{16}\,m^{-3}$, $KT_e = 2$ eV.
(b) The earth's ionosphere, with $n = 10^{12}$ m^{-3}, $KT_e = 0.1$ eV.
(c) A θ-pinch, with $n = 10^{23}$ m^{-3}, $KT_e = 800$ eV.

1.7 Applications of Plasma Physics

Plasmas can be characterized by the two parameters n and KT_e. Plasma applications cover an extremely wide range of n and KT_e: n varies over 28 orders of magnitude from 10^6 to 10^{34} m^{-3}, and KT can vary over seven orders from 0.1 to 10^6 eV. Some of these applications are discussed very briefly below. The tremendous range of density can be appreciated when one realizes that air and water differ in density by only 10^3, while water and white dwarf stars are separated by only a factor of 10^5. Even neutron stars are only 10^{15} times denser than water. Yet gaseous plasmas in the entire density range of 10^{28} can be described by the same set of equations, since only the classical (non-quantum mechanical) laws of physics are needed.

1.7.1 Gas Discharges (Gaseous Electronics)

The earliest work with plasmas was that of Langmuir, Tonks, and their collaborators in the 1920s. This research was inspired by the need to develop vacuum tubes that could carry large currents, and therefore had to be filled with ionized gases. The research was done with weakly ionized glow discharges and positive columns typically with $KT_e \simeq 2$ eV and $10^{14} < n < 10^{18}$ m^{-3}. It was here that the shielding phenomenon was discovered; the sheath surrounding an electrode could be seen visually as a dark layer. Before semiconductors, gas discharges were encountered only in mercury rectifiers, hydrogen thyratrons, ignitrons, spark gaps, welding arcs, neon and fluorescent lights, and lightning discharges. The semiconductor industry's rapid growth in the last two decades has brought gas discharges from a small academic discipline to an economic giant. Chips for computers and the ubiquitous handheld devices cannot be made without plasmas. Usually driven by radiofrequency power, partially ionized plasmas (gas discharges) are used for etching and deposition in the manufacture of semiconductors.

1.7.2 Controlled Thermonuclear Fusion

Modern plasma physics had its beginnings around 1952, when it was proposed that the hydrogen bomb fusion reaction be controlled to make a reactor. A seminal conference was held in Geneva in 1958 at which each nation revealed its classified controlled fusion program for the first time. Fusion power requires holding a 30-keV plasma with a magnetic field for as long as one second. Research was carried out by each individual country until 2007, when the ITER project was started. ITER stands for International Thermonuclear Experimental Reactor, a large experiment being built in France and funded by seven countries. Serendipitously, ITER is a Latin word meaning path or road. It is a road that mankind must take to solve the problems of global warming and oil shortage by 2050. Of all the "magnetic bottles" presented at Geneva, the USSR's TOKAMAK has survived as the leading idea and is the configuration for ITER. First plasma in ITER is scheduled for about the year 2021.

The fuel is heavy hydrogen (deuterium), which exists naturally as one part in ~6000 of water. The principal reactions, which involve deuterium (D) and tritium (T) atoms, are as follows:

$$\begin{aligned} D + D &\rightarrow {}^3\text{He} + n + 3.2\,\text{MeV} \\ D + D &\rightarrow T + p + 4.0\,\text{MeV} \\ D + T &\rightarrow {}^4\text{He} + n + 17.6\,\text{MeV} \end{aligned}$$

These cross sections are appreciable only for incident energies above 5 keV. Accelerated beams of deuterons bombarding a target will not work, because most of the deuterons will lose their energy by scattering before undergoing a fusion reaction. It is necessary to create a plasma with temperatures above 10 keV so that there are enough ions in the 40-keV range where the reaction cross section maximizes. The problem of heating and containing such a plasma is responsible for the rapid growth of the science of plasma physics since 1952.

1.7.3 Space Physics

Another important application of plasma physics is in the study of the earth's environment in space. A continuous stream of charged particles, called the solar wind, impinges on the earth's magnetosphere, which shields us from this radiation and is distorted by it in the process. Typical parameters in the solar wind are $n = 9 \times 10^6\ \text{m}^{-3}$, $KT_i = 10$ eV, $KT_e = 12$ eV, $B = 7 \times 10^{-9}$ T, and drift velocity 450 km/s. The ionosphere, extending from an altitude of 50 km to 10 earth radii, is populated by a weakly ionized plasma with density varying with altitude up to $n = 10^{12}\ \text{m}^{-3}$. The temperature is only 10^{-1} eV. The solar wind blows the earth's magnetic field into a long tail on the night side of the earth. The magnetic field lines there can reconnect and accelerate ions in the process. This will be discussed in a later chapter.

The Van Allen radiation belts are two rings of charged particles above the equator trapped by the earth's magnetic field. Here we have $n \leq 10^9\ \text{m}^{-3}$, $KT_e \leq 1$ keV, $KT_i \simeq 1$ eV, and $B \simeq 500 \times 10^{-9}$ T. In addition, there is a hot component with $n = 10^3\ \text{m}^{-3}$ and $KT_e = 40$ keV, and some ions have 100s of MeV.

