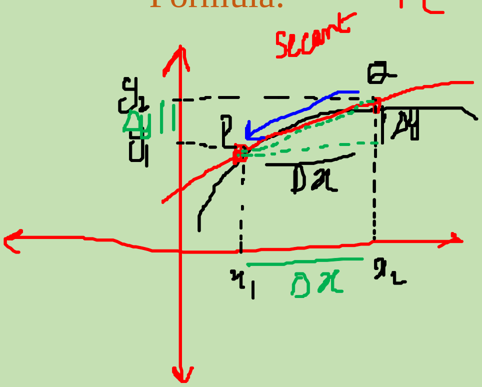
Chapter :02

Title: LIMITS AND CONTINUITY

AVERAGE RATE OF CHANGE OR SECANT SLOP:

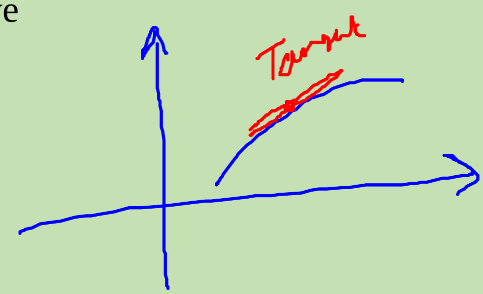
A line joining two point of a curve is a secant to the curve

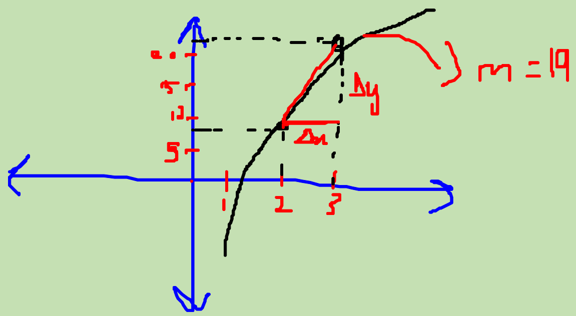
Formula: $P(x_1, y_1)$ and $Q(x_2, y_2)$



$$\text{Avg} = \frac{\Delta y}{\Delta x}$$

$$\boxed{\text{Avg} = \frac{y_2 - y_1}{x_2 - x_1}}$$





Example

1. $f(x) = x^3 + 1, \quad [2,3]$

$x_1 = 2, \; x_2 = 3$

$f(x) = y$

$f(x_1) = y_1 \;;\; f(x_2) = y_2$

$f(x_1) = f(2) = (2)^3 + 1 = 8 + 1 = 9$

$\boxed{y_1 = 9}$

$f(x_2) = f(3) = (3)^3 + 1 = 27 + 1 = 28$

$\boxed{y_2 = 28}$

$$\text{Avg} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{28 - 9}{3 - 2} = \frac{19}{1} = 19$$

$\boxed{\text{Avg} = 19}$ Or $\boxed{\text{secant slop} = 19}$

$\boxed{m_{secant} = 19}$

2. $f(x) = x^3 + 1, \quad [-1, 1]$

$x_1 = -1, \; x_2 = 1$

$f(-1) = (-1)^3 + 1 = -1 + 1 = 0$

$f(1) = (1)^3 + 1 = 1 + 1 = 2$

$$\text{Avg} = \frac{y_2 - y_1}{x_2 - x_1}$$

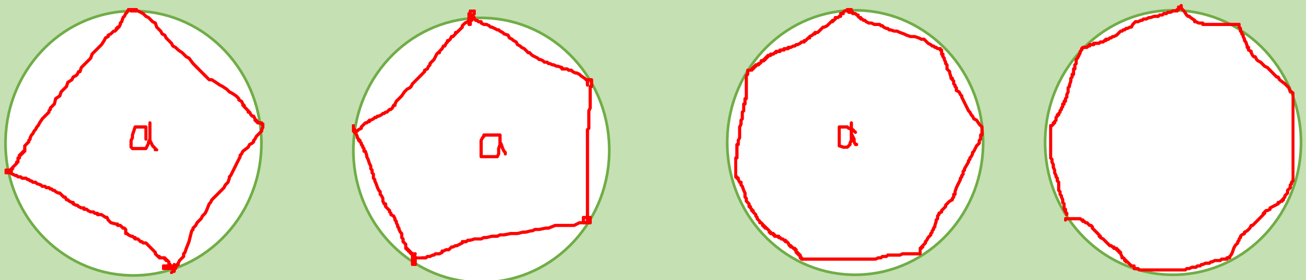
$$= \frac{2 - 0}{1 - (-1)} = \frac{2}{2} = 1$$

$\boxed{m_{secant} = 1}$

2. $g(x) = x^2; \quad [-1,1]$

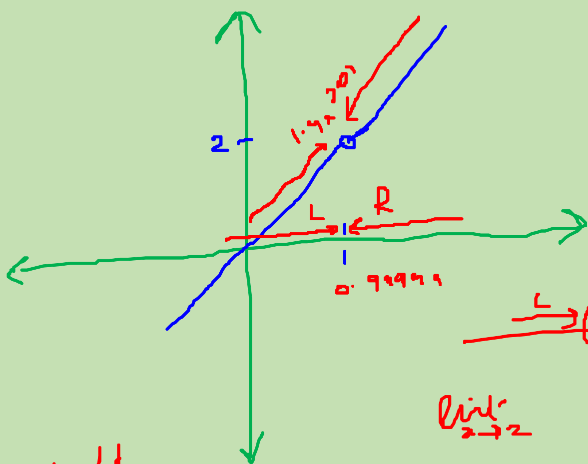
3. $g(x) = x^2; \quad [-2,0]$

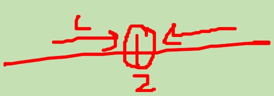
BASIC CONCEPT:



$n \to \infty \qquad a = ?$

$$\boxed{\lim_{n\to\infty} a(n) = (A)}$$





$\lim_{x\to 2}$

Left

x	1.8	1.9	1.99	1.999 $\to$ 2
$f(x)$	3.8	3.9	3.99	3.999 $\to$ 4

Right

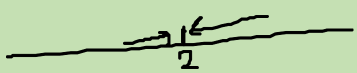
x	2.1	2.05	2.001	2.0001 $\to$ 2
$f(x)$	4.1	4.05	4.001	4.0001 $\to$ 4

$f(2) = 2^3 + 1 = 9$

$f(x) = \dfrac{x^2-4}{x-2}$

$x = 2$

$f(2) = \dfrac{2^2-4}{2-2} \to \dfrac{0}{0}$



$f(x) = \dfrac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} \;; \; x \neq 2$

$\lim_{x\to 2}$

$f(x) = x+2$

$\boxed{f(2) = 2+2 = 4}$

ONE SIDED LIMIT:

RIGHT HAND LIMIT:

A function f is said to have the right-hand limit l as $x \to a^+$

$$\boxed{\lim_{x \to a^+} f(x) = L}$$

Exp $\lim_{x \to 2^+} f(x) = 3$

LEFT HAND LIMIT:

A function f is said to have the right-hand limit l as $x \to a^-$

$$\boxed{\lim_{x \to a^-} f(x) = L}$$

$\lim_{x \to a^-} f(x) = 3$

$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$

$3 = 3$

$\boxed{\lim_{x \to a} f(x) = 3}$

If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \lim_{x \to a} f(x)$

$L = L$

$\Rightarrow \boxed{\lim_{x \to a} f(x) = L}$ exist

The relationship between one-sided and two-sided limits

Example:

1. $\lim_{x \to 2} 4 =$ 4

   $f(x) = 4$

2. $\lim_{x \to -13} 4 =$ 4

3. $\boxed{\lim_{x\to 3} x = 3}$

$f(x) = 3$

$x\ \infty\ ,\ \frac{0}{0}$

4. $\lim_{x\to 2}(5x-3) = 5(2)-3 = 10-3 = 7$

$f(x) = 5x-3$

5. $\lim_{x\to -2}\frac{3x+4}{x+5} = \frac{3(-2)+4}{-2+5} = \frac{-6+4}{+3} = -2/3$

$-2+5 = +3$

LIMITS LAW:

1. **Identity function:**

   If f is the identity function $\boxed{f(x) = x}$, then the limit at point $\boxed{x = x_0}$

   $\lim_{x\to x_0} f(x) = x_0$

2. **Constant function:**

If f is the constant function $\boxed{f(x) = k}$, then the limit at point $\boxed{x = x_0}$

$\lim_{x\to x_0} f(x) = K$

3. **Sum Rule:**

   If $\lim_{x\to a} f(x) = L$ & $\lim_{x\to a} g(x) = M$ then

$\lim_{x\to a}\left(f(x)+g(x)\right) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

$= L + M$

2

3 4 5

4. **Difference Rule:**

If $\lim_{x\to a} f(x) = L$ & $\lim_{x\to a} g(x) = M$ then

$$\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$
$$= L - M$$

5. **Constant Multiple Rule:**

If $\lim_{x\to a} f(x) = L$ then

$$\lim_{x\to a} C f(x) = C \lim_{x\to a} f(x)$$
$$= C L$$

6. **Quotient Rule:**

If $\lim_{x\to a} f(x) = L$ & $\lim_{x\to a} g(x) = M$ then

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M} \;;\; M \neq 0$$

7. **Power rule**

If $\lim_{x\to a} f(x) = L$ then

$$\lim_{x\to a} (f(x))^{r/s} = \left(\lim_{x\to a} f(x)\right)^{r/s} = (L)^{r/s}$$

Example:

(a) $\lim_{x\to c}(x^3 + 4x^2 - 3) = \lim_{x\to c} x^3 + \lim_{x\to c} 4x^2 - \lim_{x\to c} 3$

$= c^3 + 4\lim_{x\to c} x^2 - 3$

$= c^3 + 4c^2 - 3$

$=$ Ans

(b) $\lim_{x\to 6} \dfrac{x^4 + x^2 - 1}{x^2 - 5}$

$= \dfrac{\lim_{x\to 6}(x^4 + x^2 - 1)}{\lim_{x\to 6}(x^2 - 5)} = \dfrac{1333}{31}$

(c) $\lim_{x\to 2}\left(\sqrt{4x^2 - 3}\right)$

$= \lim_{x\to 2} (4x^2 - 3)^{1/2}$

$= \left(\lim_{x\to 2} 4(2)^2 - 3\right)^{1/2}$

$= \sqrt{13}$