Exploration of other planets have revealed the presence of plasmas. Though Mercury, Venus, and Mars have little plasma phenomena, the giant plants Jupiter and Saturn and their moons can have plasma created by lightning strikes. In 2013 The Voyager 1 satellite reached the boundary of the solar system. This was ascertained by detecting an *increase* in the plasma frequency there (!).

1.7.4 Modern Astrophysics

Stellar interiors and atmospheres are hot enough to be in the plasma state. The temperature at the core of the sun, for instance, is estimated to be 2 keV;

thermonuclear reactions occurring at this temperature are responsible for the sun's radiation. The solar corona is a tenuous plasma with temperatures up to 200 eV. The interstellar medium contains ionized hydrogen with $n \simeq 10^6\ \mathrm{m}^{-3}$ (1 per cc). Various plasma theories have been used to explain the acceleration of cosmic rays. Although the stars in a galaxy are not charged, they behave like particles in a plasma; and plasma kinetic theory has been used to predict the development of galaxies. Radio astronomy has uncovered numerous sources of radiation that most likely originate from plasmas. The Crab nebula is a rich source of plasma phenomena because it is known to contain a magnetic field. It also contains a visual pulsar. Current theories of pulsars picture them as rapidly rotating neutron stars with plasmas emitting synchrotron radiation from the surface. Active galactic nuclei and black holes have come to the forefront. Astrophysics now requires an understanding of plasma physics.

1.7.5 MHD Energy Conversion and Ion Propulsion

Getting back down to earth, we come to two practical applications of plasma physics. Magnetohydrodynamic (MHD) energy conversion utilizes a dense plasma jet propelled across a magnetic field to generate electricity (Fig. 1.5). The Lorentz force $q\mathbf{v} \times \mathbf{B}$, where $\mathbf{v}$ is the jet velocity, causes the ions to drift upward and the electrons downward, charging the two electrodes to different potentials. Electrical current can then be drawn from the electrodes without the inefficiency of a heat cycle.

A more important application uses this principle in reverse to develop engines for interplanetary missions. In Fig. 1.6, a current is driven through a plasma by applying a voltage to the two electrodes. The $\mathbf{j} \times \mathbf{B}$ force shoots the plasma out of the rocket, and the ensuing reaction force accelerates the rocket. A more modern device called a Hall thruster uses a magnetic field to stop the electrons while a high voltage accelerates the ions out the back, giving their momenta to the spacecraft. A separate discharge ejects an equal number of warm electrons; otherwise, the space ship will charge to a high potential.

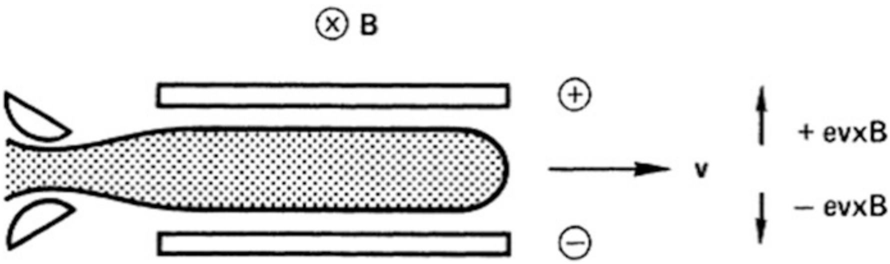


Fig. 1.5 Principle of the MHD generator

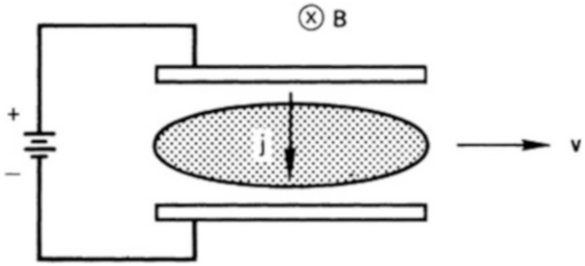


Fig. 1.6 Principle of plasma-jet engine for spacecraft propulsion

1.7.6 Solid State Plasmas

The free electrons and holes in semiconductors constitute a plasma exhibiting the same sort of oscillations and instabilities as a gaseous plasma. Plasmas injected into InSb have been particularly useful in studies of these phenomena. Because of the lattice effects, the effective collision frequency is much less than one would expect in a solid with $n \simeq 10^{29}$ m^{-3}. Furthermore, the holes in a semiconductor can have a very low effective mass—as little as 0.01 m_e—and therefore have high cyclotron frequencies even in moderate magnetic fields. If one were to calculate N_D for a solid state plasma, it would be less than unity because of the low temperature and high density. Quantum mechanical effects (uncertainty principle), however, give the plasma an effective temperature high enough to make N_D respectably large. Certain liquids, such as solutions of sodium in ammonia, have been found to behave like plasmas also.

1.7.7 Gas Lasers

The most common method to "pump" a gas laser—that is, to invert the population in the states that give rise to light amplification—is to use a gas discharge. This can be a low-pressure glow discharge for a dc laser or a high-pressure avalanche discharge in a pulsed laser. The He–Ne lasers commonly used for alignment and surveying and the Ar and Kr lasers used in light shows are examples of dc gas lasers. The powerful CO_2 laser has a commercial application as a cutting tool. Molecular lasers such as the hydrogen cyanide (HCN) laser make possible studies of the hitherto inaccessible far infrared region of the electromagnetic spectrum. The krypton fluoride (KrF) laser uses a large electron beam to excite the gas. It has the repetition rate but not the power for a laser-driven fusion reactor. In the semiconductor industry, short-wavelength ultraviolet lasers are used to etch ever smaller transistors on a chip. Excimer lasers such as the argon fluoride laser with 193 nm wavelength are used, with plans to go down to 5 nm.

1.7.8 Particle Accelerators

In high-energy particle research, linear accelerators are used to avoid synchrotron radiation on curves, especially for electrons. The 3-km long SLAC accelerator at Stanford produces 50-GeV electrons and positrons. Plasma accelerators generate plasma waves on which particles "surf" to gain energy. This technique has been applied at SLAC to double the energy from 42 to 84 GeV. A 31-km International Linear Collider is being built to generate 500 GeV colliding beams of electrons and positrons. In principle, plasma waves can add 1 GeV per cm.

1.7.9 Industrial Plasmas

Aside from their use in semiconductor production, partially ionized plasmas of high density have other industrial applications. Magnetrons are used in sputtering, a method of applying coatings of different materials. Eyeglasses can be coated with plasmas. Arcs, such as those in search lights, are plasmas. Instruments in medical research can be cleaned thoroughly with plasmas.

1.7.10 Atmospheric Plasmas

Most plasmas are created in vacuum systems, but it is also possible to produce plasmas at atmospheric pressure. For instance, a jet of argon and helium can be ionized with radiofrequency power. This makes possible small pencil-size devices for cauterizing skin. Industrial substrates can be processed by sweeping such a jet to cover a large area. Large atmospheric-pressure plasmas can also be produced for roll-to-roll processing.

Problems

1.8. In laser fusion, the core of a small pellet of DT is compressed to a density of 10^{33} m^{-3} at a temperature of 50,000,000 °K. Estimate the number of particles in a Debye sphere in this plasma.

1.9. A distant galaxy contains a cloud of protons and antiprotons, each with density $n = 10^6$ m^{-3} and temperature 100° K. What is the Debye length?

1.10. (Advanced problem) A spherical conductor of radius a is immersed in a uniform plasma and charged to a potential ϕ_0. The electrons remain Maxwellian and move to form a Debye shield, but the ions are stationary during the time frame of the experiment.

   (a) Assuming $e\phi/KT_e \ll 1$, write Poisson's equation for this problem in terms of λ_D.

   (b) Show that the equation is satisfied by a function of the form e^{-kr}/r. Determine k and derive an expression for $\phi(r)$ in terms of a, ϕ_0, and λ_D.

1.11. A field-effect transistor (FET) is basically an electron valve that operates on a finite-Debye-length effect. Conduction electrons flow from the source S to the drain D through a semiconducting material when a potential is applied between them. When a negative potential is applied to the insulated gate G, no current can flow through G, but the applied potential leaks into the semiconductor and repels electrons. The channel width is narrowed and the electron flow impeded in proportion to the gate potential. If the thickness of the device is too large, Debye shielding prevents the gate voltage from penetrating far enough. Estimate the maximum thickness of the conduction layer of an n-channel FET if it has doping level (plasma density) of $10^{22}\ \mathrm{m}^{-3}$, is at room temperature, and is to be no more than 10 Debye lengths thick. (See Fig. P1.11.)

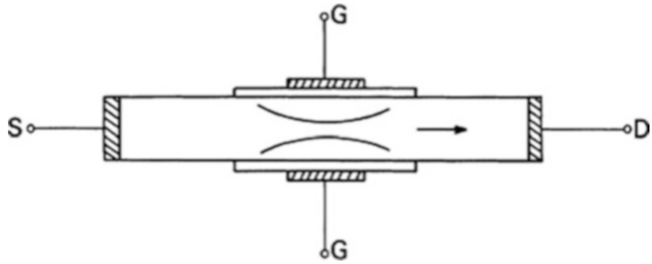


Fig. P1.11

1.12. (Advanced problem) Ionization is caused by electrons in the tail of a Maxwellian distribution which have energies exceeding the ionization potential. For instance, this potential is $E_{\mathrm{ioniz}} = 15.8$ eV in argon. Consider a one-dimensional plasma with electron velocities u in the x direction only. What fraction of the electrons can ionize for given KT_{e} in argon? (Give an analytic answer in terms of error functions.)